Dynamics of Change and Change in Dynamics

Steven M. Boker

Department of Psychology
University of Virginia

Dynamical Systems Analysis Workshop Part 5

Modern Modeling Methods
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Overview

- Dynamics of Change
- Second Order Linear Differential Equation
  - Fitting an Aggregate Latent Differential Equation (LDE)
- Individual Differences in Equilibrium
  - LDE with Individual Differences in Equilibrium
- Changes in Equilibrium
  - LDE with Change in Equilibrium
- Individual Differences in Dynamics
  - LDE with Individual Differences in Coefficients of Dynamics
- Second Level Predictors
  - LDE with Predictors of Individual Differences in Equilibrium and/or Dynamics
- Changes in Dynamics
  - LDE with Longitudinal Change in Dynamics

Dynamics of Change

- Processes occur within individuals that have lawful relations
- Examples include:
  1. Resiliency and stress in older individuals.
  2. Self–perceived mental health in recent widows.
  3. Self–disclosure and intimacy in married couples.
  4. Ovarian hormones and disordered eating in young women.
- These processes can be modeled with differential equations.

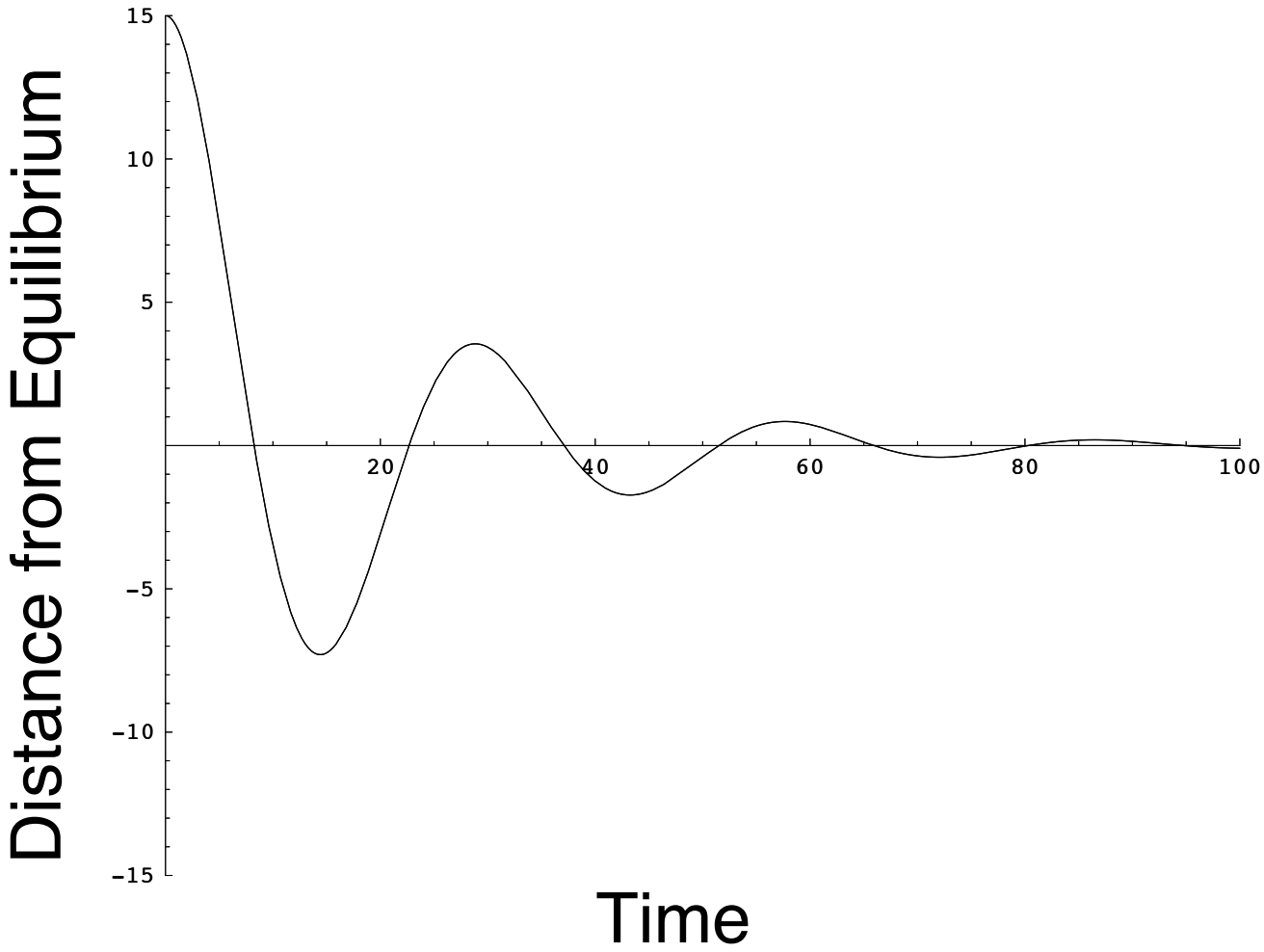
Linear Second Order Differential Equation

- A simple linear second order system is

$$\ddot{x}(t) = \eta x(t) + \zeta \dot{x}(t)$$

  where x is the displacement from an equilibrium.
- If $\zeta < 0$, negative exponential damping.
- If $\eta < 0$ and $\eta + \zeta^2/4 < 0$, oscillation of period $\lambda = \frac{2\pi}{\sqrt{-(\eta+\zeta^2/4)}}$.
- To anthropomorphize:
  - The farther x is from equilibrium, the more it wants to curve back towards equilibrium.
  - The faster x is changing, the more it wants to slow down.

Linear Second Order Differential Equation



Trajectory of the system over time

Time Delay Embedding

- Suppose a time series X has been centered around each individual's equilibrium values.
- If the time series X is ordered by occasion j within individual i then the series of all observations $x_{(i,j)}$ can be written as a vector of scores

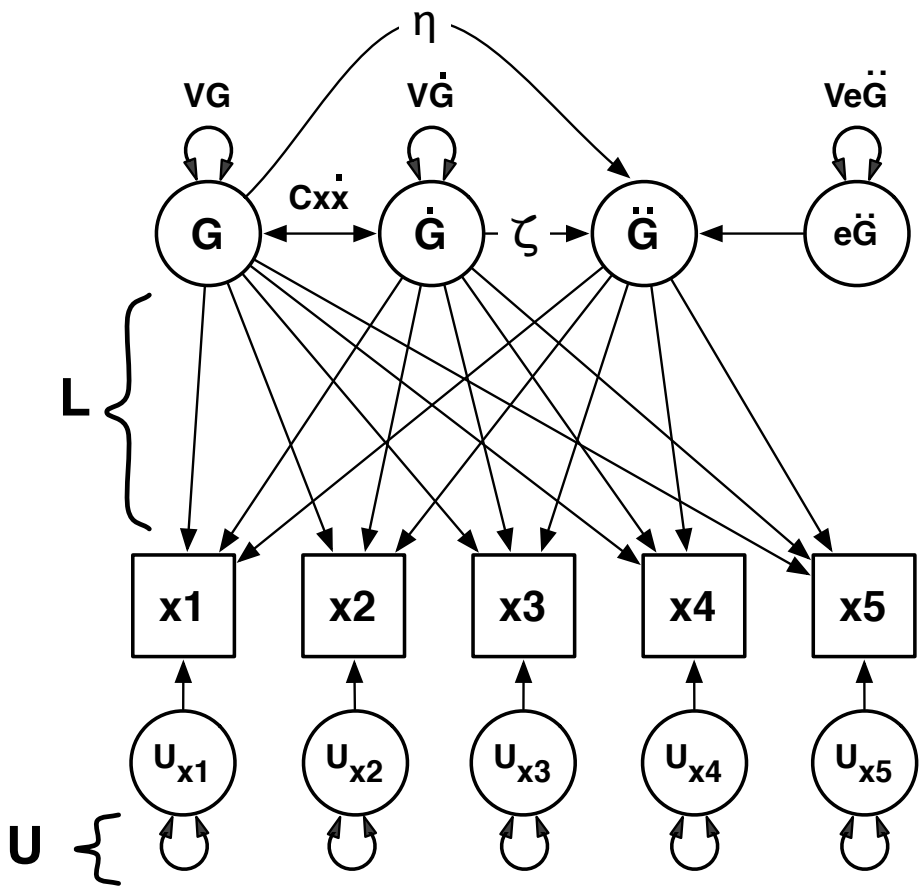
$$\begin{aligned} X &= \{x_{(1,1)}, x_{(1,2)}, \ldots x_{(1,P)}, x_{(2,1)}, x_{(2,2)}, \ldots x_{(2,P)}, \\ &\quad \ldots, x_{(N,1)}, x_{(N,2)}, \ldots x_{(N,P)}\}. \end{aligned}$$

