The Impact of Bayesian Priors on Specification Search of Structural Equation Modeling

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Outline

- Introduction to specification search methods
- Purposes of study
- Methods
- Results
- Discussion
- Future directions

Specification Search

- Theoretical models are imperfect to varying extent (Box, 1979).
- Purposes of specification search:
  - improve model fit
  - reduce specification errors
  - elicit a model best representing the population model
  - provide meaningful interpretations for data
- A conventional search method is to use modification index (MI; Sörbom, 1989).

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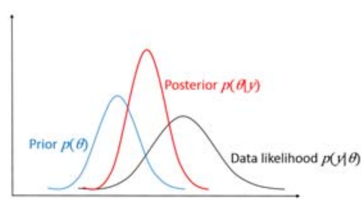
Bayesian Methods

- Bayesian Estimation
  - Posterior ∝ Prior x Likelihood
    $p(\theta|y) \propto p(\theta)p(y|\theta)$
  - Markov chain Monte Carlo (MCMC)



- Priors: non-informative, weakly informative, or informative.
- Model fit: posterior predictive checking
  - Posterior predictive *p*-value (*PPP*; if < .05, poor fit):
    $$PPP = P\left(f\left(x, \hat{\theta}_i\right) < f\left(x^{rep}, \hat{\theta}_i\right)\right)$$

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Bayesian Structural Equation Modeling (BSEM; Muthén & Asparouhov, 2012)

- BSEM starts with an over-parameterized model, and use a backward search method.
- BSEM specifies informative priors with small variances on parameters that are nearly 0 but believed not to be exactly 0.
  - i.e., cross-loadings, correlated errors, etc.
- These parameters are suggested to be freely estimated if the Bayesian credibility interval from the parameter posterior does not cover 0.
  - i.e., parameter estimates are significant.
- BSEM can be used as a search method.

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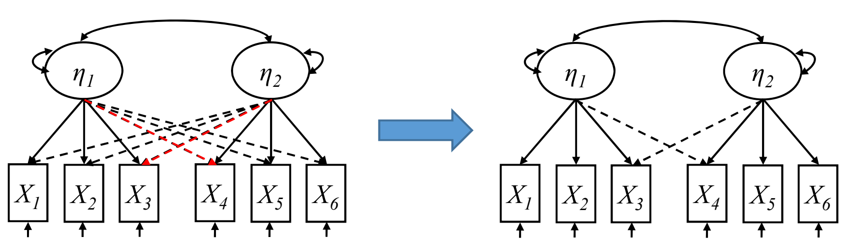
BSEM – with Cross-Loadings

- Informative priors are typically applied to cross-loadings.
- For other parameters believed to be nonzero, such as primary factor loadings, factor and residual covariance matrices, non-informative or weakly informative priors can be imposed.



Initial model | Population model

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Features of BSEM

- Advantages of BSEM
  - Incorporates prior knowledge into the posterior estimation
  - Does not dependent on the asymptotic theory or multivariate normality assumption
  - Handles under-identified models
  - Provides multiple suggestions in one analysis
- Disadvantage of BSEM
  - Potentially a long running time

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Purpose of Research

- In practice, factor structures with cross-loadings are common, mainly due to:
  - Random errors in items
  - Items measuring more than one latent construct
- Constraining small cross-loadings to be 0 may result in inflated factor correlation estimates, and more.
- A critical step in BSEM analyses is to make a good selection of priors through sensitivity analyses (Asparouhov, Muthén, & Morin, 2015).

- This study aims to investigate the impact of prior distributions in specification search of small cross-loadings in confirmatory factor analysis (CFA).

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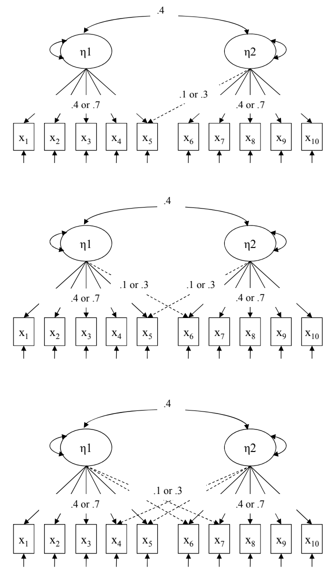
Methods

- # conditions: 84
- # replications: 2000
- Design factors

3 *Factor structures*	Two-factor models with 1, 2, and 4 cross-loadings
4 *loading specifications*	Primary loading: .4 or .7 Cross-loading: .1 or .3
7 *Sample sizes*	50, 100, 200, 400, 600, 800, 1000



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Data Analysis

- Each dataset was fit to a two-factor model with primary loadings and all possible cross-loadings.
- If cross-loadings were significant at the .05 level, they were retained in the model.
- The model was then re-specified and compared with the data-generating model.
- Estimation:
  - Gibbs sampler
  - Two MCMC chains
  - Medians as parameter estimates
  - predictive checking procedure for model fit

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Prior Distributions

- All possible cross-loadings

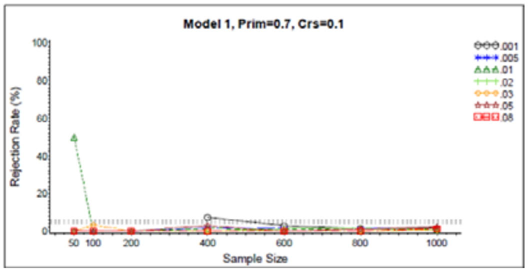
Prior	95% CI
N(0, .001)	±.06
N(0, .005)	±.14
N(0, .01)	±.20
N(0, .02)	±.28
N(0, .03)	±.34
N(0, .05)	±.44
N(0, .08)	±.55

