Intra and Interindividual Variation Modeling: Bayesian Mixed-Effects Nonstationary Latent Differential Equation Model

Mauricio Garnier-Villarreal [1] Pascal R. Deboeck [2] David K. Johnson [3] Amber Watts [3]

[1] Marquette University

[2]The University of Utah

[3]University of Kansas

May 23, 2017

Outline

Introduction

- Longitudinal analysis is a gold standard method in psychology for studying change over time (McArdle & Nesselroade, 2014).
- The nomothetic approach looks for the overall population's true processes, whereas the idiographic approach seeks to understand the unique individual process (Maxwell & Boker, 2007; Molenaar, 2004).
- Psychological research can only be considered complete until it includes the idiographic analysis as well as the nomothetic analysis (Molenaar, 2004).

Continuous Time and Differential Equations

- The problem of studying longitudinal processes is that researchers can only measure at discrete moments in time.
- Continuous-time assumes that the variable exists in an infinite number of time points from which the measured time points are a finite set.
- A common way to handle time as continuous with discrete-time measures is with the use of differential equations (DE).
- Continuous-time models intend to estimate the data-generating process (Voelkle, Oud, Davidov, & Schmidt, 2012). The generating process explains the underlying continuous process that was measured finitely.
- Differential Equation is any equation where the variable is a derivative, a derivative represents the change in a variable with respect to other variables.
- The implementation within the framework of analyzing time series and modeling intraindividual variability, the derivatives will represent change in a variable x with respect to change in time (Deboeck, Boker, & Bergeman, 2008).

Mixed-Effects Differential Equations

- Appropriate to estimate non-independent data, including the estimation of random effects, taking into account the between-subject (interindividual) variability
- Allows researchers to combine the nomothetic and idiographic approaches; it also allows researchers to intensively examine the dynamics of a single individual and to generalize the dynamics to a population of individuals (Maxwell & Boker, 2007).
- Mixed-effects DE models are commonly apply in fields of biological research (Tornoe, Agerso, Jonsson, Madsen, & Nielsen, 2004; Wang et al., 2014).
- Mixed-effects DE models properly estimate continuous-time dynamics while taking into account interindividual variability.

Differential Equations Modeling in Psychology

- DE are not commonly apply in psychological research.
- There are two models with analytical solution (Oud, 2007; Oravecz, Tuerlinckx, & Vandekerckhove, 2011).
- Also, has been presented a method to approximate the analytical solution with a clear intention to make these kinds of models available to psychologists, Latent Differential Equations (LDE), from a known framework as SEM (S. Boker, Neale, & Rausch, 2004).

Damped Linear Oscillator (DLO)

Taken from S. M. Boker and Nesselroade (2002)

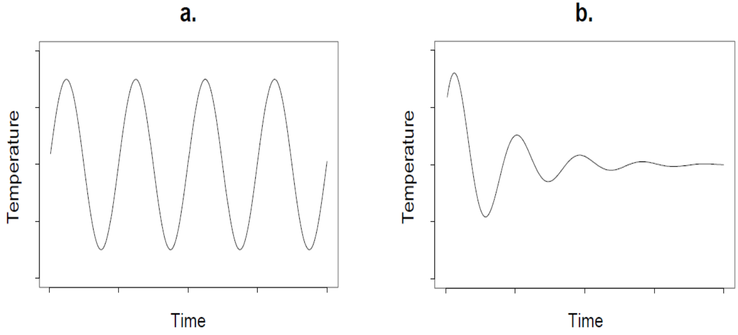


Figure 1

Idealized Plot of Temperature Over Time for Two Simple Thermostats
(a) A thermostat that only responds to difference from the desired equilibrium temperature. (b) A thermostat that responds to both difference from equilibrium and rate of change in temperature.

Damped Linear Oscillator

$$\ddot{x} = \eta x + \zeta \dot{x} + e \tag{1}$$

- The zeroth derivative represents the displacement of the variable x from its equilibrium. The first derivative represents the velocity of change. The second derivative represents the acceleration of change (Hu, Boker, Neale, & Klump, 2014).
- Parameter η is related to the frequency where $\eta < 0$ and $\eta + \zeta^2/4 < 0$. The period of the oscillation will be $\lambda = 2\pi/\sqrt{-(\eta + \zeta^2/4)}$. An approximation of the frequency, when ζ is equal to zero, will be $\omega = \frac{1}{2\pi}\sqrt{-\eta}$. ζ is the damping of an oscillating system.
- The damped linear oscillator (DLO) is a simple model to account for self-regulating systems that have a stationary equilibrium point.