Time Delay Embedding

- For N people, each of whom have been sampled P times, a 5–D time delay embedded matrix $\mathbf{X}^{(5)}$ can be constructed as

$$\mathbf{X}^{(5)} = \begin{bmatrix} x_{(1,1)} & x_{(1,2)} & x_{(1,3)} & x_{(1,4)} & x_{(1,5)} \\ x_{(1,2)} & x_{(1,3)} & x_{(1,3)} & x_{(1,5)} & x_{(1,6)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{(1,P-4)} & x_{(1,P-3)} & x_{(1,P-2)} & x_{(1,P-1)} & x_{(1,P)} \\ x_{(2,1)} & x_{(2,2)} & x_{(2,3)} & x_{(2,4)} & x_{(2,5)} \\ x_{(2,2)} & x_{(2,3)} & x_{(2,4)} & x_{(2,5)} & x_{(2,6)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{(2,P-4)} & x_{(2,P-3)} & x_{(2,P-2)} & x_{(2,P-1)} & x_{(2,P)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{(N,1)} & x_{(N,2)} & x_{(N,3)} & x_{(N,4)} & x_{(N,5)} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ x_{(N,P-4)} & x_{(N,P-3)} & x_{(N,P-2)} & x_{(N,P-1)} & x_{(N,P)} \end{bmatrix}.$$

Second Order LDE

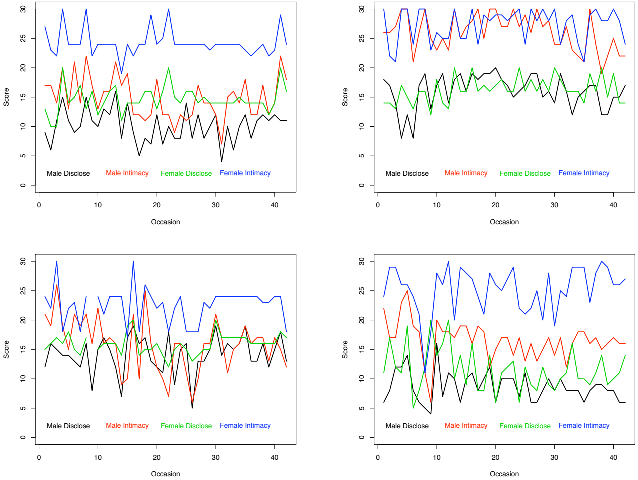


$$\ddot{G}_{ij} = \eta G_{ij} + \zeta \dot{G}_{ij} + e_{ij}$$
$$\mathbf{X}^{(5)} = \mathbf{GL} + \mathbf{U}$$

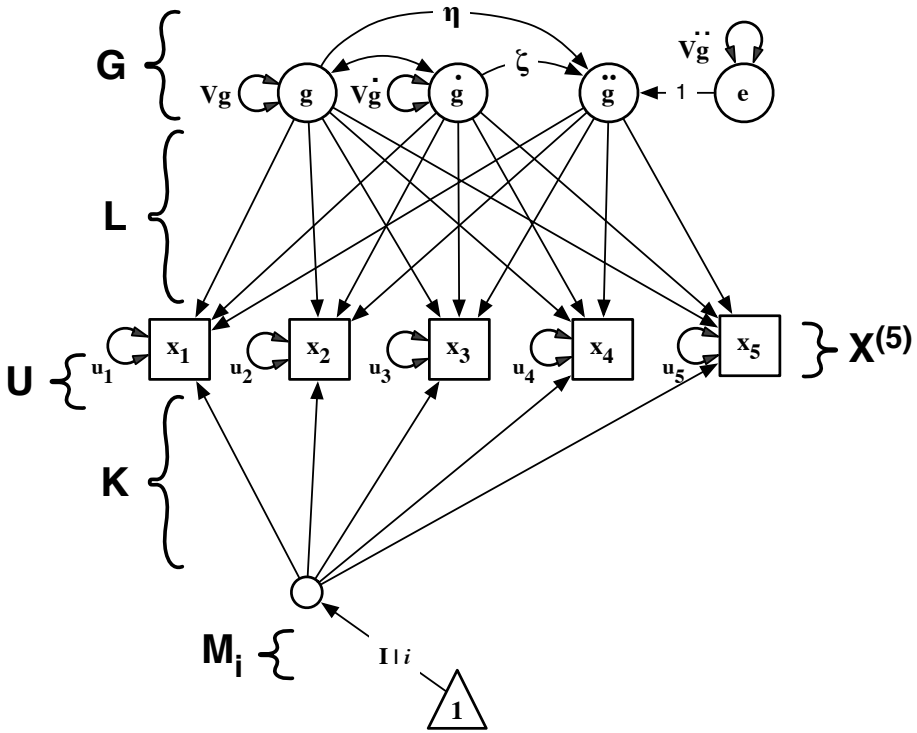
Individual Differences in Equilibrium

- There may be individual differences in equilibrium values.
- The differential equation models assume centering about the equilibrium.
- We must take these individual differences in equilibrium level into account.

Individual Differences in Equilibrium

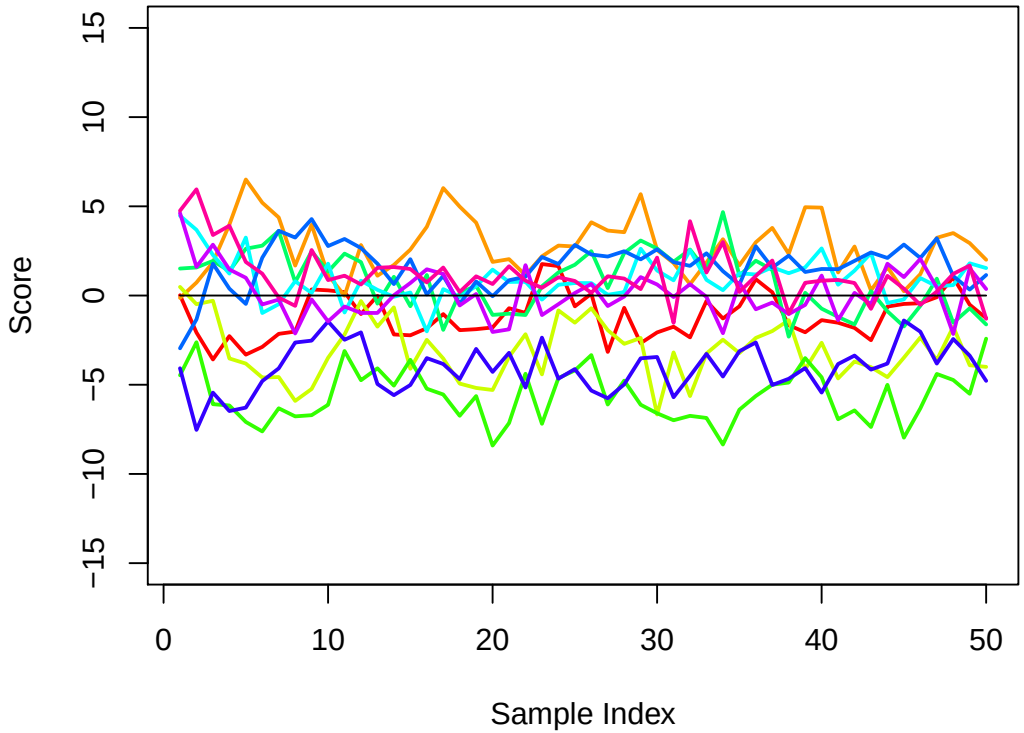


Individual Differences in Equilibrium

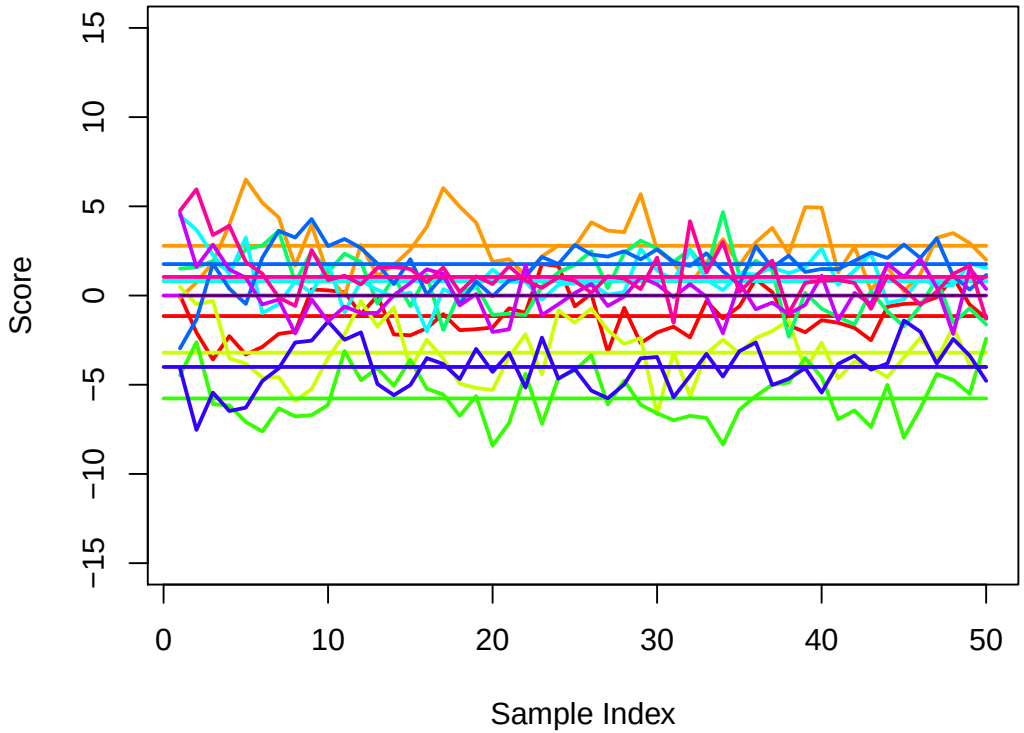


$$\ddot{G}_{ij} = \eta G_{ij} + \zeta \dot{G}_{ij} + e_{ij}$$
$$\mathbf{X}_{ij}^{(D)} = \mathbf{M}_i \mathbf{K} + \mathbf{G}\mathbf{L} + \mathbf{U}$$

