Dynamical Systems Modeling: An Application to the Regulation of Intimacy and Disclosure in Marriage

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Introduction

This chapter provides an introduction to coupled differential equations models of self–regulating dynamic systems, describes a method for estimating parameters of such models, and then works through an application of this method to self–disclosure and feelings of intimacy in a sample of married couples. The methods used include Local Linear Approximation (LLA) of derivatives (Boker & Nesselroade, 2002) and multilevel modeling (i.e., generalized linear mixed modeling or GLMM)(see Walls, Schwartz, & Jung, in press, in this volume) to account for and predict individual differences in parameters of differential equations. We have chosen to use LLA and multilevel modeling since this affords a simple and straightforward approach to the estimation of parameters of these models.

Advances in the modeling of longitudinal data have led to the development of tests of theories modeled as differential equations and based on dynamic systems interpretations of social and behavioral phenomena. One type of dynamic systems model attempts to account for self–regulation — the process by which a phenomenon maintains equilibrium by responding to information about change in the phenomenon's state. A more complex dynamical systems model allows regulation in one part of a system to influence the regulation of another part of a system. For instance, one might consider a married couple a system composed of two self–regulating members. The self–regulation of feelings of intimacy of each member of a married couple might influence the self–regulation of feelings of intimacy in the other.

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Self–Regulation and Intrinsic Dynamics

Many psychological constructs show trait–like individual differences, that is when an individual is measured on many occasions his or her mean score may be distinguishable from other individuals' mean scores. However, there may also be short–term intraindividual variability within each person's score (Nesselroade, 1991). One may reasonably inquire as to the source of this intraindividual variability. Perhaps this variation is simply some random fluctuation either due to unreliability of a measurement instrument or due to the influence of some unmeasured random variable. Or perhaps these fluctuations are due to some sort of process intrinsic to the individual — thus there is some patterning to the intraindividual variability such that the current score for a person is somehow predictive of a future score. In this case one might consider the patterned variability around the stable mean score to be an instance of an intrinsic dynamical process about an equilibrium value. One may consider such an intrinsic dynamic using the language of self–regulation (Carver & Scheier, 1998) whereby a process variable continuously changes its value so as to remain within some "comfort zone" near its set point, i.e., equilibrium value. Psychological constructs such as well–being (Bisconti, Bergeman, & Boker, 2004) or positive and negative emotions (Chow et al., in press) may exhibit this type of self–regulating behavior. Physiological variables such as hormone levels in menstrual cycles may couple with behaviorial variables such as eating behavior (Nilsson et al., 2004; Varma et al., 1999) so as to form coupled self–regulating systems.

Dynamical systems theory offers a way to formalize concepts of self–regulation. Let us call the *state* of a system the values of the indicators for a psychological construct at one moment in time and the *trajectory* of a system the continuously evolving state of the system over some interval of time. Suppose a system has a stable *equilibrium state*, in other words a fixed set–point value for the psychological construct that is, given no other information, the expected value for that construct. Now we can define a *linear dynamical system* that has a *basin of attraction* around a *point attractor* by stating that the likelihood that the system's future trajectory turns toward the equilibrium state is proportional to the displacement from the equilibrium state (see e.g., Kaplan & Glass, 1995). Thus the difference between the equilibrium value and the current value of the psychological construct is negatively proportional to the curvature in the construct's trajectory.

Figure 1 plots four trajectories that conform to a model in which the curvature (i.e., the change in the slope) is negatively proportional to the displacement from equilibrium. The slope of the trajectory for some construct x at some selected time t is the tangent to the trajectory of x at time t, that is the first derivative of x_t with respect to time and is written either as dx_t/dt, or as $\dot{x}_t$. The change in the slope of the construct x at time t (the curvature of the trajectory of x) is the second derivative of x_t can either be written as d^2x_t/dt^2, or as $\ddot{x}_t$. In this chapter we will use the notation $\dot{x}_t$ and $\ddot{x}_t$ to represent the first and second derivative of a construct x with respect to time at some selected time t.

We can now formalize a simple linear model of a construct x with a fixed point equilibrium as

$$\ddot{x}_t = \eta x_t \, , \tag{1}$$

where η is some negative valued constant that represents how quickly the trajectory turns back toward its equilibrium state when it is displaced from its equilibrium. In this way, we can see why for this system the equilibrium state is called an attractor — the farther the trajectory is displaced from the equilibrium state the more it is attracted back towards the equilibrium.

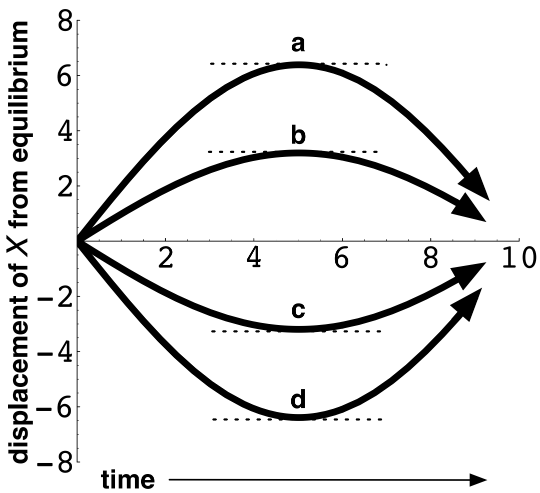

Figure 1. Four trajectories in which the curvature of the trajectory of a construct x is negatively proportional to the displacement from an equilibrium state of zero.

It can be seen why η must be a negative value for this attractor to form by examining Figure 1 in which a construct x conforming to Equation 1 has an equilibrium of $x = 0$. Look at Figure 1 and consider the top trajectory including the point labeled a. This trajectory starts at time $t = 0$ at the equilibrium ($x = 0$) and has some positive slope leading to a positive displacement from equilibrium. Note that at the point labeled a the displacement is just a bit greater than 6 and there is a high degree of curvature in the trajectory. Prior to a the slope of the trajectory is positive and after a the slope is negative. Thus, the change in the slope is negative in the neighborhood of a, and therefore the second derivative $\ddot{x}_t$ is negative. If x_t is positive and $\ddot{x}_t$ is negative then η, the constant coefficient that expresses their proportional relationship, must be negative. One may also verify that when the displacement of x from equilibrium is negative (at points c and d in Figure 1) then the second derivative is positive.

