Coupled Latent Differential Equations

Steven M. Boker

Department of Psychology
University of Virginia

Dynamical Systems Analysis Workshop Part 6

Modern Modeling Methods
May 22, 2017

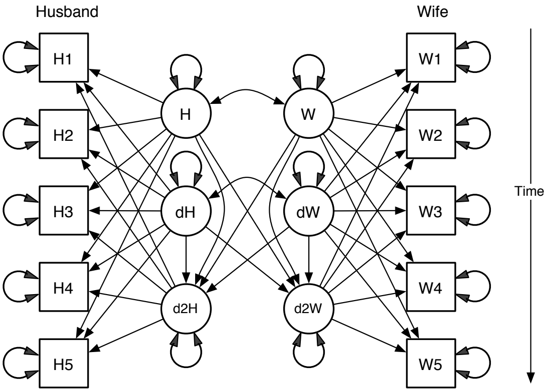
Asymmetric Direct Coupled Differential Equations

One form of coupled oscillator equations involves asymmetric direct coupling,

$$\begin{aligned}\ddot{x}_{ij} &= \eta_{ix} x_{ij} + \zeta_{ix} \dot{x}_{ij} + \eta_{ix2} y_{ij} + \zeta_{ix2} \dot{y}_{ij} + e_{ij} \\ \ddot{y}_{ij} &= \eta_{iy} y_{ij} + \zeta_{iy} \dot{y}_{ij} + \eta_{iy2} x_{ij} + \zeta_{iy2} \dot{x}_{ij} + f_{ij}\end{aligned}$$

where x_{ij} and y_{ij} are the ith person's scores at the jth occasion and $\dot{x}_{ij}$ and $\ddot{x}_{ij}$ are the first and second derivatives of x with respect to time.

Direct Coupled LDE Model



Coupled Latent Differential Equations Model with 5 occasions and asymmetric direct coupling.

Problems with Direct Coupling

- Recently, Angela Staples has demonstrated that asymmetric direct coupled oscillators can be empirically unidentified.
- One solution to this problem is to set upper and lower bounds on the coupling parameters, but this does not guarantee an identified solution.
- Another way to deal with this is to add a proportionality constraint.

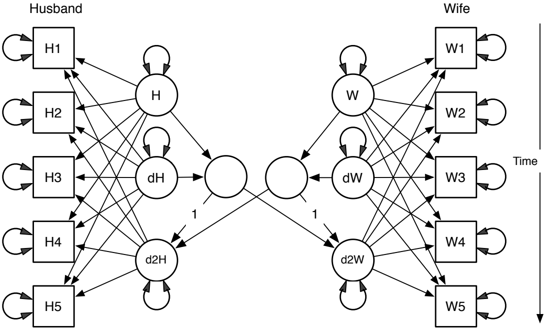
Proportionally Coupled Differential Equations

A more restrictive form of coupling is asymmetric proportional coupling,

$$\begin{aligned} \ddot{x}_{ij} &= \eta_{ix}x_{ij} + \zeta_{ix}\dot{x}_{ij} + \gamma_{xi}(\eta_{iy}y_{ij} + \zeta_{iy}\dot{y}_{ij}) + e_{ij} \\ \ddot{y}_{ij} &= \eta_{iy}y_{ij} + \zeta_{iy}\dot{y}_{ij} + \gamma_{yi}(\eta_{ix}x_{ij} + \zeta_{ix}\dot{x}_{ij}) + f_{ij} \end{aligned}$$

where x_{ij} and y_{ij} are the ith person's scores at the jth occasion and $\dot{x}_{ij}$ and $\ddot{x}_{ij}$ are the first and second derivatives of x with respect to time.

Proportionally Coupled LDE Model



Coupled Latent Differential Equations Model with 5 occasions and asymetric linear coupling.