Simulation Results



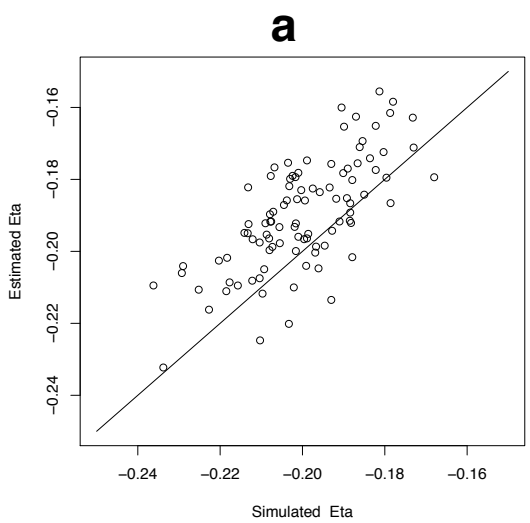
Simulation Results

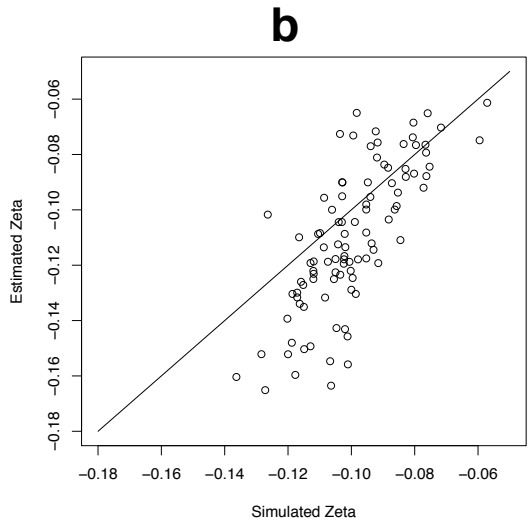


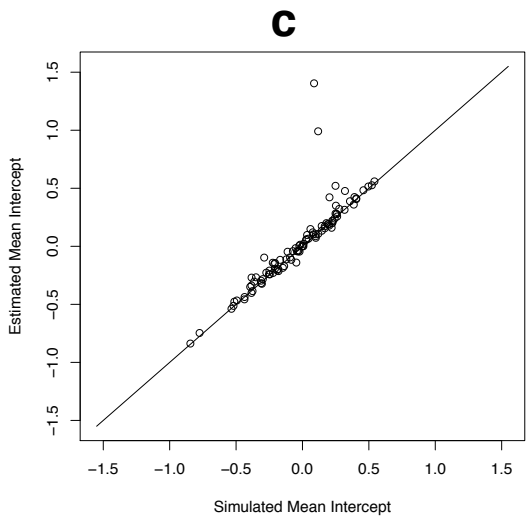
Simulation Results

	True	(SD)	Model B	(SD)
N	100		96	
Did Not Converge			1	
Extreme Outliers			3	
Eta Mean	-0.200	0.014	-0.189	0.016
Eta Variance				
Zeta Mean	-0.100	0.015	-0.110	0.026
Zeta Variance				
Eta Interaction				
Zeta Interaction				
Intercept Mean	-0.026	0.289	0.004	0.311
Intercept Variance	4.081	0.830	4.641	4.411
Slope Mean				
Slope Variance				

