What's for `dynr`: A package for linear and nonlinear DYNamic modeling in R

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Modern Modeling Methods (M^3)
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What's in a name?

BicameRal Extended Kalman Filter with Iterated Smoothing (BREKFIS)

Overview

Why dynr?

Why make a new package for this?

- OpenMx (Neale et al., in press) (linear only)
- ctsem (Driver, Oud, & Voelkle, 2015) (linear, continuous time only)
- MATLAB (single subject)
- MKFM6 (Dolan, 2005) (linear only)
- dlm (Petris, 2010) (slow)
- SsfPack (Koopman, Shephard, & Doornik, 1999) (single subject)
- MPlus 8 (not available yet, linear only)

Why dynr?

Why make a new package for this?

- Linear and nonlinear models
- Multiple subjects
- Intuitive interface
- Fast
- Great reporting
- Combine dynamics for continuous and categorical latent variables

Why dynamics?

- Goal of psychology (Hockenbury & Hockenbury, 2010)
  - To understand behavior and mental processes
- Behavior unfolds in time.
- Mental processes are fundamentally time-oriented
- Action across time is the study of dynamics.
- Development occurs *within people* and *across time*.

Why differential equations?

- Common language of many sciences
- Hypothesize *rules* that govern *individual* systems over time
  - Rules: actions, reactions, forces, influences
  - Individual: people are generally heterogeneous
- Discrete-time methods are differential equations in discrete time.

What can dynr do? Dynr Facts 1-5

1. Dynr fits discrete- and continuous-time dynamic models to multivariate longitudinal/time-series data.
2. Dynr handles linear and nonlinear dynamic models with an easy-to-use interface.
   - Dynr allows model specification in matrix and formula forms.
   - Dynr allows automatic differentiation.
3. Dynr deals with dynamic models with regime-switching properties.
   - e.g., Markov switching autoregressive (MSAR); Markov switching Kalman Filter (MSKF)
   - Caveat: Only linear measurement
4. Dynr computes in C and runs fast.
5. Dynr provides ready-to-present results through LaTex equations and plots.

Where can I get dynr?

- https://quantdev.ssri.psu.edu/avada_resources/dynr/
- In the next week or so, on CRAN!

dynr preparation

- Gather data with `dynr.data()`
- Prepare *recipes* with
  - `prep.measurement()`
  - `prep.*Dynamics()`
  - `prep.initial()`
  - `prep.noise()`
  - `prep.regimes()` (optional)
- *Mix* recipes and data into a model with `dynr.model()`
- *Cook* model with `dynr.cook()`
- *Serve* results with
  - `summary()`
  - `plot()`
  - `dynr.ggplot()`
  - `plotFormula()`
  - `printex()`

Modeling Framework

- Dynamic Model (`prep.*Dynamics()`)
  - Discrete-time
    $\mathbf{x}_i(t_{i,j}) = F_{\boldsymbol{\theta}_{f,i}}(\mathbf{x}_i(t_{i,j-1})) + \zeta_i(t_{i,j}), \zeta_i(t_{i,j}) \sim N(\mathbf{0}, \boldsymbol{\Sigma}_\zeta)$
  - Continuous-time Ordinary and Stochastic Differential Equation (ODE & SDE)
    $d\mathbf{x}_i(t) = F_{\boldsymbol{\theta}_{f,i}}(\mathbf{x}_i(t), t)dt + G_{\boldsymbol{\theta}_{f,i}}(t)d\mathbf{w}(t)$
- Measurement Model (`prep.measurement()`)
  $\mathbf{y}_i(t_{i,j}) = \boldsymbol{\mu} + \boldsymbol{\Lambda}\mathbf{x}_i(t_{i,j}) + \boldsymbol{\epsilon}_i(t_{i,j}), \boldsymbol{\epsilon}_i(t_{i,j}) \sim N(\mathbf{0}, \boldsymbol{\Sigma}_y)$
- Initial condition (`prep.initial()`)
  $\mathbf{x}_1(t_{1,j}) \sim N(\mathbf{x}_0, \mathbf{P}_0)$

What are regime-switching dynamic models?