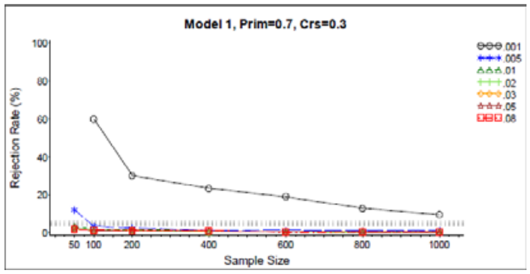
- primary loadings: $N(0, 10^{10})$
- factor covariance: $IW(0, -3)$
- residual variances: $IG(-1, 0)$

Evaluation

- Model fit
  - *PPP* rejection rates at the .05 level
- Model recovery
  - Model recovery rate: proportion of replications successfully recovered the population models.
  - Solution positive rate: proportion of replications that recovered the population model and extra parameters.
  - 95% coverage rate: proportion of replications where the 95% credibility interval covers the population value.

Rejection Rates





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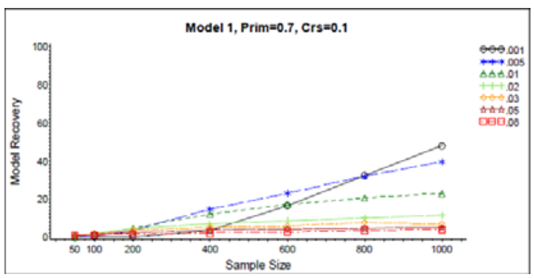
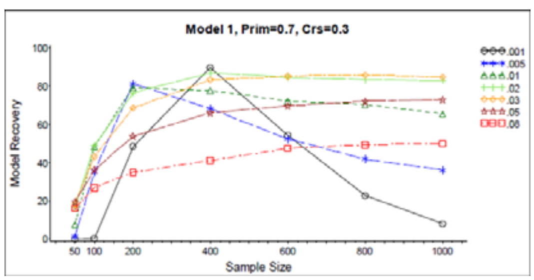
Model Recovery Rates





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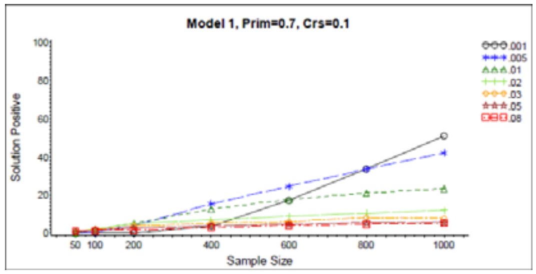
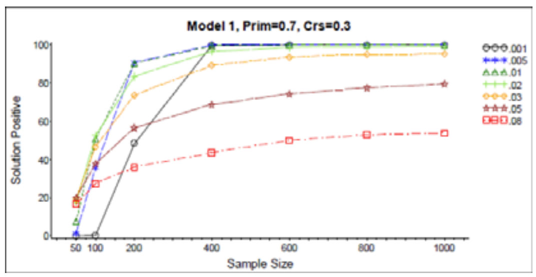
Solution Positive Rates





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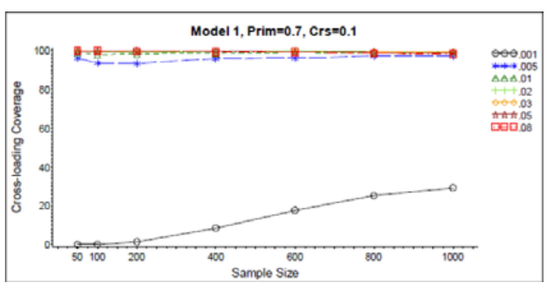
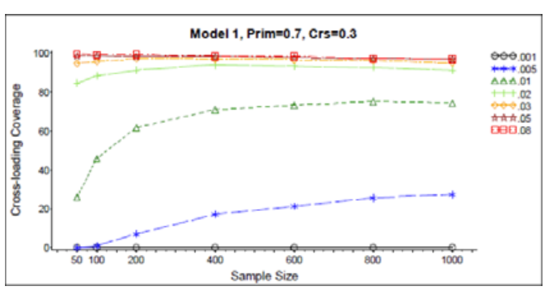
Coverage for Cross-Loadings





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Discussion

- Better model recovery was associated with a simpler model, larger cross-loading value, and greater sample size.
- Model recovery was the best, when the prior CI was roughly around the population cross-loading value.
  - > 80% for Models 1 and 2, and > 60% for Model 3.
- Impact of priors:
  - Too small variance -> more false positive recovery
  - Too large variance -> low model recovery
- Recommendation:
  - Consider an informative prior with the CI boundaries close to the true value.
  - Consider BSEM search results along with MI suggestions.
  - Examine sensitivity of search results to prior distributions.

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Future Directions

- Weakly informative priors on primary loadings and covariance matrices.
- Categorical and non-normally distributed data.
- Subsequent Bayesian searches.
- Model comparison.

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Thank you!
Questions or Comments?

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