Simulation Results



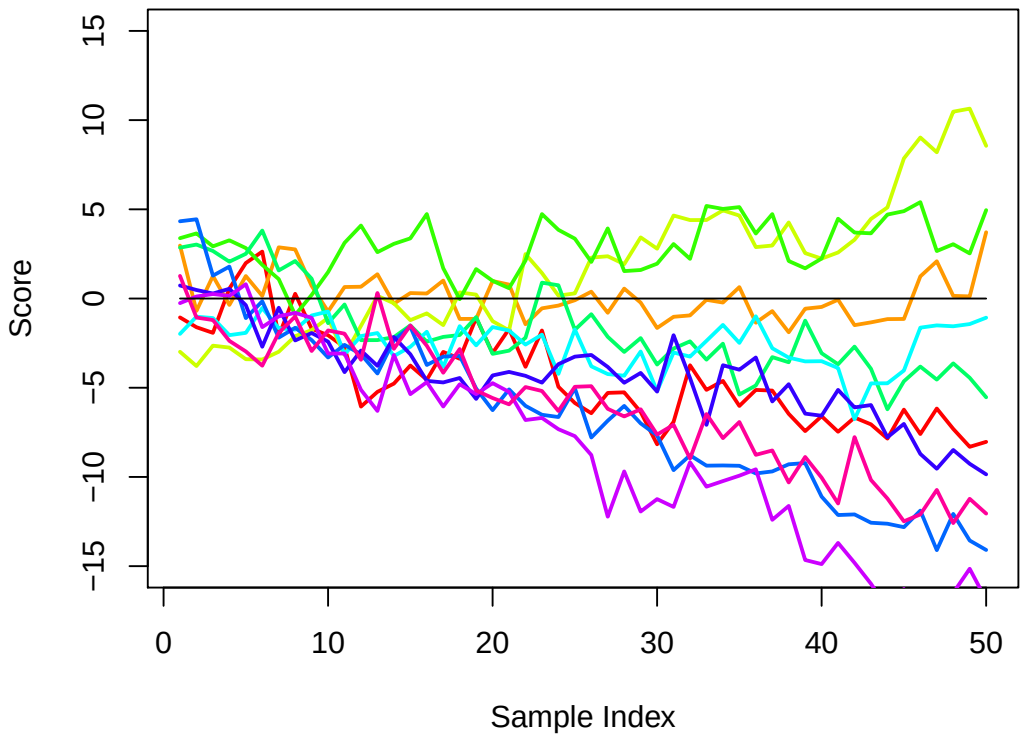




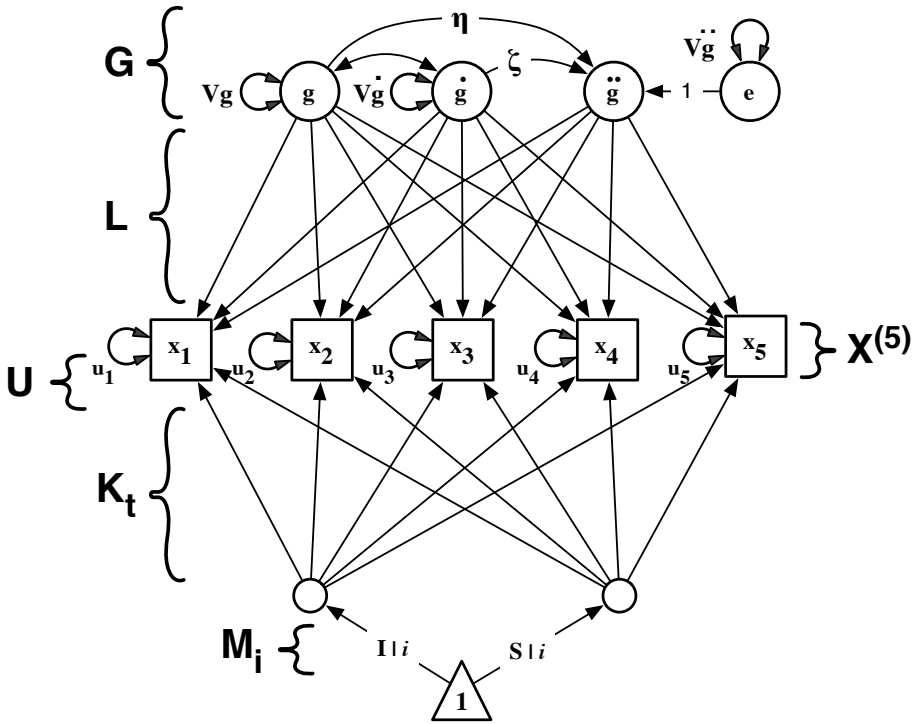
Individual Differences in Equilibrium

Go to ChangesInDynamics_AppendixB.R

When This Model Breaks Down

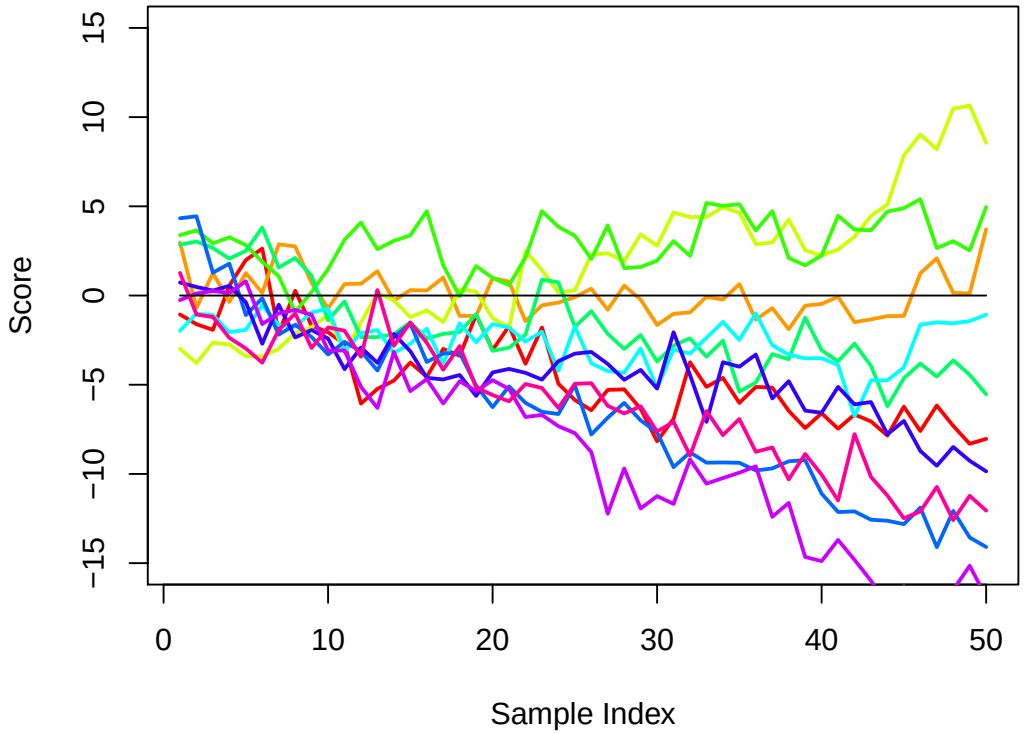


Individual Differences in Equilibrium Change

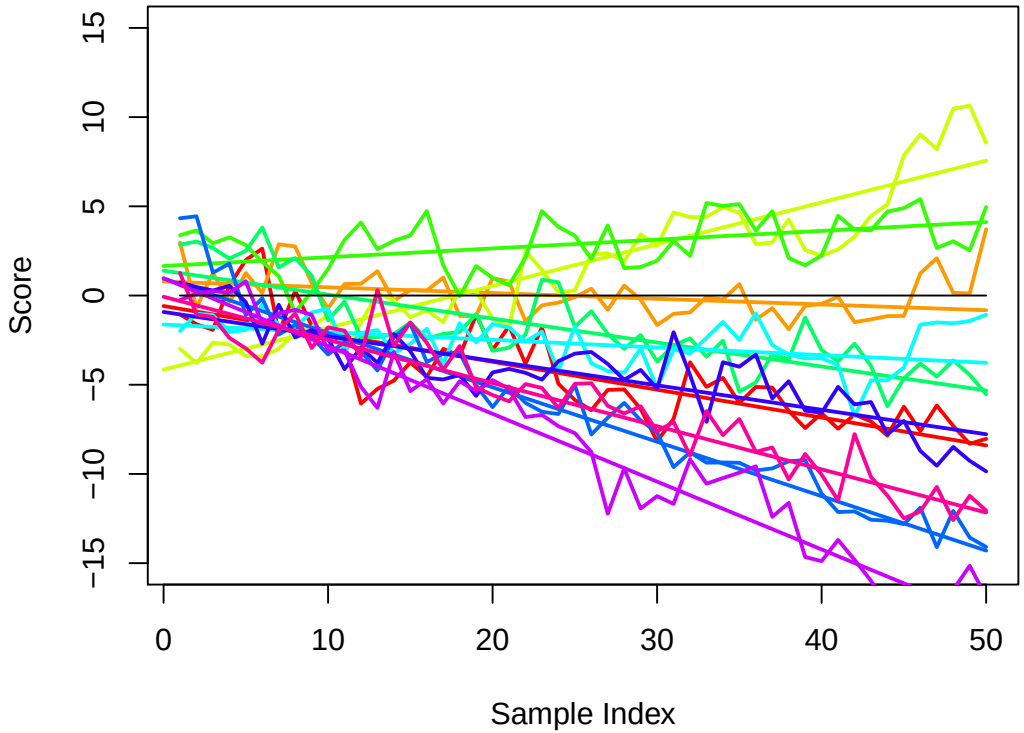


$$\begin{aligned} \ddot{G}_{ij} &= \eta G_{ij} + \zeta \dot{G}_{ij} + e_{ij} \\ \mathbf{X}_{ij}^{(D)} &= \mathbf{M}_i \mathbf{K} + \mathbf{G}\mathbf{L} + \mathbf{U}, \end{aligned}$$

Simulation Results



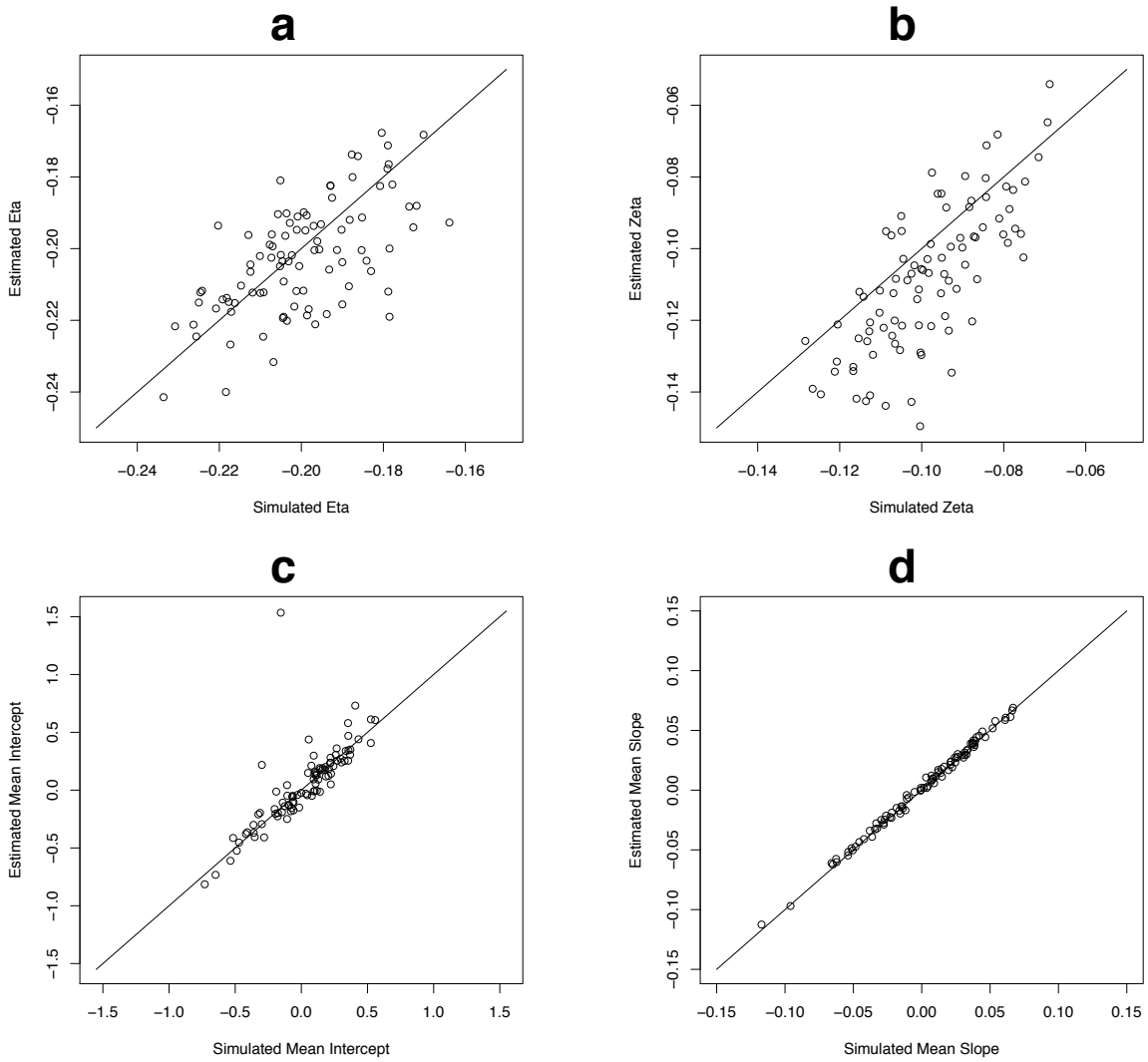
Simulation Results



Simulation Results

	Model C			
	True	(SD)	Est	(SD)
N	100		93	
Did Not Converge			5	
Extreme Outliers			2	
Eta Mean	-0.200	0.015	-0.203	0.016
Eta Variance				
Zeta Mean	-0.099	0.014	-0.110	0.022
Zeta Variance				
Eta Interaction				
Zeta Interaction				
Intercept Mean	0.021	0.272	0.044	0.332
Intercept Variance	3.992	0.967	5.492	7.946
Slope Mean	0.002	0.037	0.003	0.037
Slope Variance	0.087	0.017	0.086	0.017