State Space Embedding for LDE Model

- Suppose 3 couples were measured on 8 occasions.

$$\begin{bmatrix}
h_{11} & h_{12} & h_{13} & h_{14} & h_{15} & w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\
h_{12} & h_{13} & h_{14} & h_{15} & h_{16} & w_{12} & w_{13} & w_{14} & w_{15} & w_{16} \\
h_{13} & h_{14} & h_{15} & h_{16} & h_{17} & w_{13} & w_{14} & w_{15} & w_{16} & w_{17} \\
h_{14} & h_{15} & h_{16} & h_{17} & h_{18} & w_{14} & w_{15} & w_{16} & w_{17} & w_{18} \\
h_{21} & h_{22} & h_{23} & h_{24} & h_{25} & w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\
h_{22} & h_{23} & h_{24} & h_{25} & h_{26} & w_{12} & w_{13} & w_{14} & w_{15} & w_{16} \\
h_{23} & h_{24} & h_{25} & h_{26} & h_{27} & w_{13} & w_{14} & w_{15} & w_{16} & w_{17} \\
h_{24} & h_{25} & h_{26} & h_{27} & h_{28} & w_{14} & w_{15} & w_{16} & w_{17} & w_{18} \\
h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\
h_{32} & h_{33} & h_{34} & h_{35} & h_{36} & w_{12} & w_{13} & w_{14} & w_{15} & w_{16} \\
h_{33} & h_{34} & h_{35} & h_{36} & h_{37} & w_{13} & w_{14} & w_{15} & w_{16} & w_{17} \\
h_{34} & h_{35} & h_{36} & h_{37} & h_{38} & w_{14} & w_{15} & w_{16} & w_{17} & w_{18}
\end{bmatrix}$$

Second Order Coupled Loading Matrix

$$
\mathbf{L} = \begin{bmatrix}
1 & -2\Delta t & (-2\Delta t)^2/2 & 0 & 0 & 0 \\
1 & -1\Delta t & (-1\Delta t)^2/2 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1\Delta t & (1\Delta t)^2/2 & 0 & 0 & 0 \\
1 & 2\Delta t & (2\Delta t)^2/2 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -2\Delta t & (-2\Delta t)^2/2 \\
0 & 0 & 0 & 1 & -1\Delta t & (-1\Delta t)^2/2 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1\Delta t & (1\Delta t)^2/2 \\
0 & 0 & 0 & 1 & 2\Delta t & (2\Delta t)^2/2
\end{bmatrix}
$$

Structural Matrices (Asymmetric Direct)

$$
\mathbf{A} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\eta_H & \zeta_H & 0 & \gamma_W & \gamma_{dW} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
\gamma_H & \gamma_{dH} & 0 & \eta_W & \zeta_W & 0
\end{bmatrix}
$$

$$
\mathbf{S} = \begin{bmatrix}
V_H & C_{HdH} & 0 & C_{HW} & C_{dHW} & 0 \\
C_{HdH} & V_{dH} & 0 & C_{HdW} & C_{dHdW} & 0 \\
0 & 0 & V_{d2H} & 0 & 0 & 0 \\
C_{HW} & C_{dHW} & 0 & V_W & C_{WdW} & 0 \\
C_{HdW} & C_{dHdW} & 0 & C_{WdW} & V_{dW} & 0 \\
0 & 0 & 0 & 0 & 0 & V_{d2W}
\end{bmatrix}
$$

$$
\mathbf{R} = \mathbf{L}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{S}(\mathbf{I} - \mathbf{A})^{-1'}\mathbf{L}' + \mathbf{U}^2
$$

Coupled Univariate LDE (Asymmetric Direct)

Go to CoupledLDEUnivariateExample.R

Structural Matrices (Asymmetric Proportional)

$$
\mathbf{A} = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & \gamma_W \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \gamma_H & 1 \\
\eta_H & \zeta_H & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & \eta_W & \zeta_W & 0 & 0 & 0
\end{bmatrix}
$$