- A regime–switching longitudinal model consists of several latent (unobserved) classes–or "regimes." Within each class, a submodel is used to described the distinct change patterns associated with the class.
- Each "regime" can be thought of as one of the stages or phases of a dynamic process.
- Individuals can switch between classes or regimes over time.
- The changes that unfold within a regime are continuous in nature.
- Markov-switching, hidden markov models, latent transition analysis, probabilistic functions of Markov chains (Petrie, 1969)

Initial regime probabilities Transition probabilities

Multinomial logistic regression models are used to represent the initial regime probabilities and describe each individual i's transition in class membership from time t-1 to time t as

$$\Pr(S_{i1} = k|\mathbf{x}_{i1}, \boldsymbol{\theta}) \triangleq \pi_{k,i1} = \frac{\exp(a_{k1} + \mathbf{b}'_{k1}\mathbf{x}_{i1})}{\sum_{s_1=1}^{K} \exp(a_{s1} + \mathbf{b}'_{s1}\mathbf{x}_{i1})},$$

$$\Pr(S_{it} = k|S_{i,t-1} = j, \mathbf{x}_{it}, \boldsymbol{\theta}) \triangleq \pi_{jk,it} = \frac{\exp(a_{kt} + \mathbf{b}'_{kt}\mathbf{x}_{it})}{\sum_{st=1}^{K} \exp(a_{st} + \mathbf{b}'_{st}\mathbf{x}_{it})}$$

S_{it} = individual i's class membership at time t

K = the number of regimes

a_{kt} = the logit intercept for the kth regime at time t

$\mathbf{x}_{it}$ = a vector of covariates for person i at time t

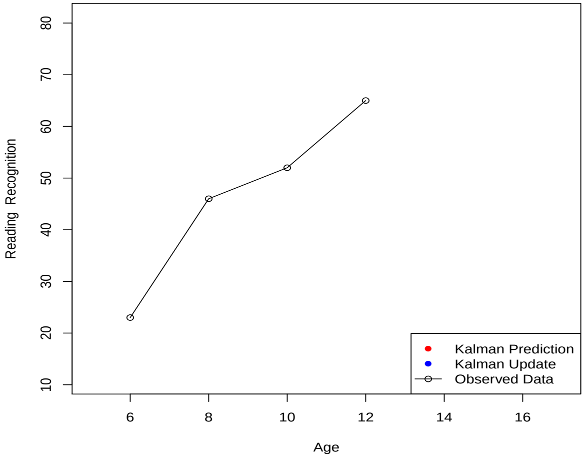
$\mathbf{b}_{kt}$ = a vector of logit slopes for the kth regime at time t

Extended Kalman Filter

1. Time update (Prediction)
2. Measurement Update (Correction)

$$
\begin{aligned}
\mathbf{v}_k &= \mathbf{y}_k - \boldsymbol{\Lambda}\hat{\mathbf{x}}_{k|k-1}, \mathbf{R}_{e,k} = \boldsymbol{\Sigma}_\epsilon + \boldsymbol{\Lambda}\hat{\mathbf{P}}_{k|k-1}\boldsymbol{\Lambda}^T \\
\mathbf{K}_k &= \hat{\mathbf{P}}_{k|k-1}\boldsymbol{\Lambda}^T\mathbf{R}_{e,k}^{-1} \\
\hat{\mathbf{x}}_{k|k} &= \hat{\mathbf{x}}_{k|k-1} + \mathbf{K}_k\mathbf{v}_k, \hat{\mathbf{P}}_{k|k} = \hat{\mathbf{P}}_{k|k-1} - \mathbf{K}_k\boldsymbol{\Lambda}\hat{\mathbf{P}}_{k|k-1}
\end{aligned}
$$