Although the negative relationship between the displacement of x and the curvature of its trajectory tends to keep the trajectory in the neighborhood of the equilibrium, fluctuations in this simple model do not decrease or increase with time. Figure 2 plots the same trajectory that passed through the point a in Figure 1, but continues for the interval from $t = 0$ to $t = 80$. Note that the points on the trajectory that have a slope of zero (i.e., a, c, and e) have the same absolute value of displacement from equilibrium. After the slope of the trajectory turns from positive to negative in the neighborhood of a, the trajectory continues to have greater and greater negative slope until point b when it crosses through equilibrium. Since the displacement is zero, the change in the slope is also zero at b. Thus, the trajectory continues along the same negative slope and diverges from the equilibrium,

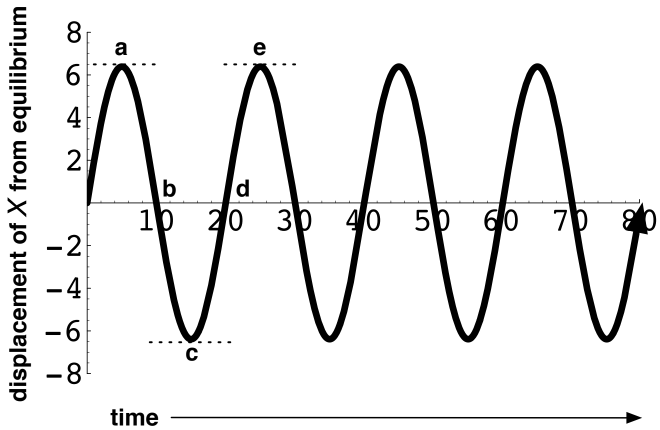

Figure 2. A trajectory in which the curvature of the trajectory of a construct x is negatively proportional to the displacement from an equilibrium state of zero, but the fluctuations are not damped.

having more and more negative displacement and consequently greater positive curvature. By the time the trajectory reaches c, the positive curvature has changed the slope from negative back to positive. But the positive curvature continues until the point d where the trajectory crosses the equilibrium again. As time progresses, the trajectory continues to "overshoot" the equilibrium by the same amount.

When the example trajectory for the construct x crosses its equilibrium, its slope is at a maximum, either as a positive slope or negative negative slope. Equation 1 formalized a self–regulation in which displacement from equilibrium induced curvature such that a trajectory moving away from equilibrium would "turn around" and move back towards equilibrium. Another way to say this is that the system responds negatively to displacement from equilbrium. Consider what might happen if the system were to respond negatively to change — that is a self–regulation mechanism that tended to reduce large absolute values of the slope. In this case, curvature would be negatively proportional to the slope as well as negatively proportional to the displacement from equilibrium. We can formalize this relationship as

$$\ddot{x}_t = \eta x_t + \zeta \dot{x}_t \ , \tag{2}$$

where η and ζ are negative constants (Thompson & Stewart, 1986).

If we plot a trajectory that conforms to Equation 2, it now damps towards equilibrium as shown in Figure 3. Although the trajectory continues to overshoot the equilibrium, observe that the points where the slope of the trajectory is zero (i.e., a, c, and e) are closer to zero as time progresses. If some momentary exogenous influence were to displace this self–regulating construct x away from its equilibrium, one might observe a pattern of return to equilibrium similar to that shown in Figure 3.

Equation 2 is an example of a *second order differential equation*, that is an equation that express relationships between a variable and its first and second derivatives. We have expressed Equations 1 and 2 as being completely deterministic. In other words, these equations do not have any residual term. Of course, this is unrealistic in real–world data.

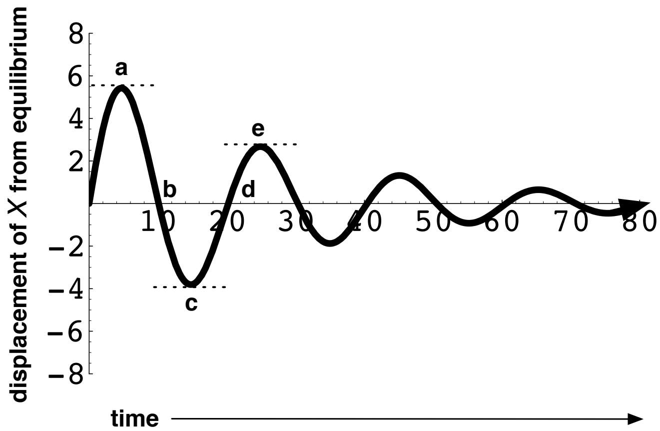

Figure 3. A trajectory in which the curvature of the trajectory of a construct x is negatively proportional to the displacement from an equilibrium and also negatively proportional to the slope. Fluctuations from equilibrium are damped and the trajectory settles to the equilibrium.

We can add a residual term e_t that conforms to standard regression assumptions (i.e., independent, normally distributed, with a mean of zero) and reexpress Equation 2 as a regression equation that would allow us to estimate the coefficients η and ζ

$$\ddot{x}_t = \eta x_t + \zeta \dot{x}_t + e_t \ . \tag{3}$$

Thus if we were to have estimates of the values of x_t, $\dot{x}_t$, and $\ddot{x}_t$ for a sample of values of time, we can estimate the values of η and ζ using multiple regression (Boker & Nesselroade, 2002). We will return to this idea in a later section, but first we will present a short digression on a model that is popular in longitudinal analysis: autoregression.

Many trajectories are possible that still conform to Equation 3 (see Nesselroade & Boker, 1994, for some examples). One such trajectory is shown in Figure 4 which can also be modeled more simply as the *first order differential equation*

$$\dot{x}_t = \alpha x_t + e_t \ , \tag{4}$$

where α is a negative constant coefficient that expresses the negative proportional relationship between the displacement of a construct x from its equilibrium at time t and the instantaneous slope of x at time t. This results in a negative exponential trajectory that returns towards equilibrium but does not overshoot. If we sample a negative exponential trajectory at discrete intervals of time τ, the value of x_t and x_{t+tau} will be a simple linear proportion β such that β is between 0 and 1.

Autoregressive models are often conceptualized as a variable having an influence on itself over time. But one may also consider these models within the framework of self–regulating dynamical systems as a process that regulates back to equilibrium without an overshoot. Equation 3 can be used to fit data generated by a model conforming to a first order autoregressive process. Thus the model in Equation 3 is more general than autoregressive modeling, although at the expense of being less parsimonious. However, if

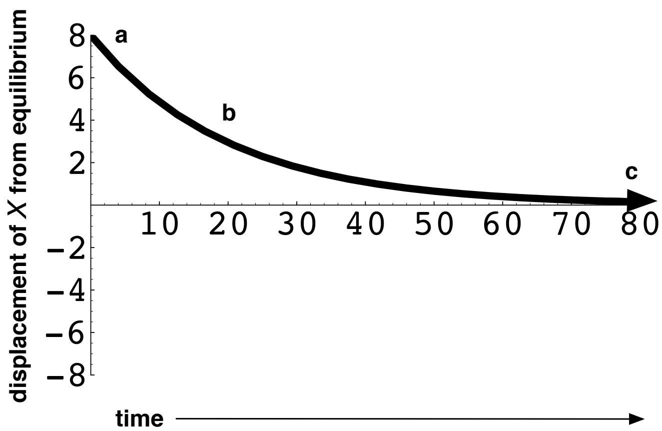

Figure 4. A trajectory in which the slope of the trajectory of a construct x is negatively proportional to the displacement from equilibrium. Any displacement from equilibrium leads to a return to equilibrium.

one suspects that a self–regulating construct might overshoot its equilibrium (i.e., oscillate), then a first order model (whether autoregressive or differential equation) is inappropriate since its self–regulation does not behave in this way.

In this chapter, we are interested in data from married couples that may show oscillation and that also may show mutual influence between two individuals. In order to build a model for these data we must first consider the possibility that two systems are coupled together. The next sections will explore some possibilities for theoretical models of coupled dynamics and how we might take into account individual differences in equilibria, self–regulation, and coupling between individuals.

Coupled Regulation and Coupled Dynamics

Suppose two self–regulating systems of the form shown in Equation 3 not only regulated themselves, but also regulated each other. In this case the definition of the system we are studying must include both of the self–regulating subsystems as well as their mutual influence on each other. How might we think of such a system?