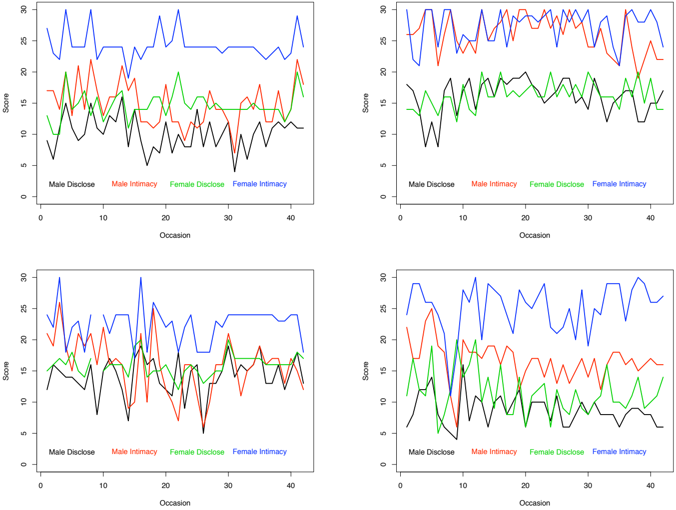
$$
\mathbf{S} = \begin{bmatrix}
V_H & C_{HdH} & 0 & C_{HW} & C_{dHW} & 0 & 0 & 0 \\
C_{HdH} & V_{dH} & 0 & C_{HdW} & C_{dHdW} & 0 & 0 & 0 \\
0 & 0 & V_{d2H} & 0 & 0 & 0 & 0 & 0 \\
C_{HW} & C_{dHW} & 0 & V_W & C_{WdW} & 0 & 0 & 0 \\
C_{HdW} & C_{dHdW} & 0 & C_{WdW} & V_{dW} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & V_{d2W} & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
\mathbf{R} = \mathbf{L}(\mathbf{I}-\mathbf{A})^{-1}\mathbf{S}(\mathbf{I}-\mathbf{A})^{-1'}\mathbf{L}' + \mathbf{U}^2
$$

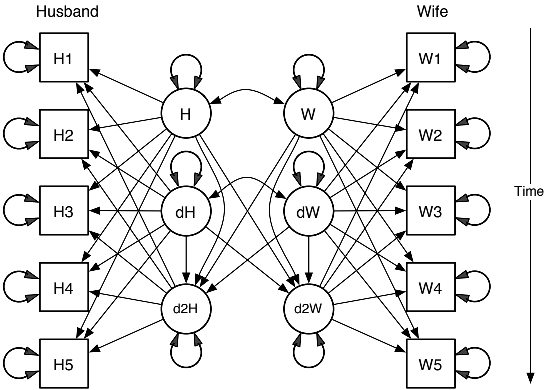
Coupled Univariate LDE (Asymmetric Proportional)

Go to CoupledPropLDEUnivariateExample.R

Intimacy and Disclosure for Four Couples



Coupled LDE Model



Coupled Latent Differential Equations Model with 5 occasions and asymetric linear coupling.

State Space Embedding for LDE Model

- Suppose 3 couples were measured on 8 occasions.