3. Optimization
   $\log L(\boldsymbol{\theta}) = \frac{1}{2}\sum_{i=1}^{n}\sum_{k=1}^{T}(-p_{i,k}\log(2\pi) - \log|\mathbf{R}_{e,k}| - \mathbf{v}_{i,k}'\mathbf{R}_{e,k}^{-1}\mathbf{v}_{i,k})$
4. Kim Smoother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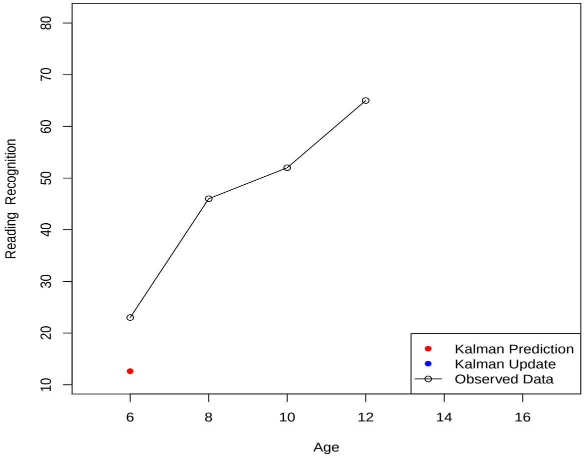
Kalman Update
Observed Data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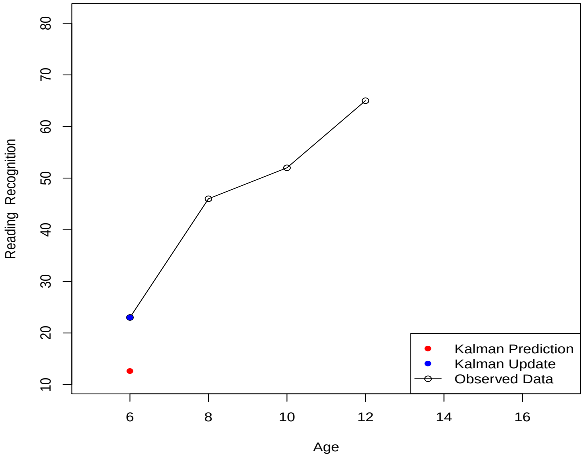
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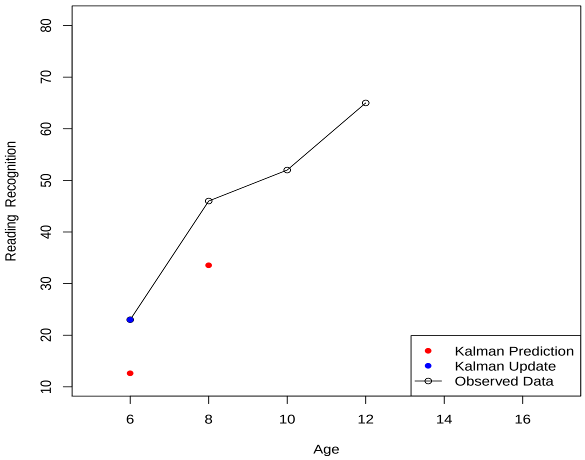
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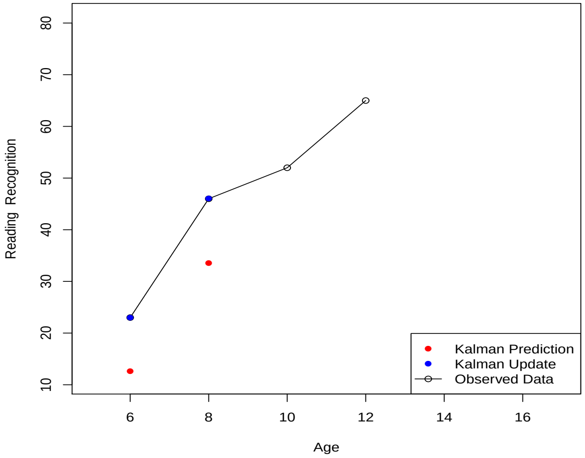