Simulation Results



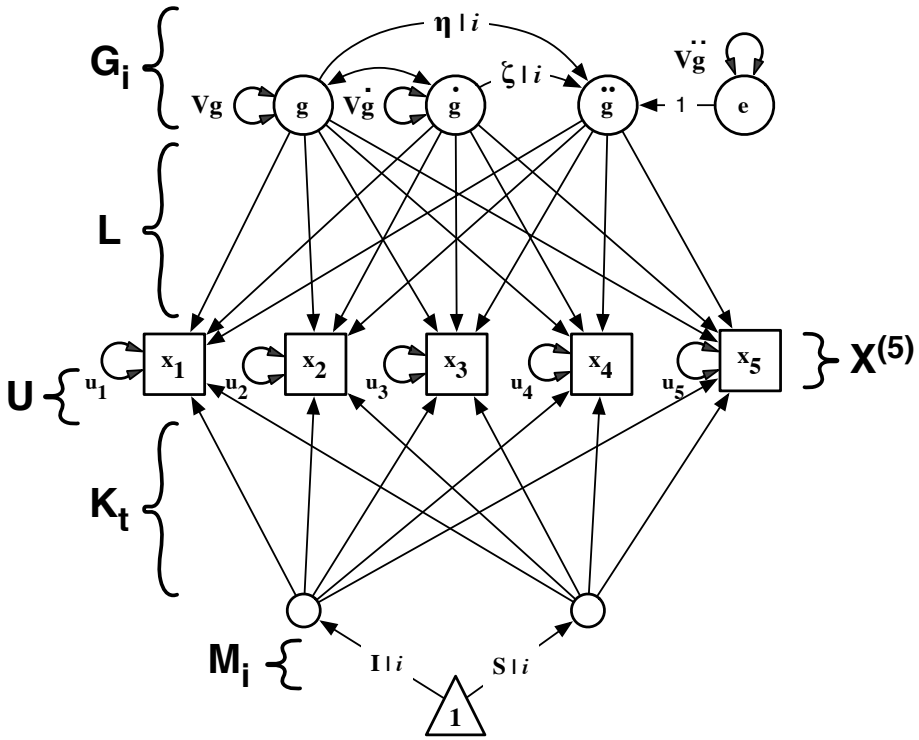
Individual Differences in Equilibrium Change

Go to ChangesInDynamics_AppendixC.R

Individual Differences in Dynamics

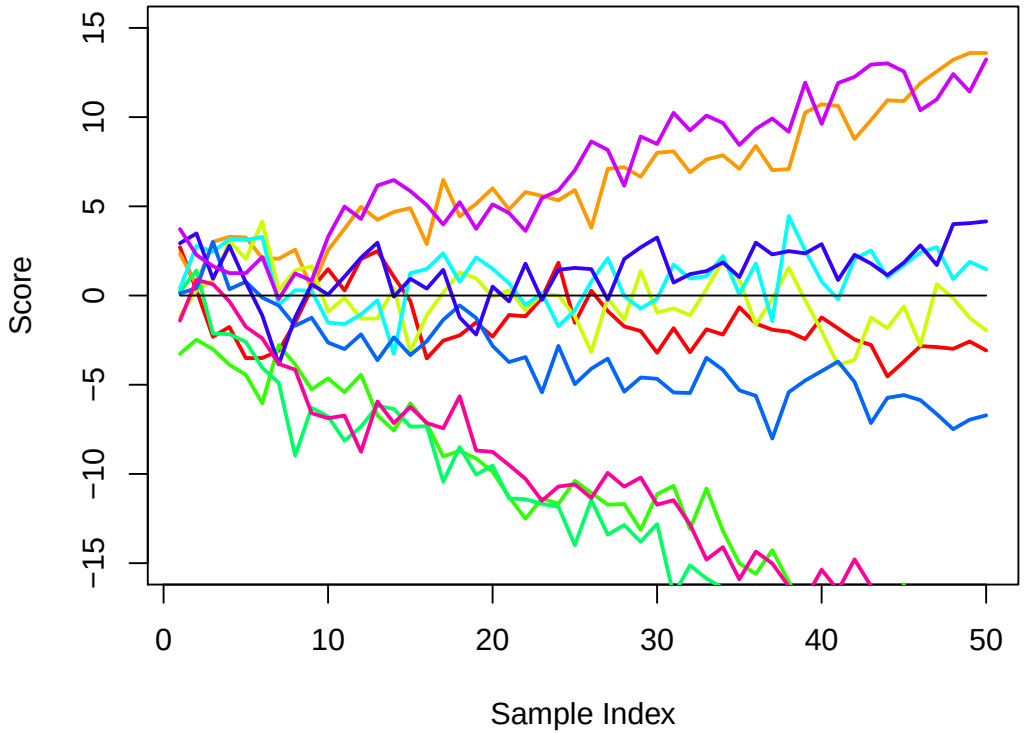
- Individuals may differ from one another in the way that they self–regulate.
- We have seen evidence of individual differences in
  1. Resiliency in older adults.
  2. Disclosure and intimacy in married couples.
  3. Fluctuations in self–perceived mental health in widows.
  4. Cyclic changes in disordered eating in young women.
- We need to account for these individual differences in dynamics.

Individual Differences in Dynamics

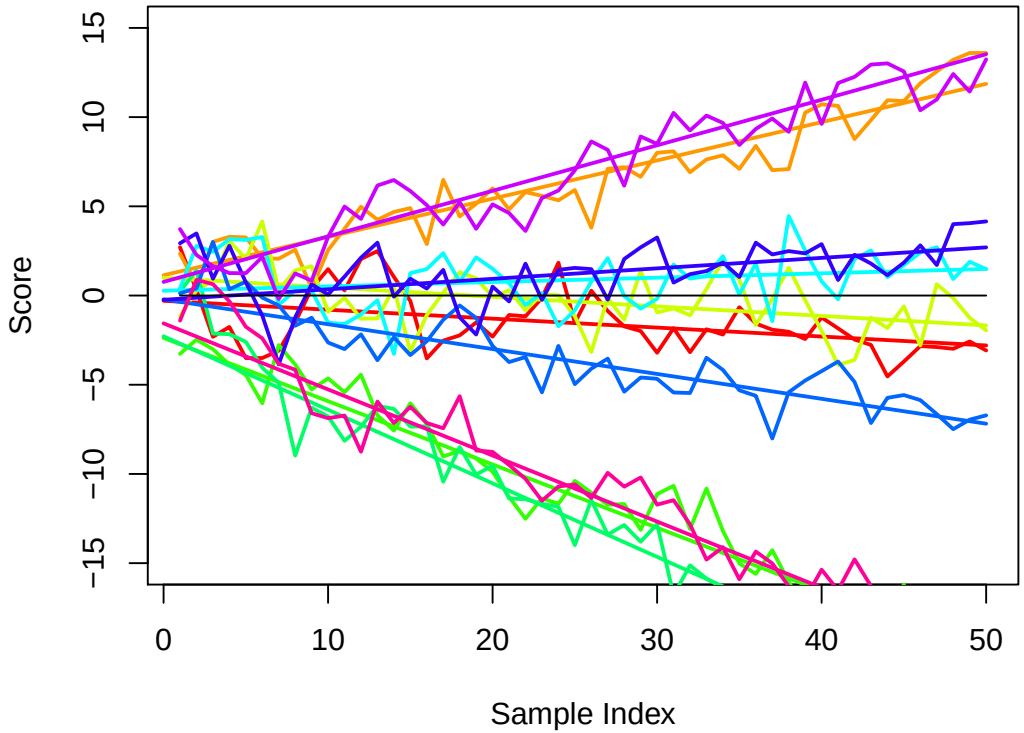


$$\begin{aligned} \ddot{g}_{ij} &= \eta_i g_{ij} + \zeta_i \dot{g}_{ij} + e_{ij} \\ \mathbf{X}^{(5)}_{ij} &= \mathbf{M}_i\mathbf{K} + \mathbf{G}_i\mathbf{L} + \mathbf{U}. \end{aligned}$$