One commonly used metaphor in dynamical systems is that of a pendulum with friction, a system that can be approximated by Equation 3. Consider a system composed of two pendulums X and Y as illustrated in Figure 5. Each pendulum may have its own length, thereby determining the frequency at which it would swing if no outside influence were in effect. Each pendulum might also have its own friction at the pivot. The acceleration (i.e., second derivative) due to gravity G is constant, but the acceleration in the direction in which each pendulum can move is proportional to its displacement from equilibrium. Thus, for instance $\ddot{x}$ is proportional to x as is formalized in Equation 3.

Now suppose we add a coupling between the two pendulums in the form of a linear spring. In this way, there might be an additional contribution to the second derivative of each pendulum from the displacement and velocity (first derivative) of the other pendulum.

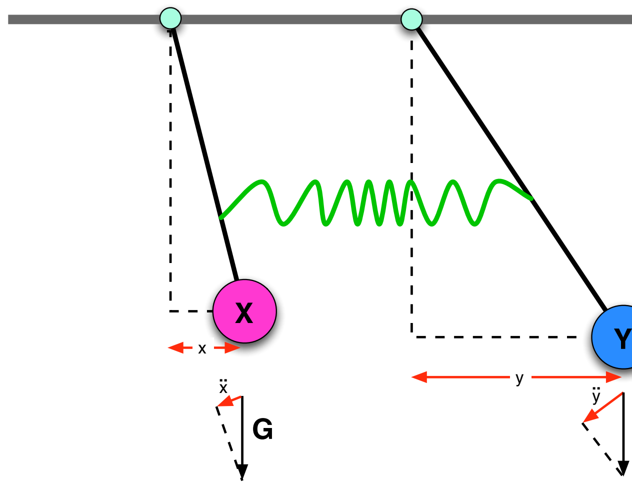

Figure 5. A system composed of two pendulums coupled together with a linear spring. Each pendulum may have its own length rod and its own friction at the pivot. The acceleration due to gravity G is constant, but its effect on each pendulum is proportional to the displacement from equilibrium.

We might formalize this relationship as

$$\ddot{x}_t = \eta_x x_t + \zeta_x \dot{x}_t + \gamma_x(\eta_y y_t + \zeta_y \dot{y}_t) + e_{xt} \quad (5)$$
$$\ddot{y}_t = \eta_y y_t + \zeta_y \dot{y}_t + \gamma_y(\eta_x x_t + \zeta_x \dot{x}_t) + e_{yt} \quad (6)$$

where η_x and ζ_x are the frequency and damping coefficients for the x variable, η_y and ζ_y are the frequency and damping coefficients for the x variable, and γ_x and γ_y are the coupling strengths for x and y respectively. Note that in this model the same coefficients η_y and ζ_y are used in the equation for regulating x as well as y. This suggests that the same mechanism for self–regulation is used within a variable as well as in coupling of the variables together.

There is one major difference between the system modeled in Equations 5 and 6 and the pendulums example in Figure 5. In the pendulums example, the strength of the spring's pull on x is exactly matched by the strength of the spring's pull on y. Thus, the coupling is symmetric in Figure 5. But in a system such as a married couple one need not assume that $\gamma_x = \gamma_y$, that is the influence of the husband on the wife may not be the same as the influence of the wife on the husband. In fact, one may not even wish to assume that these effects have the same sign! Thus, the system in Equations 5 and 6 may potentially exhibit *asymmetric coupling* — a difficult system to build from pendulums and springs.

The coupled dynamical system defined by Equations 5 and 6 can be simulated numerically in order to gain some understanding of its behavior. Figure 6 plots the results of a simulation calculated using the numerical integration function *NDSolve* in the Mathematica (Wolfram Research, 2003) software. The two variables x and y are set to have different intrinsic dynamics: the ratio $\eta_x/\eta_y = 1/1.28$. In addition, ζ_x is negative while ζ_y is positive. Thus one could think of the dynamics of x as being inhibitory whereas the dynamics of y are excitatory. The x and y variables are coupled together with equal and positive cou-

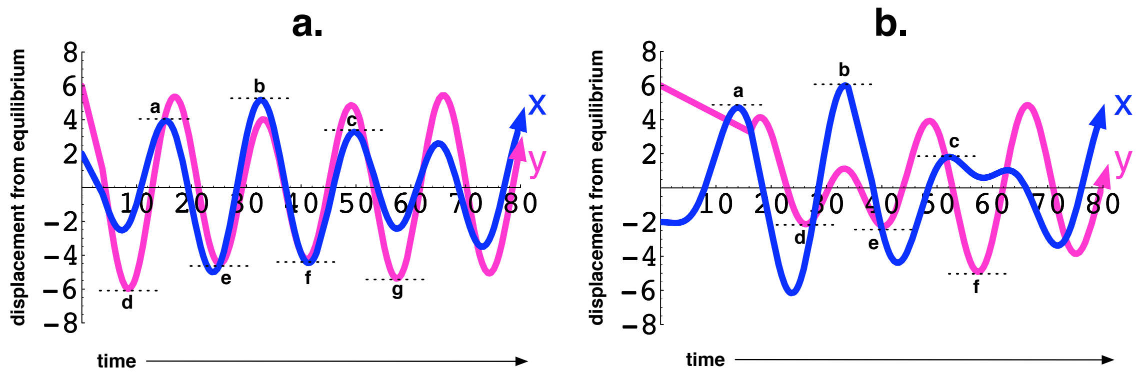

Figure 6. Two instances of coupled trajectories, x and y, that exhibit self–regulation as well as mutual influence. Although each figure has parameters that conform to the linear system from Equation 2, differences in initial conditions can result in apparently irregular trajectories.

pling coefficients $\gamma_x = \gamma_y = 0.3$ leading to moderately strong coupling. The only difference between the trajectories in Figure 6–a and Figure 6–b is in their initial conditions.

Notice how, in Figure 6–a the trajectory from the variable x passes through maxima at points a, b, and c. But at first these maxima are increasing from a to b and then the maxima are decreasing from b to c. Similarly the minima for y, points d, e, f and g, first appear to be damping to zero from d to e to f and then increase again from f to g. There is not a clear monotonic pattern of increasing or decreasing fluctuations over time. This results from the opposite sign of the two damping parameters where $\zeta_x < 0$ and $\zeta_y > 0$. Also note that even though the intrinsic frequency of y is faster than x ($\eta_x > \eta_y$), y does not appear to oscillate faster than x. Observe that at the point a, the maximum of x occurs prior to the maximum of y. Later at point c, the maximum of y occurs prior to the maximum of x. Then on the next maximum x precedes y again. The two systems are coupled together into a mutually dependent, self–regulated and other–regulated frequency.

Now consider the two simulated trajectories x and y in Figure 6–b. These two trajectories are generated by the system of equations with the same coefficient values as the two trajectories in Figure 6–a, and yet the trajectories appear remarkably different. The only difference between Figure 6–a and Figure 6–b is that the x trajectory is started at time $t = 0$ at a value $x_0 = 2$ in Figure 6–a and $x_0 = -2$ in Figure 6–b. In this case, the first three maxima at a, b and c for the trajectory of x vary more widely than they did previously. After the maximum at c the trajectory does not cross the equilibrium before it begins to diverge from equilibrium again. The trajectory for y also differs markedly from the previous figure including an initial period where the trajectory appears to be following a negative exponential before it changes to an oscillation.