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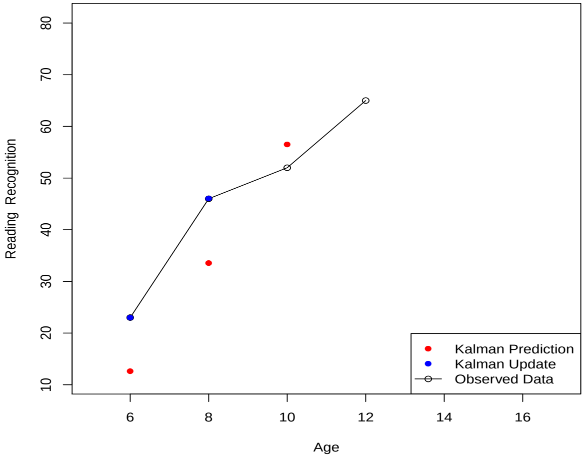
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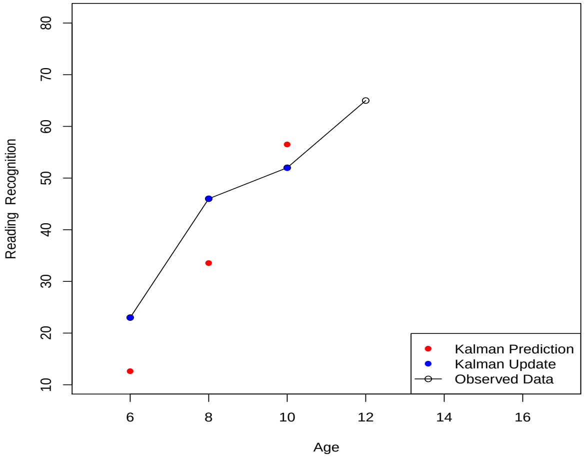
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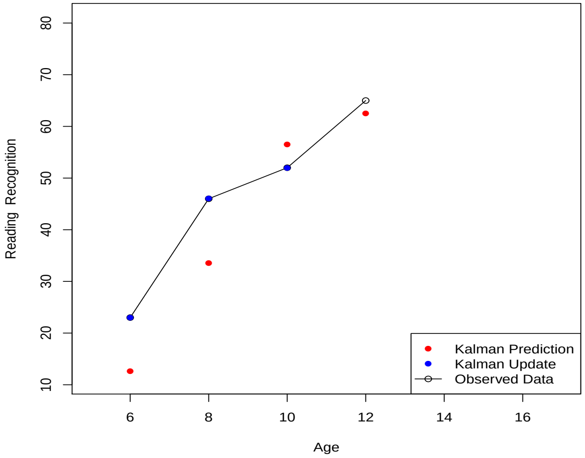
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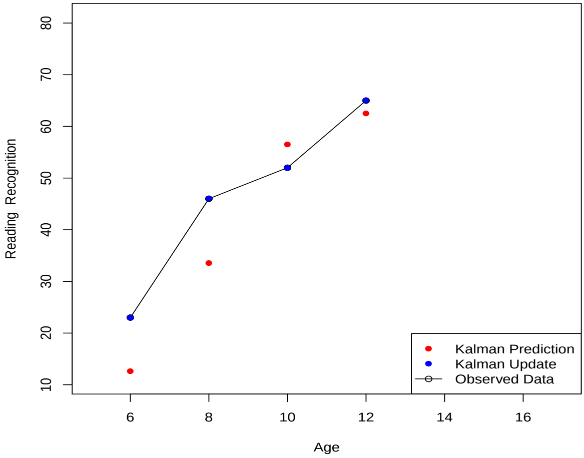
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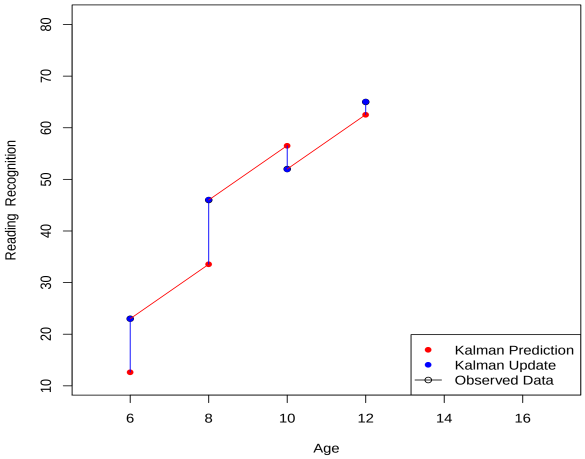
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Extended Kalman Filter

