Windowed Cross Correlation

Steven M. Boker

Department of Psychology
University of Virginia

Dynamical Systems Analysis Workshop Part 7

Modern Modeling Methods
May 22, 2017

Outline

1. Stationarity and nonstationarity.
2. Variability of association.
3. Relaxing the assumption of stationarity.
4. Cross–correlation.
5. Windowed cross–correlation.
6. Peak picking.
7. Example application.

Human Processes

- Many human phenomena evolve over time.
  - Developmental processes (months to decades)
  - Psychological processes (seconds to days)
  - Psychophysical processes (milliseconds)
  - Biochemical processes (microseconds)
- In order to measure these processes we obtain many observations on one or more variables for one or more individuals.
- Most statistical techniques assume that any sample over time is a representative sample, i.e. the variables are stationary.

Human Processes

- But what if the organism can adapt to a changing environment?
  - Muscle growth in response to physical exercise.
  - Learning to perform a cognitive task.
  - Child's reactions to parental affect.
  - Posture and gesture during conversation.
- The assumption of stationarity may be violated.
  - That "violation" may be exactly the phenomenon of interest.

Stationarity

- When any equal–N contiguous subsample of a time–series gives estimates of the statistical properties of the full time–series that conform to some fixed probability distribution, the time–series is said to be stationary.
- Thus, one may start sampling a variable X at any moment in time and proceed for N measurements and one will derive the same statistical properties as if one started sampling at any other time.

Stationarity

- Benefits of the assumption of stationarity.
  - Your sample of occasions is representative of the whole behavior.
  - It doesn't matter when you start sampling.
  - Including more occasions of measurement in your analysis is always better since this leads to more reliable estimates of statistical properties.
  - You should fit models to all available occasions of measurement at once since this leads to more reliable parameter estimates.
  - If you split occasions of measurment into two halves, any difference in model parameter estimates is due to unreliability.

Nonstationarity

- When equal–N contiguous subsamples of a time–series give time dependent estimates of the statistical properties of the full time–series, it is said to be nonstationary.
- Consequences of nonstationarity.
  - Each sample of occasions *may not* be representative.
  - It *does* matter when you start sampling.
  - Including more occasions of measurement in your analysis *may not* be better since this leads to less sensitive estimates of changes in statistical properties.
  - If you split occasions of measurment into two halves, differences in model parameter estimates may be due to a combination of nonstationarity and unreliability.
- Nonstationarity may be reliable and should be estimated.

Variability of Association

- Variability of a variable.
- Association between variables.
- Association between variables separated by a time lag.
- Variability in association between variables.
  - Variation in strength of association for a time lag.
  - Variation in strength of maximum association.
  - Variation in time lag of maximum association.

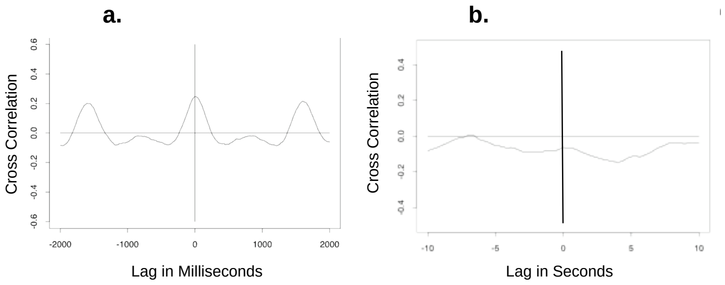
Cross–correlation

Suppose we wish to cross–correlate two time series each containing N observations $X = \{x_1, x_2, x_3, \ldots x_N\}$ and $Y = \{y_1, y_2, y_3, \ldots y_N\}$. If we assume stationarity and choose a time lag τ, the cross–correlation between X at time t and Y at time $t + \tau$ is a function r of X, Y, t and τ that can be defined as

$$r(X, Y, t, \tau) = \frac{1}{N - \tau} \sum_{i=1}^{N-\tau} \frac{(x_i - \bar{X})(y_{i+\tau} - \bar{Y})}{\mathrm{sd}(X)\mathrm{sd}(Y)} ,$$

where $\bar{X}$ and $\bar{Y}$ are the sample means and $\mathrm{sd}(X)$ and $\mathrm{sd}(Y)$ are the sample standard deviations of X and Y respectively.

Crosscorrelation and Dyadic Coordination



Overall lagged cross–correlations for (a) whole body movements of two individuals dancing during a 40 second trial and (b) head movements of two individuals during a 5 minute dyadic conversation

Crosscorrelation and Dyadic Coordination

- High cross–correlations between individuals' average body movements during dance.
  - People coordinate their movements during dance.
- Low cross–correlations between individuals' head motions during conversations.
  - People don't coordinate their movements during conversation.
- Or perhaps conversation involves nonstationarity?