$$
\begin{bmatrix}
h_{11} & h_{12} & h_{13} & h_{14} & h_{15} & w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\
h_{12} & h_{13} & h_{14} & h_{15} & h_{16} & w_{12} & w_{13} & w_{14} & w_{15} & w_{16} \\
h_{13} & h_{14} & h_{15} & h_{16} & h_{17} & w_{13} & w_{14} & w_{15} & w_{16} & w_{17} \\
h_{14} & h_{15} & h_{16} & h_{17} & h_{18} & w_{14} & w_{15} & w_{16} & w_{17} & w_{18} \\
h_{21} & h_{22} & h_{23} & h_{24} & h_{25} & w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\
h_{22} & h_{23} & h_{24} & h_{25} & h_{26} & w_{12} & w_{13} & w_{14} & w_{15} & w_{16} \\
h_{23} & h_{24} & h_{25} & h_{26} & h_{27} & w_{13} & w_{14} & w_{15} & w_{16} & w_{17} \\
h_{24} & h_{25} & h_{26} & h_{27} & h_{28} & w_{14} & w_{15} & w_{16} & w_{17} & w_{18} \\
h_{31} & h_{32} & h_{33} & h_{34} & h_{35} & w_{11} & w_{12} & w_{13} & w_{14} & w_{15} \\
h_{32} & h_{33} & h_{34} & h_{35} & h_{36} & w_{12} & w_{13} & w_{14} & w_{15} & w_{16} \\
h_{33} & h_{34} & h_{35} & h_{36} & h_{37} & w_{13} & w_{14} & w_{15} & w_{16} & w_{17} \\
h_{34} & h_{35} & h_{36} & h_{37} & h_{38} & w_{14} & w_{15} & w_{16} & w_{17} & w_{18}
\end{bmatrix}
$$

Fixed Loading Matrix

$$
\mathbf{L} = \begin{bmatrix}
1 & -2\Delta t & (-2\Delta t)^2/2 & 0 & 0 & 0 \\
1 & -1\Delta t & (-1\Delta t)^2/2 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & 1\Delta t & (1\Delta t)^2/2 & 0 & 0 & 0 \\
1 & 2\Delta t & (2\Delta t)^2/2 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & -2\Delta t & (-2\Delta t)^2/2 \\
0 & 0 & 0 & 1 & -1\Delta t & (-1\Delta t)^2/2 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1\Delta t & (1\Delta t)^2/2 \\
0 & 0 & 0 & 1 & 2\Delta t & (2\Delta t)^2/2
\end{bmatrix}
$$

Structural Matrices

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \eta_H & \zeta_H & 0 & \gamma_W & \gamma_{dW} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \gamma_H & \gamma_{dH} & 0 & \eta_W & \zeta_W & 0 \end{bmatrix}$$

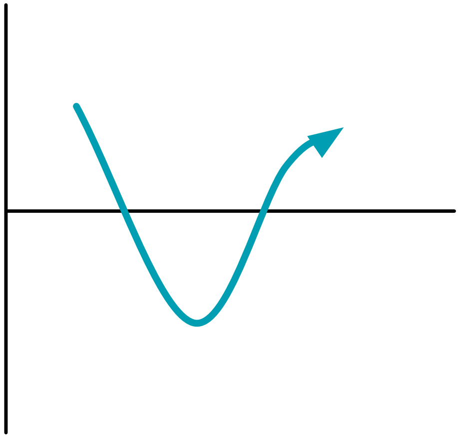
$$\mathbf{S} = \begin{bmatrix} V_H & C_{HdH} & 0 & C_{HW} & C_{dHW} & 0 \\ C_{HdH} & V_{dH} & 0 & C_{HdW} & C_{dHdW} & 0 \\ 0 & 0 & V_{d2H} & 0 & 0 & 0 \\ C_{HW} & C_{dHW} & 0 & V_W & C_{WdW} & 0 \\ C_{HdW} & C_{dHdW} & 0 & C_{WdW} & V_{dW} & 0 \\ 0 & 0 & 0 & 0 & 0 & V_{d2W} \end{bmatrix}$$

$$\mathbf{R} = \mathbf{L}(\mathbf{I} - \mathbf{A})^{-1}\mathbf{S}(\mathbf{I} - \mathbf{A})^{-1'}\mathbf{L}' + \mathbf{U}^2$$