- Prediction (Discrete Time)

$$\hat{\mathbf{x}}_{t|t-1} = F(t, \hat{\mathbf{x}}_{t-1|t-1})$$

$$P_{t|t-1} = \frac{\partial F(t, \hat{\mathbf{x}}(t))}{\partial \hat{\mathbf{x}}} P_{t-1|t-1} \frac{\partial F(t, \hat{\mathbf{x}}(t))}{\partial \hat{\mathbf{x}}}^{\mathsf{T}} + Q$$

- Prediction (Continuous Time)

$$\frac{d\hat{\mathbf{x}}}{dt} = F(t, \hat{\mathbf{x}}_{t-1|t-1})$$

$$D\mathbf{P}(t) = \frac{\partial F(t, \hat{\mathbf{x}}(t))}{\partial \hat{\mathbf{x}}} \mathbf{P}(t) + \mathbf{P}(t) (\frac{\partial F(t, \hat{\mathbf{x}}(t))}{\partial \hat{\mathbf{x}}})^T + \mathbf{Q}(t)$$

- Solve these differential equations using Runge-Kutta or adaptive ODE solver.

Linear Harmonic Oscillator

- fit to data from broad domains within psychology
- resilience of older adults (Boker, Montpetit, Hunter, & Bergeman, 2010)
- coupling of smoking and drinking behavior in adolescents (Boker & Graham, 1998)
- response of widows to the loss of their spouse (Bisconti, Bergeman, & Boker, 2004, 2006)
- fluctuations and self-regulation of emotions (S. Chow, Ram, Boker, Fujita, & Clore, 2005)
- mother-infant synchrony during playful, face-to-face interactions (Zentall, Boker, & Braungart-Rieker, 2006)
- coupling of weather and mood in patients with bipolar disorder (Boker, Leibenluft, Deboeck, Virk, & Postolache, 2008)
- dynamics of romantic partners (Steele & Ferrer, 2011)

Damped and Forced Harmonic Oscillator

- As a second-order system

$$\frac{d^2x}{dt^2} = -kx - c\frac{dx}{dt} + \zeta \tag{1}$$

- As a vector of first-order systems

$$\begin{pmatrix} \frac{dx}{dt} \\ \frac{d^2x}{dt^2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -k & -c \end{pmatrix} \begin{pmatrix} x \\ \frac{dx}{dt} \end{pmatrix} + \begin{pmatrix} 0 \\ \zeta \end{pmatrix} \tag{2}$$

- The Measurement Model

$$y = \begin{pmatrix} 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ \frac{dx}{dt} \end{pmatrix} + \epsilon \tag{3}$$

Linear Harmonic Oscillator

Example code

```
# measurement
# this is the factor loadings matrix
#Lambda in SEM notation or C in OpenMx notation
meas <- prep.measurement(
  # starting values and fixed values
      values.load=matrix(c(1, 0), 1, 2),
      # parameter names or fixed
      params.load=matrix(c('fixed', 'fixed'), 1, 2),
      state.names=c("Position","Velocity"),
      obs.names=c("y1"))
```

Linear Harmonic Oscillator

Example code

```
# define the differential equation
dynamics <- prep.matrixDynamics(
        values.dyn=matrix(c(0, -0.1, 1, -0.2), 2, 2),
        params.dyn=matrix(
          c('fixed', 'spring', 'fixed', 'friction'),
          2, 2), #uses params 'spring' and 'friction'
        isContinuousTime=TRUE)
```

Linear Harmonic Oscillator

Example code

```
# Prepare for cooking
# put all the recipes together
model <- dynr.model(dynamics=dynamics,
  measurement=meas,
  noise=ecov, initial=initial, data=data,
  outfile="LinearSDE.c")
# Look with LaTeX
printex(model)
```

Linear SDE: printex {dynr}

The measurement model is given by:

$$\begin{bmatrix} y1(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} Position(t) \\ Velocity(t) \end{bmatrix} + \epsilon,$$

$$\epsilon \sim N\left(\begin{bmatrix} 0.00 \end{bmatrix}, \begin{bmatrix} mnoise \end{bmatrix} \right)$$

The dynamic model is given by:

$$\begin{bmatrix} dPosition(t) \\ dVelocity(t) \end{bmatrix} = (\begin{bmatrix} 0 & 1 \\ spring & friction \end{bmatrix} \begin{bmatrix} Position(t) \\ Velocity(t) \end{bmatrix})dt + dw(t),$$

$$dw(t) \sim N\left(\begin{bmatrix} 0.00 \\ 0.00 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & dnoise \end{bmatrix} \right)$$

The initial condition of the dynamic model is given by:

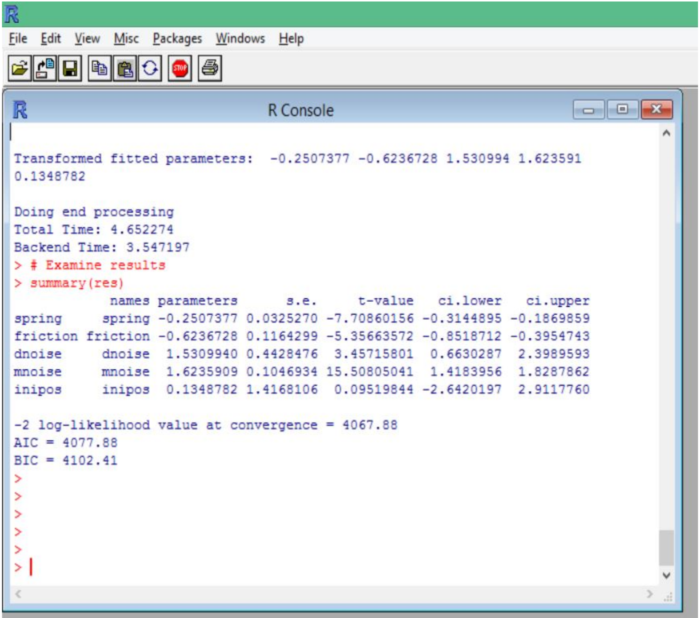
$$\begin{bmatrix} Position(0) \\ Velocity(0) \end{bmatrix} \sim N\left(\begin{bmatrix} inipos \\ 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right)$$

Linear Harmonic Oscillator

Example code

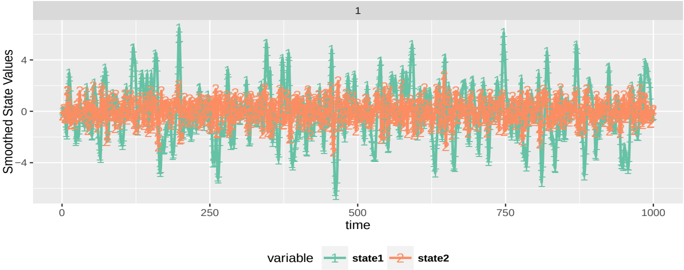
```
# Cook
# Estimate free parameters
res <- dynr.cook(model)
# Examine results
summary(res)
```

summary


File Edit View Misc Packages Windows Help
R Console
Transformed fitted parameters: -0.2507377 -0.6236728 1.530994 1.623591
0.1348782
Doing end processing
Total Time: 4.652274
Backend Time: 3.547197
> # Examine results
> summary(res)
names parameters s.e. t-value ci.lower ci.upper
spring spring -0.2507377 0.0325270 -7.70860156 -0.3144895 -0.1869859
friction friction -0.6236728 0.1164299 -5.35663572 -0.8518712 -0.3954743
dnoise dnoise 1.5309940 0.4428476 3.45715801 0.6630287 2.3989593
mnoise mnoise 1.6235909 0.1046934 15.50805041 1.4183956 1.8287862
inipos inipos 0.1348782 1.4168106 0.09519844 -2.6420197 2.9117760
-2 log-likelihood value at convergence = 4067.88
AIC = 4077.88
BIC = 4102.41

Output: plot {dynr}

```
plot(res, model)
```



Dynamic Model without Noise

$$d(\text{Position}(t)) = (\text{Velocity}(t))dt$$

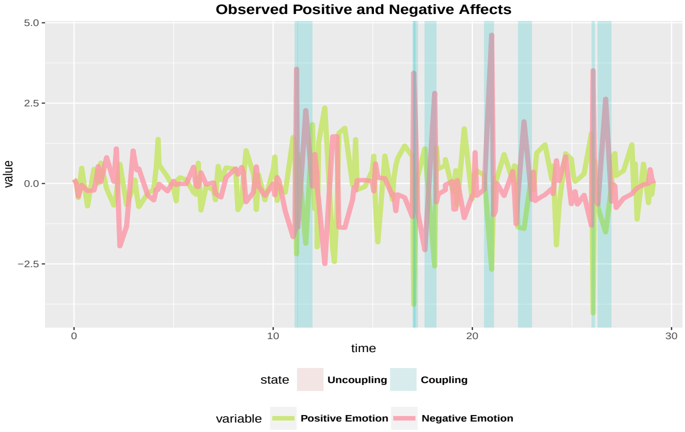
$$d(\text{Velocity}(t)) = (-0.25 \times \text{Position}(t) + -0.62 \times \text{Velocity}(t))dt$$