Measuring Nonstationarity

- Give estimates of both strength and time lag of maximum association between variables.
- Track changes in these two estimates over the duration of the experiment.
- Be flexible in the tradeoff between reliability of estimates and sensitivity to change.
- Exclude time scales that are either too short or too long to be of interest.

Windowed Cross–Correlation (WCC)

Given two data vectors each with N observations, $\mathbf{X} = \{x_1, x_2, x_3 \ldots x_N\}$ and $\mathbf{Y} = \{y_1, y_2, y_3 \ldots y_N\}$, a window size w_{max}, a time lag τ on the interval $-\tau_{max} \leq \tau \leq \tau_{max}$ and an elapsed time t from the beginning of the time series, a window can be drawn from each time series Wx and Wy as follows

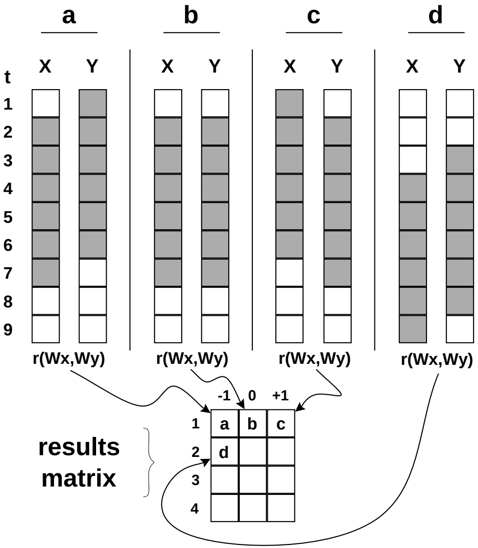
$$
\begin{aligned}
Wx &= \left\{ \begin{array}{ll} \{x_t, x_{t+1}, \ldots x_{t+w_{max}}\} & \text{if } \tau \leq 0 \\ \{x_{t-\tau}, x_{t+1-\tau}, \ldots x_{t+w_{max}-\tau}\} & \text{if } \tau > 0 \end{array} \right\} \\
Wy &= \left\{ \begin{array}{ll} \{y_{t+\tau}, y_{t+1+\tau}, \ldots y_{t+w_{max}+\tau}\} & \text{if } \tau \leq 0 \\ \{y_t, y_{t+1}, \ldots y_{t+w_{max}}\} & \text{if } \tau > 0 \end{array} \right\} .
\end{aligned}
$$

Windowed Cross–Correlation (WCC)

Now the cross–correlation between the windows Wx and Wy can be defined as

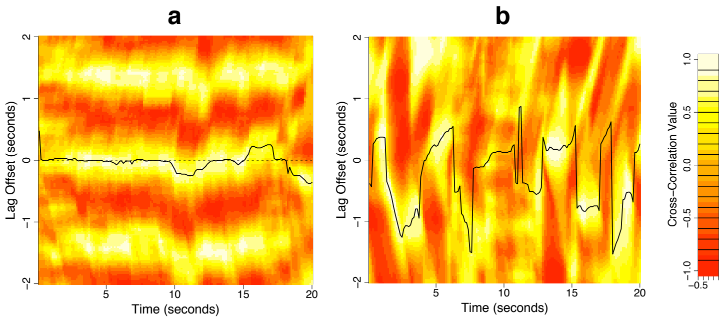
$$r(Wx, Wy) = \frac{1}{w_{max}} \sum_{i=1}^{w_{max}} \frac{(Wx_i - \overline{Wx})(Wy_i - \overline{Wy})}{\mathrm{sd}(Wx)\mathrm{sd}(Wy)} ,$$

where $\overline{Wx}$ and $\overline{Wy}$ are the means and $\mathrm{sd}(Wx)$ and $\mathrm{sd}(Wy)$ are the standard deviations of the windows Wx and Wy respectively.


a
b
c
d
t
X
Y
1
2
3
4
5
6
7
8
9
r(Wx,Wy)
-1
0
+1
results matrix

Windowed Cross–Correlation Parameters

1. w_{max} is window size.
2. w_{inc} is increment between rows of results matrix.
3. τ_{max} is maximum lag value.
4. τ_{inc} is increment between columns of results matrix.