In dynamical systems terminology, Figure 6–b exhibits the effect of *transient dynamics*, that is behavior that is due to some exogenous influence (in this case our arbitrarily setting the values of x and y at time $t = 0$) as well as being due to the dynamics of the system itself. In the real world of psychological systems we must expect that exogenous influences will occur frequently, creating transient perturbations in the observed trajectories of measured psychological constructs. For this reason, in self–regulating systems it is essential to consider the possibility that apparently differing trajectories might not belong to different classes as could be concluded if one we had used a latent growth curve model on the trajectories in Figures 6–a and 6–b. Instead, it might be that similar intrinsic dynamics (i.e., similar self–regulatory mechanisms) might produce very different trajectories given different exogenous influences. One method for testing this hypothesis lies in the use of *state space embedding* for modeling the covariances between derivatives in order to estimate coefficients of differential equation models of self–regulation.

Time Delay Embedding

In order to estimate coefficients in a differential equation model, for instance Equation 3, one must find a way to either reparameterize the model so that it does not contain derivatives or find a way to use the data to estimate the effects of the derivatives on one another. In each of these cases one must have multiple occasions of measurement on any one individual; in other words a time series. Suppose we have measured the individual i on the variable x at p occasions separated by equal intervals of time s. A time series for individual i is a vector of observations $\mathbf{x} = \{x_1, x_2, x_3, \ldots x_{p-1}, x_p\}$. The intrinsic dynamics of x are evidenced by the ordered sequence of observations; If we were to randomize the sequence, the resulting vector would have the same distribution of scores, but any evolving self–regulation of these observations would be lost. It is the way the x_1 leads to x_2 and x_2 to x_3 that captures the self–regulation of the system. This simple observation led to some formal theorems (Whitney, 1936; Takens, 1985) that show that it is, in theory, possible to recover the dynamics of a system from short, ordered sequences of observations as long as a time delay constant and number of embedding dimensions is properly chosen. We will demonstrate this idea using a simple example.

Time–delay embedding (also known in the physics literature as *state space embedding*) is a method for creating a data set that will allow the estimation of coefficients of differential equation models such those presented in the previous sections. The essential idea is easier than it might sound. One begins with a time series vector of observations $\mathbf{x} = \{x_1, x_2, x_3, \ldots x_{p-1}, x_p\}$, chooses a time delay constant, τ, and a number of embedding dimensions d and then produces a *state space matrix*. The embedding dimension d is the number of columns in the state space matrix, in other words how many observations are in a "short sequence of observations". The time delay constant τ represents how many observations to skip forward to obtain the next observation in a "short sequence of observations".

As an example, let us suppose we have 40 observations in a vector $\mathbf{x}$ and we choose a time delay of $\tau = 2$ and embedding dimension $d = 3$. From our time series vector $\mathbf{x}$ we then create the embedded state space matrix $\mathbf{X}$ as

$$\mathbf{X} = \begin{bmatrix} x_1 & x_3 & x_5 \\ x_2 & x_4 & x_6 \\ x_3 & x_5 & x_7 \\ x_4 & x_6 & x_8 \\ \vdots & \vdots & \vdots \\ x_{34} & x_{36} & x_{38} \\ x_{35} & x_{37} & x_{39} \\ x_{36} & x_{38} & x_{40} \end{bmatrix} . \tag{7}$$

Each row of $\mathbf{X}$ contains three observations, a "short sequence of observations" and could be considered to be a point in a 3–dimensional space: a *state space*. The ordering of the columns is important, since it preserves the time–ordered nature of each short sequence of observations. However, the ordering of the rows of this matrix does not matter. If we have chosen the time delay and embedding dimension correctly (see Sauer, Yorke, & Casdagli, 1991), all of the sequential information will be contained in the relationship between the columns. Although there still is sequential dependence between the rows as they are ordered above, that sequential dependence contributes nothing additional to the estimation of parameters for a differential equations model. This means that we might, for instance select a random sample of rows and reorder it and still be able to obtain estimates of the coefficients of a differential equation that had generated these data (see Boker & Nesselroade, 2002, for a more lengthy discussion).

There are several methods that can be used to estimate differential equation models from state space embedded data. Stochastic Differential Equations methods (see e.g., Arminger, 1986; Singer, 1998; Oud & Jansen, 2000) transform the model into an integral form so that coefficients can be estimated directly from the time delayed observations. While this has some advantages, estimation of such models involve complications arising from integration of the error term. Another approach is to transform the embedded state space matrix so that explicit estimates of derivatives are obtained and then use standard statistical techniques to obtain parameter estimates (see e.g., Boker & Graham, 1998; Boker, 2001). This is the approach that will be used in the current chapter. A third method called Latent Differential Equations (LDE) generates latent variable estimates of the derivatives and then estimates coefficients from the covariances between these latent variables (see Boker, Neale, & Rausch, 2004, for an introduction). Each of these methods has their advantages and disadvantages. We will focus on using the second approach, Local Linear Approximation (LLA) to transform our state space embedded data matrix into explicit estimates of derivatives and discuss how we chose the time delay and embedding dimension for this problem. We will then use multilevel modeling to estimate the coefficients of a differential equation model of coupling between husbands and wives. We chose LLA for the estimation due to the simplicity of the approach and its ability to use standard multilevel estimation routines. We chose to use a multilevel model for two reasons: We expect that there may be considerable differences between marriages in the dynamics of self– and other–regulation, and we wish to see if we can account for these individual differences using marital satisfaction as a predictor.

Accounting for Individual Differences in Dynamics

Dynamical systems modeling focuses on how the current state of a system of variables leads to the future state of those variables. In this way, a dynamical systems analysis is concerned with how change in a system evolves — and this is why differential equations are frequently used to specify dynamical models. Individual differences in a dynamical system might be manifested in several ways. First, there might be differences in equilibria. One might think of this as individual differences in central tendency of a variable over time, and these could be estimated for instance in a mean and slope multilevel model where each individuals' central tendency is changing slowly over time. Second, there may be individual differences in the mechanism that regulates how a variable fluctuates about the equilibrium in the absence of other influences. These differences could be estimated as individual differences in the coefficients of a differential equation model of the intrinsic dynamics of a variable. Finally, there might also be individual differences in how variables are coupled together. Some participants might be more reactive or responsive to outside influences than others. Or, in a coupled set of differential equations, there might be individual differences in the strength of the coupling parameter.

Prior to specifying a multilevel differential equations model, we will need to account for individual differences in equilibrium values by centering each person's time series around their respective equilibrium values. One way to accomplish this is to fit a slope and intercept model to each person's time series and save the residuals from the predicted slope and intercept as input to the differential equations analysis. These slopes and intercepts will not be used in the current analysis since here we are focusing on the short term dynamics rather than on individual differences in equilibria. But reliable differences in equilibrium values might be also be informative, and if so, one might use methods such as those described elsewhere in this volume (Walls et al., in press) to model them.