Measurement Model without Noise

$$y1 = \text{Position}$$

Example 2: Motivation

Data from the Affective Dynamics and Individual Differences study (Emotions and Dynamic Systems Laboratory, 2010). Participants provided daily affect ratings 5 times a day at random intervals over 30 days.



Regime-switching Nonlinear Dynamic Factor Analysis Model

$$
\begin{aligned}
PE_{it} &= a_P PE_{i,t-1} + b_{PN,c_{it}} NE_{i,t-1} + \zeta_{PE,it} \\
NE_{it} &= a_N NE_{i,t-1} + b_{NP,c_{it}} PE_{i,t-1} + \zeta_{NE,it} \\
b_{PN,c_{it}} &= \begin{cases} 0 & \text{if } c_{it} = 0 \\ b_{PN0}\left(\frac{exp(abs(NE_{i,t-1}))}{1+exp(abs(NE_{i,t-1}))}\right) & \text{if } c_{it} = 1, \end{cases}
\end{aligned} \tag{4}
$$

$$
b_{NP,S_{it}} = \begin{cases} 0 & \text{if } c_{it} = 0 \\ b_{NP0}\left(\frac{exp(abs(PE_{i,t-1}))}{1+exp(abs(PE_{i,t-1}))}\right) & \text{if } c_{it} = 1, \end{cases} \tag{5}
$$

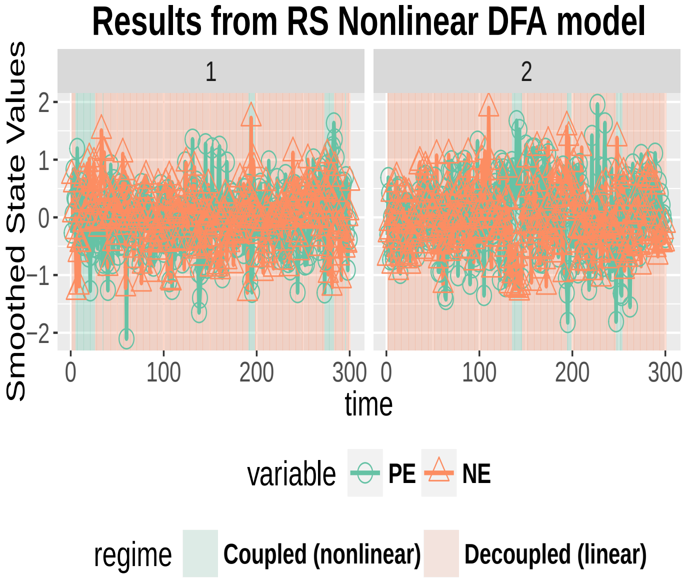
(S.-M. Chow & Zhang, 2013). Model adapted from an earlier model presented by (S.-M. Chow, Tang, Yuan, Song, & Zhu, 2011).

Regime-switching Nonlinear Dynamic Factor Analysis Model

Dynr Facts

- Dynr fits discrete-time dynamic models too.
- Dynr allows specification of nonlinear dynamic models in formula forms.
- Dynr deals with dynamic models with regime-switching properties.
- Dynr provides ready-to-present results through LaTex equations and plots.