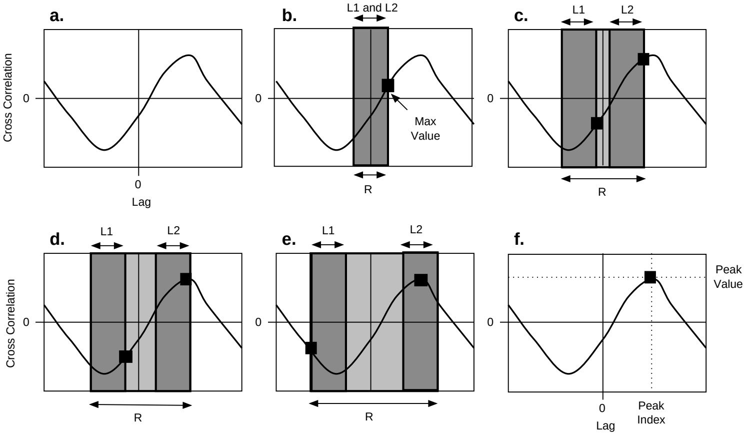
Density plots of WCC from (a) body velocities in dance, and (b) from head velocity in conversation.

1. Dance data exhibits stationarity.
2. Conversation data exhibits nonstationarity but still has a discernable pattern.

Peak Picking

1. A search region is defined centered on a lag of zero.
2. The max value is located in the current search region.
3. A local region of size L_{size} is defined within the search region and containing the maximum value.
4. If the local region can be centered on the maximum value and values within the local region are monotonically decreasing on each side of the maximum, then a peak has been found.
5. If no peak, the search region is increased in size and the process is repeated from step 1.

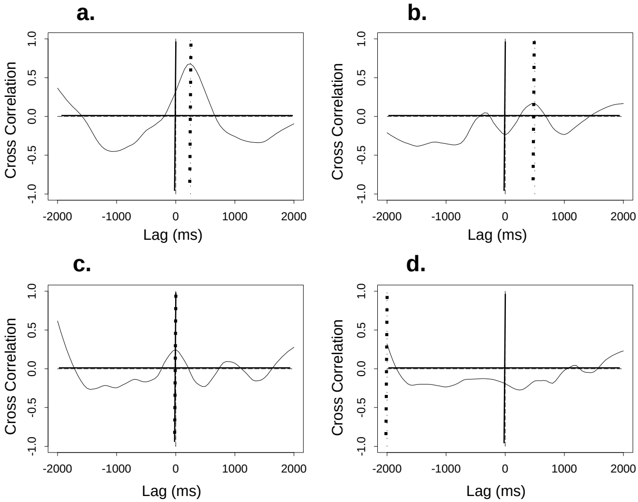
Peak Picking



Peak Picking Problems

- Four types of problems in finding a peak
  1. High frequency noise must be smoothed
  2. Peak might be a local maximum.
  3. Peak might be local maximum of opposite sign.
  4. Might be no peak.

Peak Picking Problems



Example Application: Dyadic Conversation

- Face–to–face conversation includes coordinated head movements.
- Dominance and gender have been hypothesized to have a role in these movements.
- We motion tracked head movements in 128 dyadic conversations.
- Participants were selected by gender and for low and high dominance scores on a personality measure.
- 4 participants in a quad: HF, HM, LF, LM

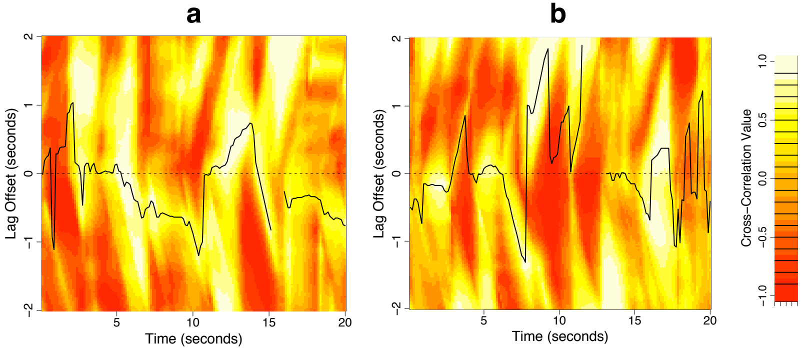
Methods

- Each participant in two 7 minute conversations.
- High dominant participants interviewed low dominant participants for a job.
- Middle 320 minutes of conversation were broken into 20 second segments.
- Angular velocity of head sensor was calculated for horizontal and vertical movement.
- WCC and peak picking was run on each segment.
- Data were analyzed using a multilevel model grouping by quad.