We now create a differential equations model of the residuals calculated above by adapting Equations 5 and 6 to account for individual differences in coefficients using a multilevel modeling framework. Suppose we were to be interested in the dynamics of husbands' feelings of intimacy and how they were influenced by the wives' feelings of intimacy. A second order linear differential equation for the self–regulation and spousal regulation of Husbands' Intimacy could be written such that

$$
\begin{aligned}
\ddot{x}_{ij} &= \eta_{ix} x_{ij} + \zeta_{ix} \dot{x}_{ij} + \eta_{iy} y_{ij} + \zeta_{iy} \dot{y}_{ij} + e_{ij} \\
\eta_{ix} &= c_{00} + u_{0i} \\
\zeta_{ix} &= c_{10} + u_{1i} \\
\eta_{iy} &= c_{20} + u_{2i} \\
\zeta_{iy} &= c_{30} + u_{3i}
\end{aligned} \tag{8}
$$

where x_{ij} is the ith couple's Husband Intimacy score and y_{ij} is the ith couple's Wife Intimacy score at the jth occasion[1]. We continue to use $\dot{x}$ to indicate the first derivative

[1] Although the notation used here for the multilevel models differs from the notation presented elsewhere in this volume, this notation is popular in many areas of psychology, especially educational psychology. We feel that it simplifies the presentation of the ideas while maintaining an accurate account of the modeling. The equivalence between this notation used here and the generalized linear mixed models notation is explained in detail in the first chapter of this volume (Walls et al., in press)

and $\ddot{x}$ to indicate the second derivative of a variable with respect to time. In the second level model, the constants c_{00}, c_{10}, c_{20}, and c_{30} represent the mean value of the respective random coefficient and u_{0i}, u_{1i}, u_{2i}, and u_{3i} represent the unique contribution to that random coefficient for the ith couple. A second model of the same form would be needed to specify the self–regulation and spousal regulation of Wive's Intimacy

Specifying the dynamics of a coupled variable in this way has the advantage that once derivatives are estimated from the data, coefficients can be easily estimated using standard mixed effects software such as SAS PROC MIXED or the lme() function in R (Pinheiro & Bates, 2000). The general advantages of multilevel modeling accrue to this specification as well, including accounting for dependency within a couple and accounting for non–balanced missing at random incomplete data. The major disadvantage of this specification is that the system is not solved simultaneously for all self–regulated variables and so the solution may be suboptimal. This could lead to bias in the parameter estimates and might obscure potentially reliable coupling effects. However, at this point, we do not yet have a viable alternative to the current specification. We suspect that such an alternative may be developed in the near future.

Example: Daily Intimacy and Disclosure in Married Couples

The experience of intimacy is the outcome of an interpersonal process of self–revealing disclosure to which the partner responds in a supportive, understanding way (Laurenceau, Rivera, Schaffer, & Pietromonaco, 2004; Reis & Shaver, 1988). An implicit aspect of the intimacy process is that each partner in a relationship has a desired level of intimacy and connectedness that can be conceived as an equilibrium range. In addition, amount of self-disclosure to a partner, which is often a way to start the intimacy process, may also be regulated with respect to a disclosure equilibrium level. Day–to–day experience of intimacy fluctuates around this desired level and the amount of self–disclosure to a partner, which is often a way to start the intimacy process, is regulated with respect to an intimacy equilibrium level.

Some relationship theorists have referred to a phenomenon of intimacy regulation in close relationships, a dyadic process reflecting a balance of the intimacy and autonomy needs of each partner (Prager & Roberts, 2004). As noted, an inherent assumption in this balance is the idea that intimacy is not consistently increasing over the course of a marriage, but rather, fluctuates in accordance to a desired level (Laurenceau, Feldman Barrett, & Rovine, in press). Nevertheless, one spouses level of intimacy may change not only as a function of their own desired intimacy level but also change as a function of the other spouses intimacy level. Close relationships, such as marriage, exhibit a high degree of this type of interdependence — where the thoughts, feelings, and behaviors of one partner influence the thoughts, feelings, and behaviors of the other (Kelley et al., 1983). In a well–adjusted marriage, the regulation of the experience of intimacy towards ones desired equilibrium level should be facilitated so as to prevent each partner from experiencing long-term extremes in levels of intimacy (i.e., too much or too little). This contention has been observed by Prager and Roberts (2004) regarding intimacy regulation, noting that: "Well–functioning couples make continuous dynamic adjustments in their behavior to avoid emphasizing one pole — intimacy or autonomy — at the expense of the other (p.55)." Is there an empirical way to capture and examine parameters that influence the dynamics of a putative dyadic intimacy process? Considering a marital dyad as the system, we attempt to model a self–regulating process in each individual partner and a coupling between these dynamic processes.

Example Data

Analysis was conducted on a sample of 96 couples who were married for an average of 9.32 years (SD = 9.50, range = .17 - 52.5). Data were collected using a daily-diary sampling method whereby each spouse independently completed a structured diary assessing day-to-day variation in variables tapping intimacy in marriage each evening for 42 consecutive days. Diaries of this type allow participants to give more accurate and focused accounts of actual, everyday social activity and capture the dynamic nature of the process of intimacy that appears static with the use of more conventional, cross-sectional designs.

Procedure

The married couple participants were recruited from a region of central Pennsylvania to participate in a "study on daily experiences in marital relationships." Advertisements were placed in the local area newspaper and flyers were posted at various public locations.

One research assistant was assigned to each couple and visited their home three times over the course of the study. At the first visit, the research assistant obtained informed consent, collected demographic information, and administered cross-sectional measures. Next, spouses were instructed to complete independently a daily diary questionnaire during the evening on each of 42 consecutive days (6 weeks). The research assistant explained the procedure for completing the diary and defined various terms on the diary form. Each partner was given a written set of diary study instructions and definitions for reference throughout the study.

To help preserve confidentiality and ensure response integrity and honesty, each spouse was given a set of 42 adhesive labels with which to seal closed each completed daily-diary form. Spouses were instructed to fold each completed diary form in thirds and use the adhesive label to seal it shut. At the end of the first visit, the members of each couple were given a sufficient number of diaries to take them through the mid-point of the 42-day recording period (i.e., 21-days) and a tentative appointment for the second visit was made. The research assistant phoned couples the following evening and spoke to each spouse individually in order to answer any questions that may have come up about the diary procedures. Couples were also called on a weekly basis to help ensure they were following the study procedure and completing diaries appropriately and to remind couples of the importance of completing the diaries independently.

The second visit was conducted at approximately the mid-point of the 42-day recording period. At this visit, the research assistant collected each spouse's completed diaries for the first half of the recording period and scheduled a tentative final visit. Upon completion of the final week of diary recordings, the research assistant visited each couple a final time at their home to collect the completed diaries for the second half of the recording period, to provide couples with remuneration for their participation in the study.

Measures

A daily diary measure was constructed to assess the variables theorized as per Reis and Shaver's (1988) interpersonal process model of intimacy and was modeled after the diary form used by Laurenceau, Barrett, and Pietromonaco (1998). Responses to diary items were all rated using 5–point Likert scales (e.g., 1 = very little, 5 = a great deal). Being part of a larger diary form, only the diary variables relevant to the current study were reported here:

Intimacy. Spouses rated the amount of closeness that they experienced across the marital interactions with their spouse that day. We chose to use the term closeness rather than intimacy to ensure that participants were rating the degree of psychological, rather than physical or sexual proximity. Based on the identified strong link between intimacy and perceptions of a partners responsiveness (Reis & Shaver, 1988), we also included items assessing perceived partner responsiveness. Spouses rated the degree to which he/she felt understood by their partner (one item), validated by their partner (one item), accepted by their partner (one item), and cared for by their partner (one item) across daily marital interactions. Based on factor analysis, it appeared that these 5 items hung well together and were aggregated to create a single daily intimacy score.