Simulation Results



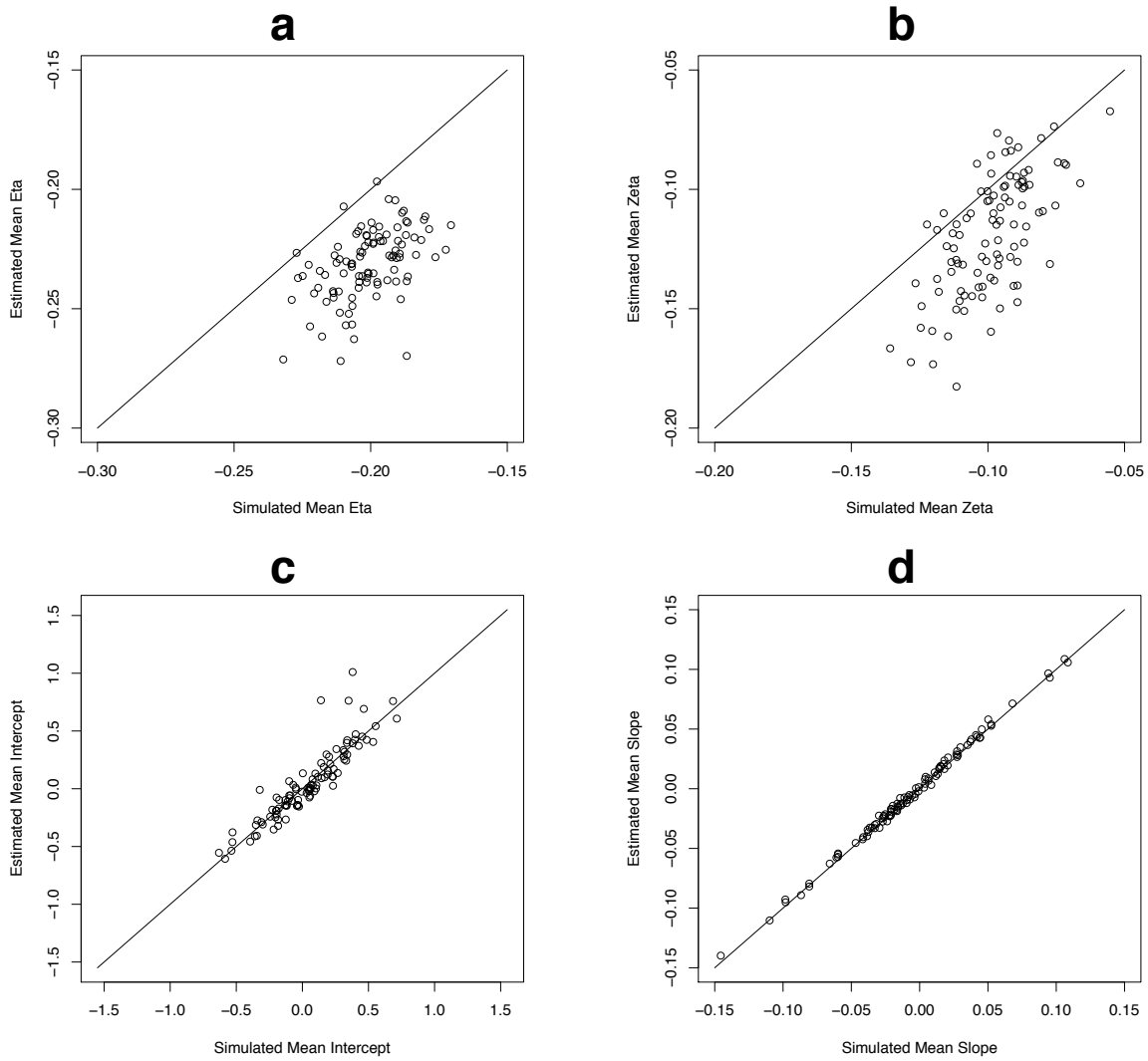
Simulation Results



Simulation Results

	Model D			
	True	(SD)	Est	(SD)
N	100		99	
Did Not Converge			0	
Extreme Outliers			1	
Eta Mean	-0.201	0.013	-0.231	0.015
Eta Variance	0.010	0.002	0.016	0.004
Zeta Mean	-0.099	0.014	-0.119	0.025
Zeta Variance	0.010	0.002	0.034	0.014
Eta Interaction				
Zeta Interaction				
Intercept Mean	0.037	0.281	0.042	0.312
Intercept Variance	3.845	0.787	4.863	3.417
Slope Mean	-0.007	0.044	-0.005	0.044
Slope Variance	0.088	0.018	0.088	0.017