Multilevel Model

$$
\begin{aligned}
y_{ij} &= b_{i0} + b_1 x_{ij} + b_2 z_{ij} + e_{ij} \\
b_{i0} &= c_0 + u_{i0}
\end{aligned}
$$

where y_{ij} represents the selected outcome variable (mean peak WCC, mean peak lag, variance of peak lag, or derivative with respect to time of the peak lag) for quad i in conversation section j. The predictor variables x_{ij} and z_{ij} are the sex of the high–dominant person and low–dominant person respectively coded as male=1 and female=0. The intercept term b_{i0} was composed of an overall fixed effect c_0 and a component u_{i0} unique to quad i.



Plots of 20 seconds of windowed cross–correlation (WCC) of head vertical angular velocity (head nod) from (a) a high–dominant female speaking with a low–dominant female and (b) the same high–dominant female speaking with a low–dominant male

Mean Peak WCC

	AIC	BIC
Vertical	-2897.113	-2868.997
Horizontal	-3000.470	-2972.355

	Value	SE	DF	t	p
Vertical					
Intercept	0.5440	0.00545	2014	99.73	< 0.0001
Male High	0.0154	0.00519	2014	2.96	0.0031
Male Low	0.0051	0.00519	2014	0.99	0.3215
Horizontal					
Intercept	0.5497	0.00521	2014	105.45	< 0.0001
Male High	0.0123	0.00507	2014	2.43	0.0150
Male Low	0.0009	0.00507	2014	0.17	0.8662

Observations=2048, Groups=32

Mean Lag of Peak WCC

	AIC	BIC
Vertical	-3785.686	-3757.570
Horizontal	-3807.876	-3779.76

	Value	SE	DF	*t*	*p*
Vertical					
Intercept	0.2152	0.00482	2014	44.69	< 0.0001
Male High	0.0013	0.00417	2014	0.30	0.7632
Male Low	0.0023	0.00417	2014	0.56	0.5742
Horizontal					
Intercept	0.2190	0.00428	2014	51.11	< 0.0001
Male High	-0.0047	0.00416	2014	-1.14	0.2564
Male Low	0.0010	0.00416	2014	0.24	0.8109

Observations=2048, Groups=32

Variability of Lag of Peak WCC

	AIC	BIC
Vertical	33752.58	33780.69
Horizontal	33868.88	33897.00

	Value	SE	DF	t	p
Vertical					
Intercept	2989.0	41.47	2014	72.08	< 0.0001
Male High	138.4	40.49	2014	3.42	0.0006
Male Low	128.4	40.49	2014	3.17	0.0015
Horizontal					
Intercept	3055.8	38.41	2014	79.57	< 0.0001
Male High	114.5	41.81	2014	2.74	0.0062
Male Low	64.9	41.81	2014	1.55	0.1209

Observations=2048, Groups=32

Derivative of Lag of Peak WCC

	AIC	BIC
Vertical	146.2651	160.4067
Horizontal	126.2958	140.4374

	Value	SE	DF	*t*	*p*
Vertical					
Intercept	0.1094	0.07076	94	1.55	0.1255
Male High	0.0469	0.06019	94	0.78	0.4381
Male Low	0.0313	0.06019	94	0.52	0.6049
Horizontal					
Intercept	0.1309	0.06105	94	2.14	0.0347
Male High	-0.0586	0.05836	94	-1.00	0.3180
Male Low	-0.0117	0.05836	94	-0.20	0.8413

Observations=2048, Groups=32

Conclusions

- The presence of a high–dominant male in the conversation led to a small ($r = 0.01$) but reliable increase in the coordination between participants' head movements.
- On average, low dominant individuals tended to be in the lead by a small (3ms) interval of time. High variability in the lag of the peak WCC ($\sigma \approx$ 350ms) indicated a high degree of nonstationarity in the cross–correlation between participants' head angular velocities.
- The degree of variability in the lag for head nods and for head shakes was statistically indistinguishable from one another.

Conclusions

- The degree of variability in the lag of peak WCC for head nods was significantly predicted by the sex of both high–dominant and low–dominant participants such that for each male present in the conversation, the variability increased.
- The degree of variability in the lag for head shakes significantly increased in the presence of a high–dominant male.
- The median first derivative with respect to time for the time lag of horizontal angular WCC was significantly larger than zero, indicating that slow changes in lag tended to start when the high–dominant participant was leading while fast changes in lag tended to start when the low–dominant participant was leading.

Thank You

Boker, S. M., Xu, M., Rotondo, J. L., & King, K. (2002). Windowed cross–correlation and peak picking for the analysis of variability in the association between behavioral time series. *Psychological Methods*, *7*(1), 338–355.