Disclosure. Spouses rated the amount that they disclosed facts and information (one item), the amount that they disclosed their thoughts (one item), and the amount that they disclosed their feelings (one item) across all the interactions that they had with their spouse during the day. Spouses also rated the amount they perceived that their partner disclosed facts and information (one item), the amount of perceived disclosure of their partner's thoughts (one item), and the amount of perceived disclosure of feelings (one item) across all the interactions that they had with their spouse during the day. A disclosure summary variable was created using the sum of these six items.

Dyadic Adjustment Scale (DAS). The DAS is a commonly administered, 32-item self–report measure used to assess global marital satisfaction (Spanier, 1976). Scores range from 0 to 151 with higher scores indicating greater marital satisfaction, and this measure was completed by spouse prior to beginning the 42–day diary recording period. The mean DAS score for husbands in this sample was 112.64 ($SD = 12.73$), while the mean of the wives was 113.92 ($SD = 14.34$). A matched pairs t–test indicated that husbands and wives did not differ in their levels of global marital satisfaction ($t(95) = -1.06$, $p = .29$). Cronbach's alphas for the husband and wife DAS scales were .90 and .91, respectively.

Modeling the Example Data

Daily diaries were completed by 96 couples for 42 consecutive days. Two couples were exclude due to low response rates. Of the remaining 94 couples, the overall complete husband and wife response rate was 97% for the disclosure measures and 96% for the intimacy measures. Figure 7 plots the husband and wife scores for intimacy and disclosure from four example couples. By inspection, it does not seem unreasonable that each individudal may have a preferred equilibrium value or set point for each of these scores. In addition, we note that there appear to be short term *sychronization events* in which the husbands' and wives' scores seem to be displaced far from equilibrium on the same day. These events appear to be relatively short term, in that the scores return to near equilibrium within an interval of a few days. Given our inspection of the plotted data, we decided that it was not unreasonable to test a model in which husbands' and wives' scores were coupled together such that they regulated each other as well as themselves.

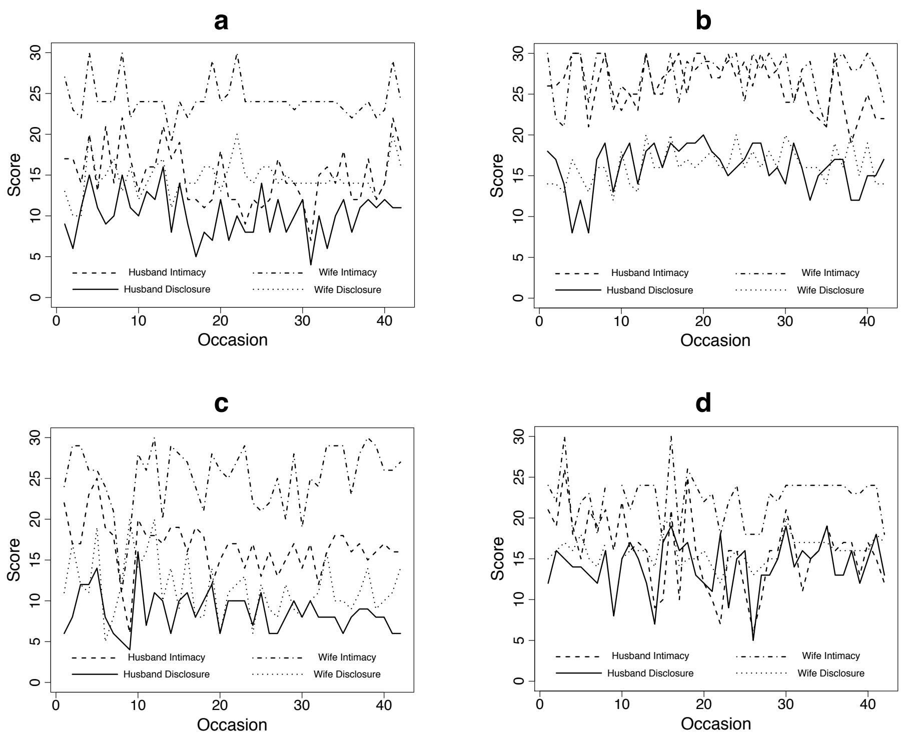

Figure 7. Daily husband and wife intimacy and disclosure time series for four couples. Note that there appear to be individual differences in equilibrium values as well as between–couple differences in the variability around the equilibrium.

An intercept–only mixed effects model grouping by dyad was fit separately to husbands and wives scores for intimacy and disclosure as

$$\begin{aligned} x_{ij} &= b_i + e_{ij} \\ b_i &= c_0 + u_i \end{aligned} \tag{9}$$

where x_{ij} is the score (e.g., wives' intimacy) for individual i at occasion j, b_i is the intercept for individual i, c_0 is the mean value for the intercepts, and u_i is the unique within–individual contribution to the intercept. Two values of interest in this equation are the standard deviation of u_i, the between–persons variance in intercepts and the standard deviation of e_{ij}, a measure of the mean within–person variability. As shown in Table 1, the standard deviations of the between–person differences (e.g., wives' intimacy between person SD=3.66) were approximately the same as the within–person variability (e.g., wives' intimacy residual

SD=3.86) for each of the variables. This suggests that although there may be individual differences in equilibrium values, the intraindividual variability is of approximately equivalent magnitude in this sample and thus it is warranted to test whether the substantial within–person component of variance is patterned as a self–regulatory system.

Table 1: Intercept only mixed effects models grouped by dyad and each predicting one variable.

Variable	Intercept	S.E.	Between S.D.	Within S.D.
Husband Intimacy	21.50	0.40	3.84	3.32
Wife Intimacy	21.62	0.38	3.66	3.87
Husband Disclosure	13.07	0.27	2.56	2.56
Wife Disclosure	12.83	0.28	2.64	2.94

In this study, there is no single event that we can use to align these intensively measured variables in time. Thus, there is no way to meaningfully assign $t = 0$, as this may be different for each couple or could even change for a couple during the course of the study. Outside events and daily stressors such as problems at work or sickness of a parent might occur at unpredictable intervals and these stressors might influence the intimacy and disclosure scores for a couple. For this reason, a state space model is a much better choice for these data than would be a growth curve model. A state space model is relatively insensitive to the timing of influences exogenous to the system whereas a growth curve model will incorporate the unpredictable intervals of the external stressors as part of the individual differences in trajectories.

We elected to use Local Linear Approximation (LLA) to explicitly estimate derivatives and then to use random coefficients (i.e. mixed effects or HLM) modeling to estimate parameters. First and second derivatives for a time series $\mathbf{x} = \{x_1, x_2, x_3, \ldots, x_p\}$ can be estimated using LLA by first removing the linear trend from each individual's data and then creating a three column time–delay embedded state space matrix $\mathbf{X}$ of order $(p - 2\tau) \times 3$ from these residuals as discussed above. The derivatives $\dot{x}_k$ and $\ddot{x}_k$ for the kth row of the matrix $\mathbf{X}$ can be calculated as

$$\dot{x}_k = (x_{k3} - x_{k1})/2\tau \tag{10}$$

and

$$\ddot{x}_k = (x_{k3} + x_{k1} - 2x_{k2})/\tau^2 \tag{11}$$

where τ is the lag offset used to create the embedded states space matrix $\mathbf{X}$ (Boker & Nesselroade, 2002).