Husbands' and Wives' Disclosure

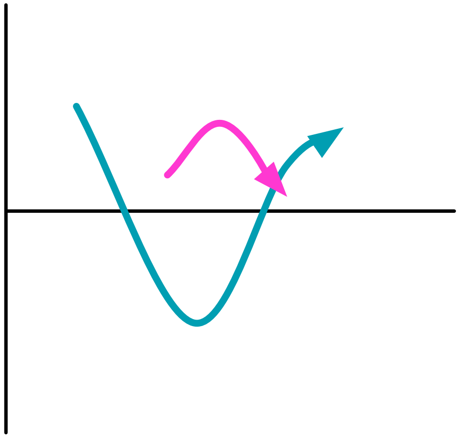
	Wife	
$W \rightarrow d2W$	-1.362	(5.4 days)
$dW \rightarrow d2W$	-.051	
$H \rightarrow d2W$	.423	
$dH \rightarrow d2W$	.127	
	Husband	
$H \rightarrow d2H$	-.928	(6.5 days)
$dH \rightarrow d2H$	-.134	
$W \rightarrow d2H$	.199	
$dW \rightarrow d2H$	.469	
DOF	1831	
$-2LL$	9081	

Moderation–like Effect of Coupling



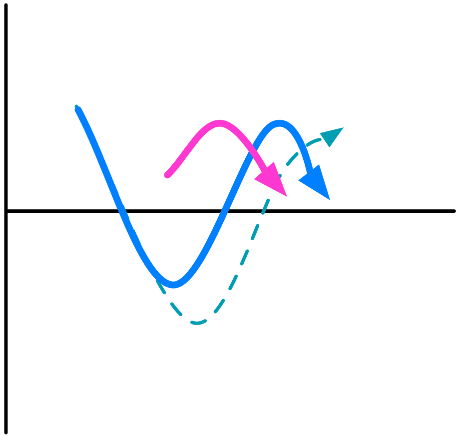
Solo self–regulation.

Moderation–like Effect of Coupling



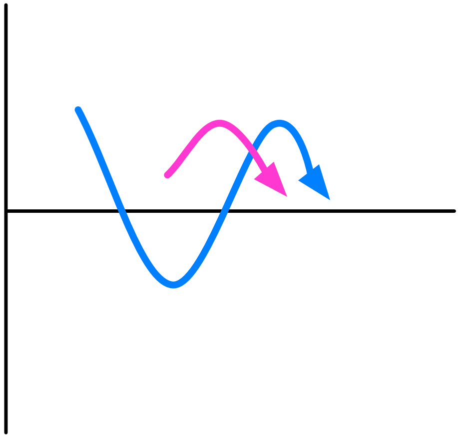
Add in the spouse's trajectory.

Moderation–like Effect of Coupling



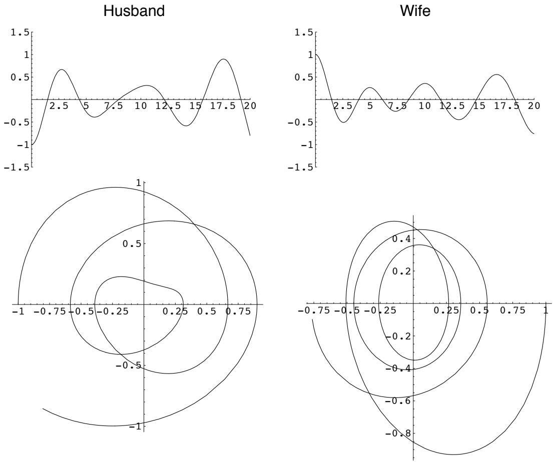
Self–regulation with positive coupling.

Moderation–like Effect of Coupling



Self–regulation with positive coupling.

Disclosure System Integrated



Conclusions from Coupled LDE

- Wives' frequency was faster than Husbands' frequency.
- Wives' Disclosure was positively coupled mostly to Husbands' Disclosure.
- Husbands' Disclosure was positively coupled mostly to rate of change in Wives' Disclosure.
- If Husbands and wives start 180 degrees out of phase, the model predicts it would take 10 to 12 days to phase synchronize in the absence of external influences.