Output: dynr.ggplot {dynr}

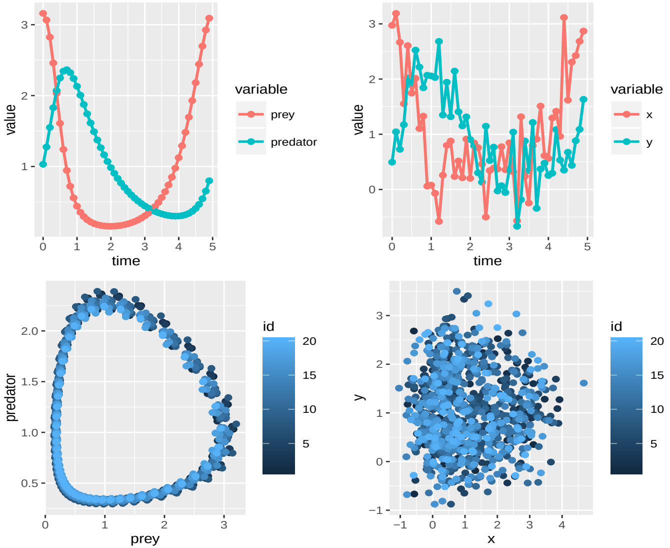


Regime-switching Predator and Prey Model

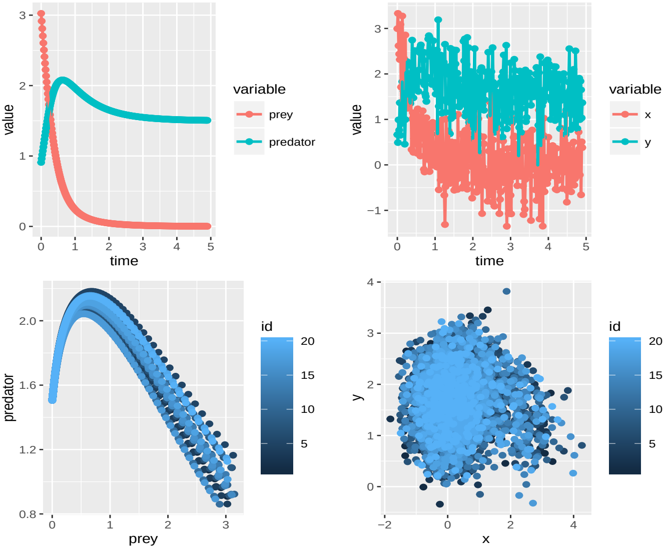
The dynamic model is given by:

$$
\begin{aligned}
&\text{Regime 1:}\\
\frac{d(prey)}{dt} &= a \times prey - b \times prey \times predator,\\
\frac{d(predator)}{dt} &= -c \times predator + d \times prey \times predator,\\
&\text{Regime 2:}\\
\frac{d(prey)}{dt} &= a \times prey - e \times prey^2\\
&\quad - b \times prey \times predator,\\
\frac{d(predator)}{dt} &= f \times predator - c \times predator^2\\
&\quad + d \times prey \times predator
\end{aligned}
$$

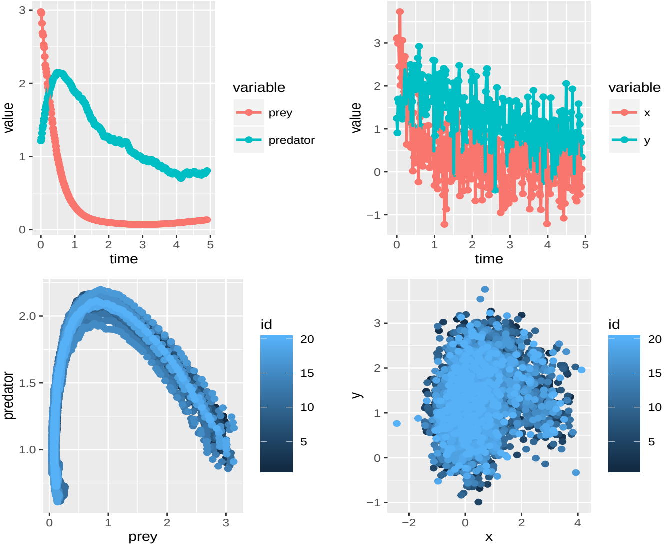
One-regime Predator-and-prey Model



One-regime extension of the Predator-and-prey Model



Regime-switching Predator and Prey Model



Regime-switching Predator and Prey Model

Dynr Facts

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- Dynr allows for automatic differentiation.
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  - `plotFormula()`
  - `printex()`

Thank you for your attention!

Acknowledgements

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- NSF grant BCS-0826844
- Sukruth Reddy, Zhaohua Lu, Timothy Brick, Meng Chen, Peifeng Yin
- The QuantDev Group

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