Once these derivatives are calculated, a second order linear differential equation mixed effects model can be fit as

$$\begin{aligned} \ddot{x}_{ij} &= \eta_{ix} x_{ij} + \zeta_{ix} \dot{x}_{ij} + e_{ij} && (12) \\ \eta_{ix} &= c_{00} + u_{0i} \\ \zeta_{ix} &= c_{10} + u_{1i} \end{aligned}$$

(13)

where x is one of the four variables: Husband's Disclosure, Husband's Intimacy, Wive's Disclosure or Wive's Intimacy. In order to choose a value of τ that is appropriate for these data, we fit the model in Equation 12 to each of the four variables using values for $\tau = \{1, 2, \ldots, 8\}$. The resulting mean explained variance (r^2) over 93 individuals' data and the lower 95% confidence interval for the mean explained variance is plotted for each value of τ in Figure 8. The horizontal line at $r^2 = 0.656$ is the expected value of r^2 for uncorrelated measurement error. Thus, we reject the hypothesis that the residual intraindividual variability from a linear trend is measurement error when $\tau \geq 4$.

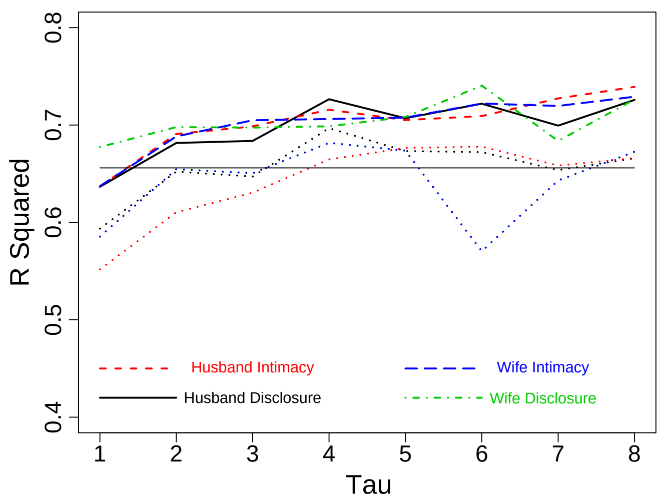

Figure 8. Mean explained variance (r^2) within–individual for univariate damped linear oscillator models for husbands' and wives' disclosure and intimacy. Dotted lines lower 95% confidence interval for (r^2) for each variable. Horizontal line is expected value for uncorrelated measurement error.

However, note that the largest gain in mean r^2 occurs between $\tau = 1$ and $\tau = 2$. The mean r^2 for all measures are near their peak value by $\tau = 2$. Previous simulations have suggested that minimum bias estimates for frequency (η) parameters are achieved at the minimum τ when the r^2 first nears its maximum value. Combining this observation with the previous observation that high displacement co–occurring "events" happened over a period of just 3 to 5 days suggests that $\tau = 2$ may be the best choice. Larger values of τ would tend to obscure these short–term episodes.

This analysis provides evidence that these data are unlikely to be measurement error when modeled with an uncoupled linear oscillator. However, we might suspect that an uncoupled model is incomplete since it ignores the possibility of husbands and wives regulating each other's behavior. A better model would include coupling parameters. We next present the results of a coupled model for each of the four variables.

Husbands' intimacy regulation was modeled using a mixed effects model. After time delay embedding using a delay of $tau = 2$, we calculated derivatives using LLA predicted the second derivative of husbands' intimacy, $\ddot{x}$ as

$$\begin{aligned}
\ddot{x}_{ij} &= \eta_{ix} x_{ij} + \zeta_{ix} \dot{x}_{ij} + \eta_{iy} y_{ij} + \zeta_{iy} \dot{y}_{ij} + e_{ij} \\
\eta_{ix} &= c_{00} + c_{01} z_i + u_{0i} \\
\zeta_{ix} &= c_{10} + c_{11} z_i + u_{1i} \\
\eta_{iy} &= c_{20} + c_{21} w_i + u_{2i} \\
\zeta_{iy} &= c_{30} + c_{31} w_i + u_{3i}
\end{aligned} \tag{14}$$

where x_{ij} is the intimacy score for husband i on day j, y_{ij} is the intimacy score for wife i on day j, and $\dot{x}_{ij}$ and $\dot{y}_{ij}$ are their respective first derivatives. The variable z_i is the marital satisfaction score for husband i and the variable w_i is the marital satisfaction score for wife i. The constant coefficients c_{00}, c_{10}, c_{20}, and c_{30} are the intercept values (fixed effects) for the random coefficients η_{ix}, ζ_{ix}, η_{iy}, and ζ_{iy} respectively. The constant coefficients c_{01}, c_{11}, c_{21}, and c_{31} represent the effects of husbands' and wives' marital satisfaction on their respective random coefficients. And finally, u_{0i}, u_{1i}, u_{2i}, and u_{3i} are the unique contributions of each husband or wife on his or her random coefficient.

Once the data have been time delay embedded into a three dimensional state space, the rows are reduced by three for every missing observation. Thus in our example, the total observations across the 93 individuals is 3006 rather than the 3722 complete observations in the data set. This reduction of the state space matrix by incompleteness in the data is a problem that must be considered prior to using a dynamical systems method: one missing observation creates two missing first derivatives and three missing second derivatives.

In the hope of clarifying the discussion, we have adopted the following notation for the parameters of subsequent models. Frequency parameters (i.e., η parameters) are denoted Husband Intimacy (HI), Husband Disclosure (HD), Wife Intimacy (WI), and Wife Disclosure (WD). Damping parameters (i.e., ζ parameters) are similarly denoted dHI, dHD, dWI, and dWD for the husbands' and wives' intimacy and disclosure scores respectively. Finally, interactions with the DAS scale are denoted HI×HDAS, dHI×HDAS, HD×HDAS, dHD×HDAS, WI×WDAS, dWI×WDAS, WD×WDAS, and dWD×WDAS for the husbands' and wives' DAS scores interacting with their respective intimacy and disclosure scores.

The results of fitting the mixed effects model from Equation 14 (to derivatives calculated with LLA using a lag of $\tau = 2$ on the Husband intimacy residuals after removing a linear trend) are presented in Table 2. The Husband η parameter (HI) value is different from zero suggesting a lawful patterning of oscillation in Husband intimacy from day–to–day. The Husband η effect is consistent with the intimacy trajectory showing the greatest curvature when the displacement is farthest from equilibrium. There is no evidence of intrinsic damping as reflected in a nonsignificant Husband ζ parameter (dHI). Examining the coupled effects of wives intimacy regulation on husband intimacy curvature, higher levels of Wives intimacy (WI) is associated with less Husband intimacy curvature, indicating the Husbands intimacy would turn around and move back towards equilibrium less quickly than usual. Interestingly, this effect is moderated by marital satisfaction, where greater Wife marital satisfaction has a negative effect on Wife η (WI×WDAS). Thus, the effect of greater wife displacement on husband curvature for wives reporting higher satisfaction is diminished, allowing husbands intimacy to turn around back towards equilibrium as per his intrinsic η parameter. The damping parameters reflected no significant effects on Husband curvature.