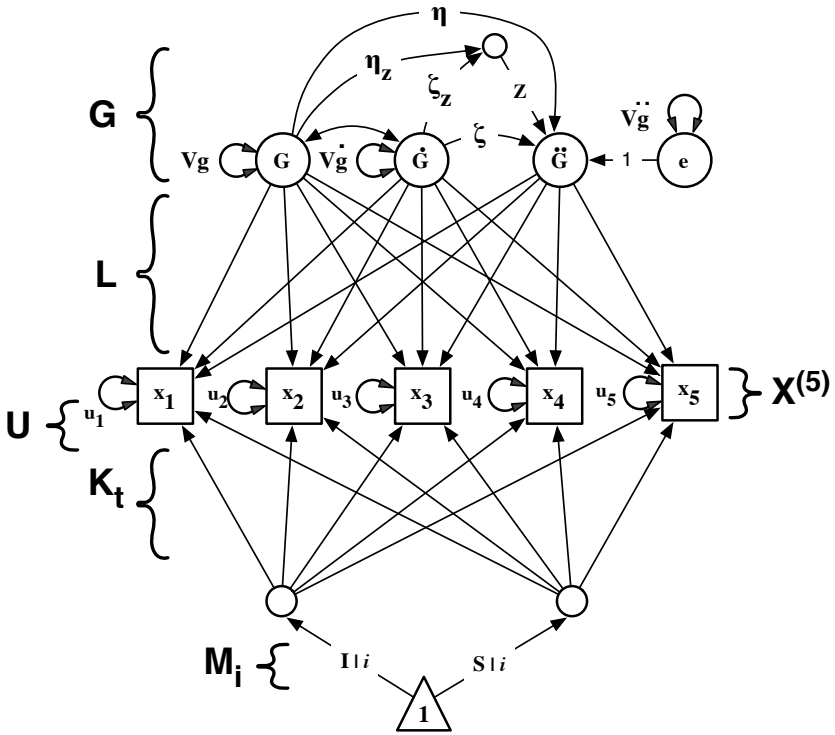
Simulation Results



Individual Differences in Dynamics

Go to ChangesInDynamics_AppendixD.R

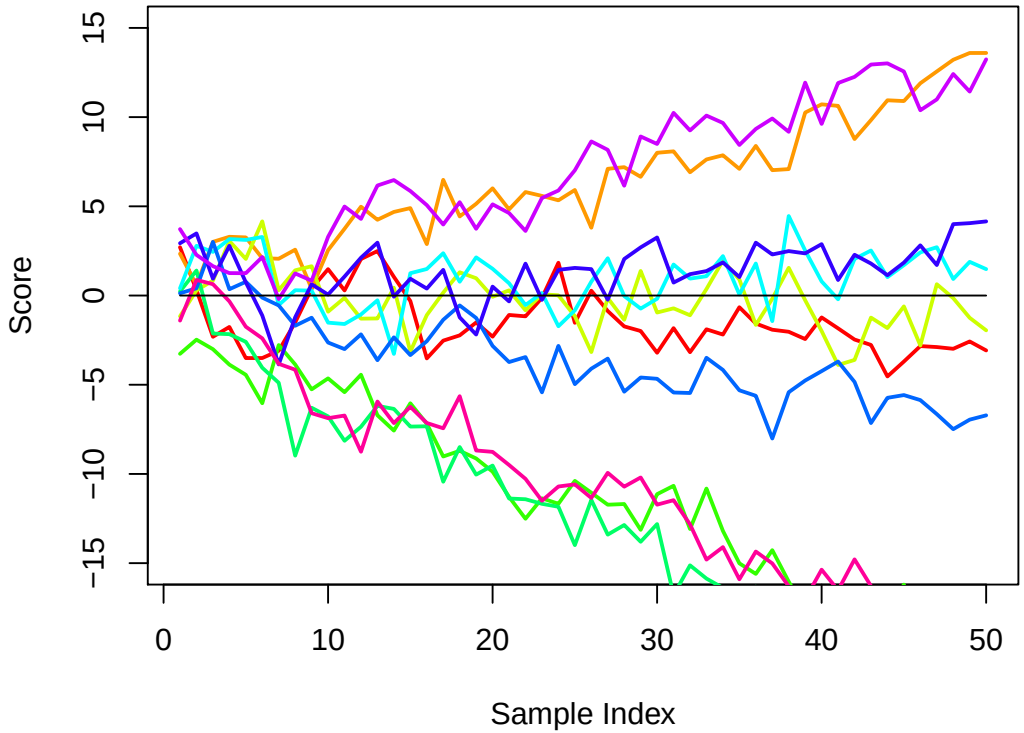
Second Level Predictors



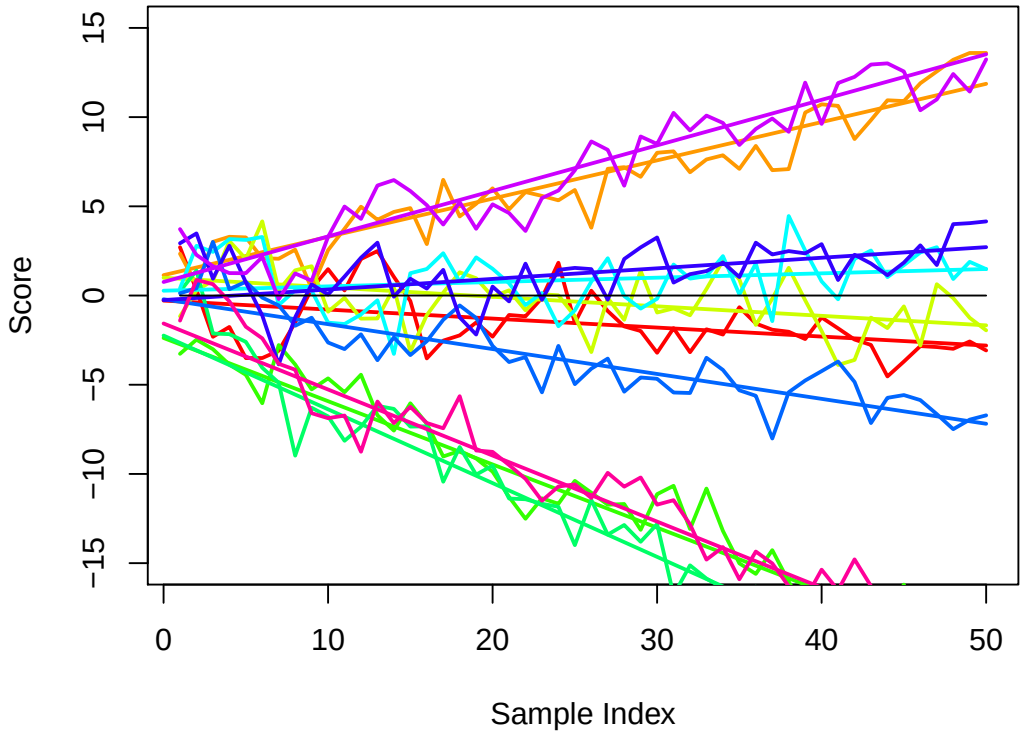
$$\begin{aligned} \ddot{g}_{ij} &= \eta_i g_{ij} + \zeta_i \dot{g}_{ij} + e_{ij} \\ \eta_i &= \eta + \eta_z z_i \\ \zeta_i &= \zeta + \zeta_z z_i \end{aligned} \tag{1}$$

$$\mathbf{X}^{(5)}_{ij} = \mathbf{M}_i + \mathbf{GL} + \mathbf{U} \tag{2}$$

Simulation Results



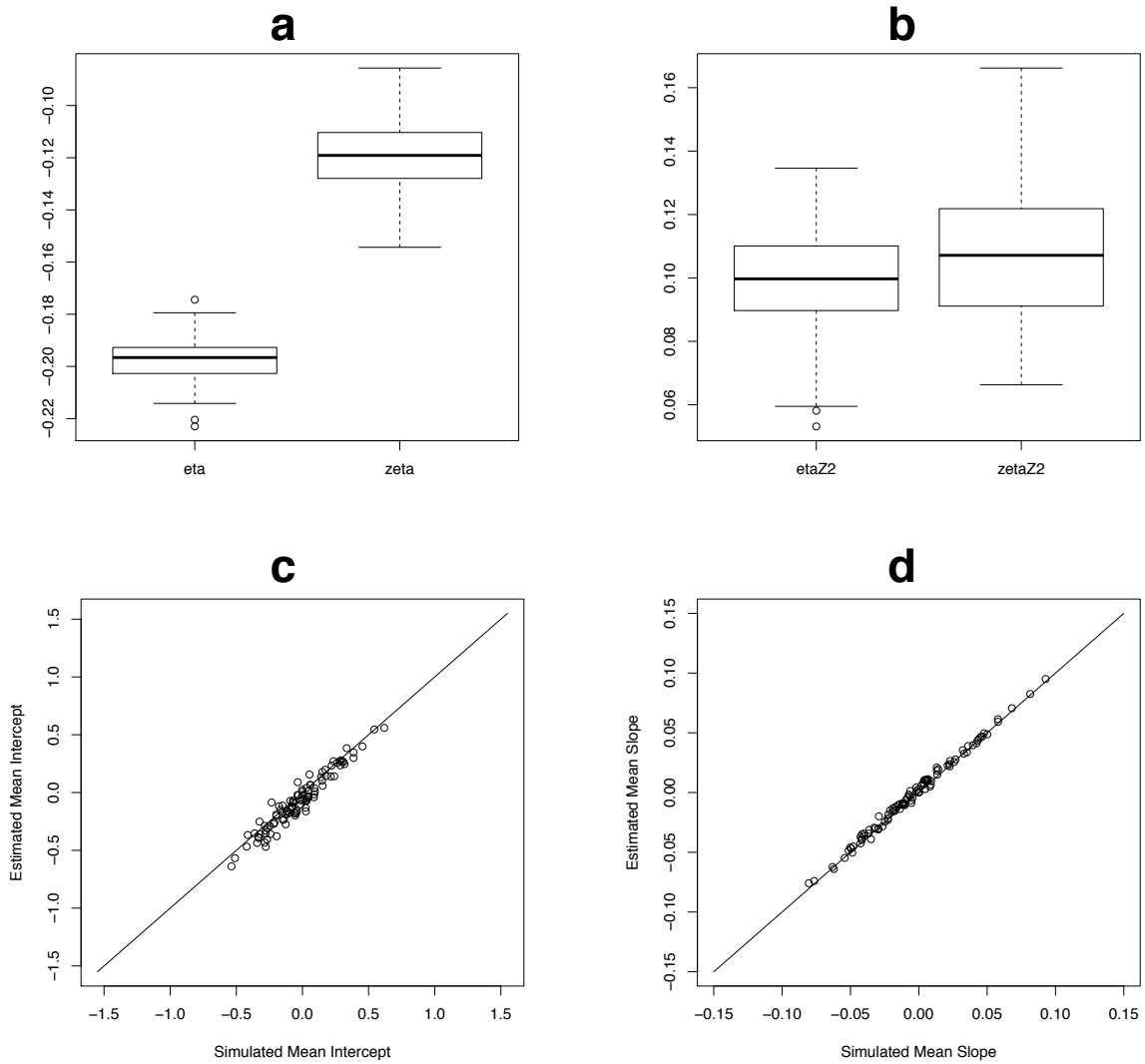
Simulation Results



Simulation Results

	Model E			
	True	(SD)	Est	(SD)
N	100		100	
Did Not Converge			0	
Extreme Outliers			0	
Eta Mean	-0.200	0.000	-0.198	0.008
Eta Variance				
Zeta Mean	-0.100	0.000	-0.119	0.014
Zeta Variance				
Eta Interaction	0.100	0.000	0.098	0.016
Zeta Interaction	0.100	0.000	0.108	0.020
Intercept Mean	-0.027	0.225	-0.073	0.238
Intercept Variance	3.892	0.779	4.089	0.787
Slope Mean	-0.004	0.034	-0.002	0.034
Slope Variance	0.088	0.018	0.087	0.017