Table 2: Mixed effects model predicting second derivative of husbands' intimacy score using intrinsic intimacy self–regulation, wives' intimacy regulation and marital satisfaction scores. (AIC= 9129, BIC= 9243, N= 3006, Groups= 94, mean r^2=0.693, $\tau = 2$, $\eta\tau^2$=-1.614)

	Value	SE	DF	t	p
dHI	-0.0012	0.1440	2905	-0.01	0.993
HI	-0.4034	0.0789	2905	-5.11	< 0.001
dWI	0.0490	0.1330	2905	0.37	0.713
WI	0.1216	0.0482	2905	2.52	0.012
HI×HDAS	-0.0008	0.0006	2905	-1.22	0.224
dHI×HDAS	0.0000	0.0012	2905	0.02	0.984
WI×WDAS	-0.0011	0.0004	2905	-2.64	0.008
dWI×WDAS	-0.0007	0.0011	2905	-0.63	0.526

Examining the model for Husbands disclosure contained in Table 3, husband η (HD) reflects a significant oscillation parameter. However, unlike the results for Husband intimacy, Wife disclosure regulation parameters are not coupled to Husband disclosure curvature. Moreover, marital satisfaction is not a moderator of η or ζ effects.

Table 3: Mixed effects model predicting second derivative of husbands' disclosure score using intrinsic disclosure self–regulation, wives' disclosure regulation and marital satisfaction scores. (AIC= 7385, BIC= 7500, N= 3006, Groups= 94, mean r^2=0.683, $\tau = 2$, $\eta\tau^2$=-1.738)

	Value	SE	DF	t	p
dHD	0.0982	0.1388	2905	0.707	0.4793
HD	-0.4346	0.0750	2905	-5.789	0.0000
dWD	0.0081	0.1174	2905	0.069	0.9446
WD	0.0352	0.0423	2905	0.831	0.4057
HD×HDAS	-0.0004	0.0006	2905	-0.647	0.5173
dHD×HDAS	-0.0009	0.0012	2905	-0.780	0.4351
WD×WDAS	-0.0004	0.0003	2905	-1.238	0.2156
dWD×WDAS	-0.0002	0.0010	2905	-0.220	0.8254

The model for Wife intimacy contained in Table 4 reflects the same pattern of results as for Husband intimacy. This model demonstrated a significant Eta parameter, indicating patterned oscillations around an equilibrium. Husband intimacy showed a coupled effect on Wife curvature, where higher levels of Husbands intimacy (HI) is associated with less Wife intimacy curvature, keeping Wife intimacy higher than she would want it to be. The strength of this coupled effect was moderated by Husband satisfaction, with greater levels of Husband satisfaction (HI×HDAS) being associated with greater Wife intimacy curvature. Damping was not a significant parameter in this model.

Table 4: Mixed effects model predicting second derivative of wives' intimacy score using intrinsic intimacy self–regulation, husbands' intimacy regulation and marital satisfaction scores. (AIC= 9812, BIC= 9926, N= 3006, Groups= 94, mean r^2=0.690, $\tau = 2$, $\eta\tau^2$=-1.955)

	Value	SE	DF	t	p
dWI	0.0607	0.1481	2905	0.410	0.6818
WI	-0.4887	0.0769	2905	-6.354	0.0000
dHI	0.1273	0.1620	2905	0.785	0.4321
HI	0.1230	0.0604	2905	2.035	0.0418
WI×WDAS	-0.0001	0.0006	2905	-0.213	0.8312
dWI×WDAS	-0.0007	0.0012	2905	-0.546	0.5851
HI×HDAS	-0.0011	0.0005	2905	-2.191	0.0285
dHI×HDAS	-0.0008	0.0014	2905	-0.600	0.5479

In contrast to the results predicting Husbands Disclosure curvature, Table 5 shows that Wives disclosure was coupled to Husbands disclosure (HD) and the strength of this coupling was predicted by Husband satisfaction (HD×HDAS).

Table 5: Mixed effects model predicting second derivative of wives' disclosure score using intrinsic disclosure self–regulation, husbands' disclosure regulation and marital satisfaction scores. (AIC= 8264, BIC= 8378, N= 3006, Groups= 94, mean r^2=0.701, $\tau = 2$, $\eta\tau^2$=-2.55)

	Value	SE	DF	t	p
dWD	0.0623	0.1343	2905	0.463	0.6431
WD	-0.6381	0.0747	2905	-8.539	0.0000
dHD	-0.0233	0.1627	2905	-0.143	0.8862
HD	0.1320	0.0652	2905	2.022	0.0432
WD×WDAS	0.0011	0.0006	2905	1.672	0.0946
dWD×WDAS	-0.0006	0.0011	2905	-0.543	0.5866
HD×HDAS	-0.0013	0.0005	2905	-2.218	0.0266
dHD×HDAS	0.0002	0.0014	2905	0.150	0.8802

Discussion

An undamped linear oscillator performed well as a dynamic process model of Husbands and Wives intimacy and disclosure trajectories. Some degree of mutual dependence between husband and wife scores might be expected, but may not be equal. We discovered *symmetric coupling* between husband and wife intimacy, whereby the strength of the coupling was moderated by marital satisfaction. Findings also revealed *asymmetric coupling* between spouse disclosure scores, such that Husbands disclosure was not coupled to Wives disclosure, but Wives disclosure was coupled to Husband disclosure. Damping was not a significant parameter in any of these models.

In some ways, it may not be surprising that an undamped linear oscillator would be a reasonable model for the trajectory of intimacy, rather than a model with damping to an equilibrium range. Intimacy is the outcome of an interpersonal process where the experience of intimacy for one partner (A) is dependent upon self–revealing acts by Partner A, supportive responses by the other partner (B), the perception of responsiveness by A from B, the timing of the exchanges between A and B, and influences outside of A and B (e.g., intimacy–facilitating vs. non–facilitating situation). The number of potential inputs to the process may lead to a situation where intimacy is unlikely to remain at an equilibrium level long, but rather is constantly oscillating around it. If an individual shoots upward past equilibrium until she is feeling too intimate, then she may actively engage in intimacy–distancing tactics (e.g., reductions in self–disclosure, inattention to partner attempts at responsiveness) to allow the regulation process to come down toward equilibrium. Our findings suggest that individuals with spouses who are highly satisfied with their marriages also have spouses who perhaps facilitate intimacy regulation towards equilibrium.

Based on theory in the close relationships literature (Reis & Shaver, 1988), intimacy is a construct that shows both qualities of constancy and change. Constancy is reflected in from an assumed desired level of intimacy that can be considered an equilibrium range that may be different across individuals. Change is reflected in the day–to–day variability in the experience of intimacy that fluctuates around an individuals equilibrium range. Moreover, as a likely consequence of the inherent interdependence that exists in close relationships, such as marriage, we found mutual influence (symmetric coupling) between the self–regulating dynamics of both spouses intimacy trajectories. We believe that the current application of coupled differential equations models of dynamic systems to intimacy in married couples is a way to parameterize the argument that intimacy is best conceptualized as a process reflecting variability, change, and fluctuation over time.

A bottom line conclusion from this work is that in couples reporting greater marital adjustment, intimacy regulation is facilitated. If the goal of an intimacy regulation system is to stay within an equilibrium range, then our findings suggest that this regulation is more apparent in satisfied couples. This type of dynamic mutual influence may be exemplified in no better context than that of close relationships, such as marriage.

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