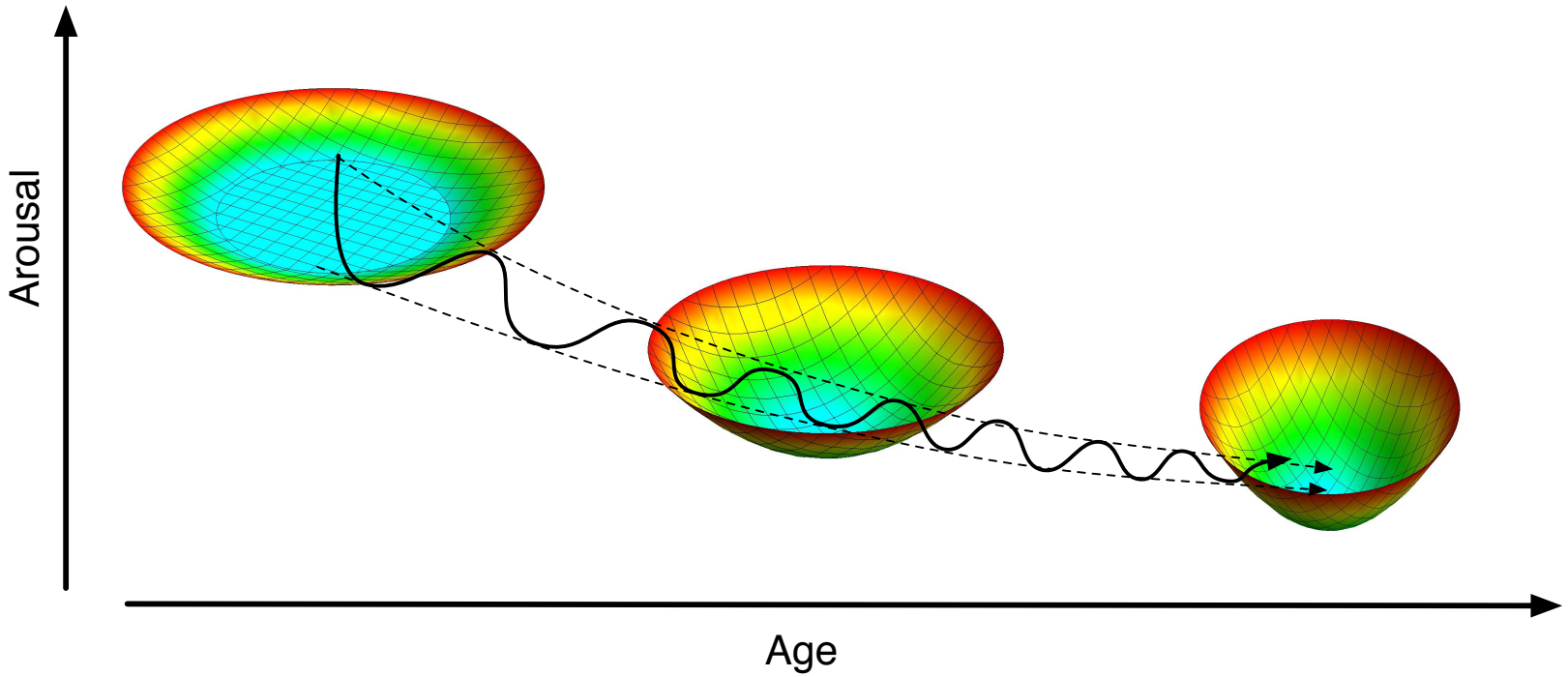
Simulation Results



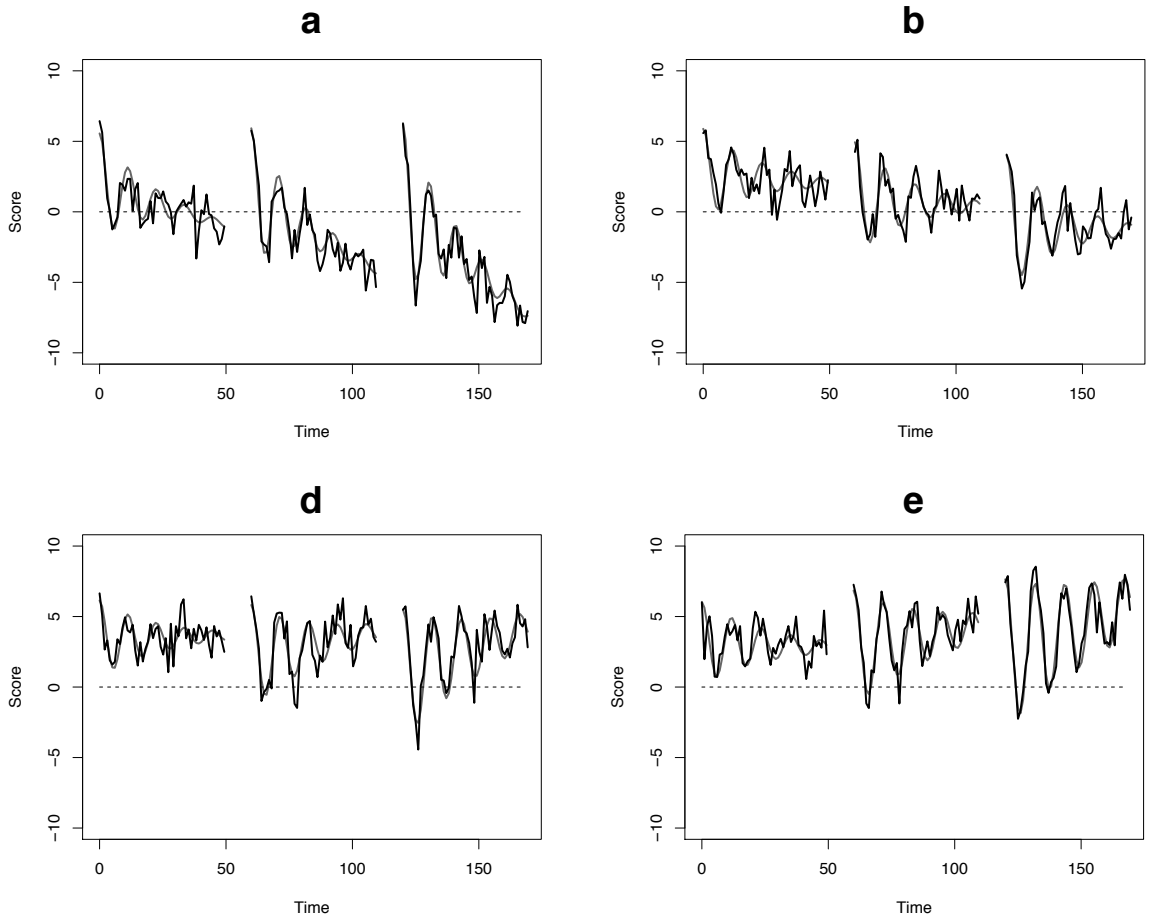
Second Level Predictors

Go to ChangesInDynamics_AppendixE.R

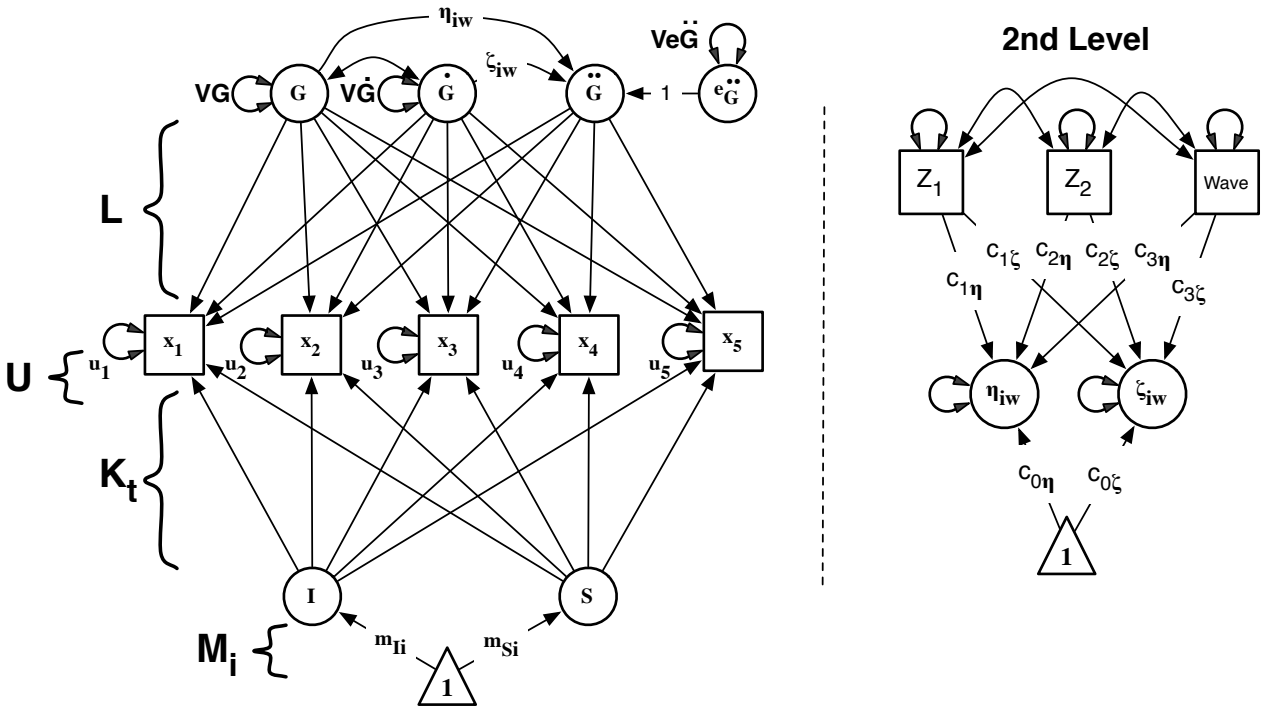
Longitudinal Change in Dynamics and Equilibrium



Longitudinal Change in Dynamics and Equilibrium



Longitudinal Change in Dynamics and Equilibrium



$$
\begin{aligned}
\ddot{G}_{ij} &= \eta_i G_{ij} + \zeta_i \dot{G}_{ij} + e_{ij} \\
\eta_i &= c_{0\eta} + c_{1\eta} Z_{1i} + c_{2\eta} Z_{2i} + c_{3\eta} Wave_i + \varepsilon_{i\eta} \\
\zeta_i &= c_{0\zeta} + c_{1\zeta} Z_{1i} + c_{2\zeta} Z_{2i} + c_{3\zeta} Wave_i + \varepsilon_{i\zeta} \\
\mathbf{X}_i^{(5)} &= \mathbf{M}_i \mathbf{K}_t + \mathbf{GL} + \mathbf{U}
\end{aligned}
$$

Summary

- Dynamics of change can be modeled as differential equations.
- Individual differences in equilibrium value must be taken into account.
- Equilibrium value may change over time within–individual.
- There may be individual differences in parameters of the dynamics of change.
- Change in dynamics can be modeled given longitudinal bursts of data.

Thank You

Boker, S. M., Staples, A., & Hu, Y. (2016). Dynamics of change and changes in dynamics. *Journal of Person-Oriented Research*, *2*(1–2), 34–55.