Latent Differential Equations (LDE)

- LDE approximates the derivatives as latent constructs in the framework of an SEM, and it follows the same structure as a latent growth model (S. Boker et al., 2004).

$$L = \begin{bmatrix} 1 & -2\Delta\tau & (-2\Delta\tau)^2/2 \\ 1 & -1\Delta\tau & (-1\Delta\tau)^2/2 \\ 1 & 0 & 0 \\ 1 & 1\Delta\tau & (1\Delta\tau)^2/2 \\ 1 & 2\Delta\tau & (2\Delta\tau)^2/2 \end{bmatrix} \quad (2)$$

- $\Delta\tau$ is the elapsed time between adjacent lagged columns in the time-delay embedded matrix

Nonstationary and individual parameters in LDE

- For a process to be stationary it means that the estimated parameters may not change over time (S. Boker et al., 2004; S. M. Boker, Staples, & Hu, 2016).
- Nonstationarity can be present in any of the parameters of the model.
- DLO assumes that both η and ζ do not change. Assumes a stable equilibrium state—nonstationarity would show if the equilibrium increased or decreased over time (slope).
- S. M. Boker et al. (2016) presented some of these needed developments of the LDE.

Bayesian Structural Equation Modeling (BSEM)

- BSEM identifies the model through data augmentation; this way θ is identify by setting priors which constrain the data space (Gelman et al., 2013; Song & Lee, 2012). A common way to set θ is to limit the distribution to $N \sim (0, 1)$.

$$\boldsymbol{\Sigma} = \boldsymbol{\Lambda}\boldsymbol{\Psi}\boldsymbol{\Lambda}' + \boldsymbol{\Theta} \tag{3}$$

$$y_{ij} = \alpha_j + \lambda_j \theta_{ik} + \epsilon_j \tag{4}$$

Bayesian Mixed-Effects Nonstationary Latent Differential Equation Model (BMNLDE)

- This project intends to develop and test a Bayesian Mixed-Effects Nonstationary Latent Differential Equation Model (BMNLDE).
- This model is a Bayesian implementation and an extension of the model presented by S. M. Boker et al. (2016).
- A framework to include mixed-effects into LDE, estimating both subject and sample parameters, including nonstationarity.
- This can be extended to multiple groups, or coupled models.
- Bayesian estimation will allow us direct inference of the parameters.
- BMNLDE estimating a DLO is expressed by the two following equations.

$$y_{id} = \alpha_d + \lambda_{1d} g_{ik} + \lambda_{2d} b_{jz} + \epsilon \quad (5)$$

$$\ddot{g}_i = \eta_j g_i + \zeta_j \dot{g}_i + \epsilon_{\ddot{g}} \quad (6)$$

BMNLDE

- Nonstationary and individual parameters in LDE
- S. M. Boker et al. (2016) included the estimation of an equilibrium intercept and slope (nonstationary).

$$\lambda_{2d} = JH + C \tag{7}$$

$$C = \begin{bmatrix} 1 & 1 & 1 & 1 & 1 \\ -2\Delta\tau & -\Delta\tau & 0 & \Delta\tau & -2\Delta\tau \end{bmatrix} \tag{8}$$

$$H = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ \Delta\tau & \Delta\tau & \Delta\tau & \Delta\tau & \Delta\tau \end{bmatrix} \tag{9}$$

For the jth occasion of measurement within individual i's data, we must create the matrix J in the following form

$$J = \begin{bmatrix} 0 & 0 \\ 0 & j \end{bmatrix} \tag{10}$$

BMNLDE

$$y_{id} \sim N(\mu_d, \epsilon^2) \tag{11}$$

$$\frac{1}{\epsilon^2} \sim \gamma(1, 0.5) \tag{12}$$

$$(x_i, \dot{x}_i) \sim MVN(\mu_x, \Sigma_x) \tag{13}$$

$$\Sigma_x \sim Inverse - Wishart(I, df) \tag{14}$$

$$\ddot{x}_i \sim N(\mu_{\ddot{x}}, \sigma^2_{\ddot{x}}) \tag{15}$$

$$\frac{1}{\sigma^2_{\ddot{x}}} \sim \gamma(1, 0.5) \tag{16}$$

BMNLDE

$$b_{jz} \sim MVN(\mu_b, \Sigma_b) \tag{17}$$

$$\mu_b \sim N(0, 100) \tag{18}$$

$$\eta_j \sim N(\mu_\eta, \sigma_\eta^2) \tag{19}$$

$$\zeta_j \sim N(\mu_\zeta, \sigma_\zeta^2) \tag{20}$$

$$\mu \sim N(-0.1, 5) \tag{21}$$

$$\sigma^2 \sim U(0, 1) \tag{22}$$

Simulation Study

- A simulation study evaluated the performance of the proposed BMNLDE model.
- Tested different values for three of the four random parameters, varying values for η_j, ζ_j, and b_{js} (equilibrium slope).
- The parameters that varied are the sample means (μ_η, μ_ζ, and μ_{bs})

Simulation Study

- 50 subjects with 50 data points each.
- 100 replications.
- The elements of the simulated data that will not vary are

Table: Parameters for simulated data that are fixed across conditions

parameter	value
μ_{bi}	0
σ_{bi}	2
σ_{bs}	0.3
σ_{η}	0.1
σ_{ζ}	0.1
ρ_{g}	0
ρ_{b}	0.3

Simulation Study

- The three parameters that will vary across the condition will have four values each

Table: Parameters for simulated data that are vary across conditions

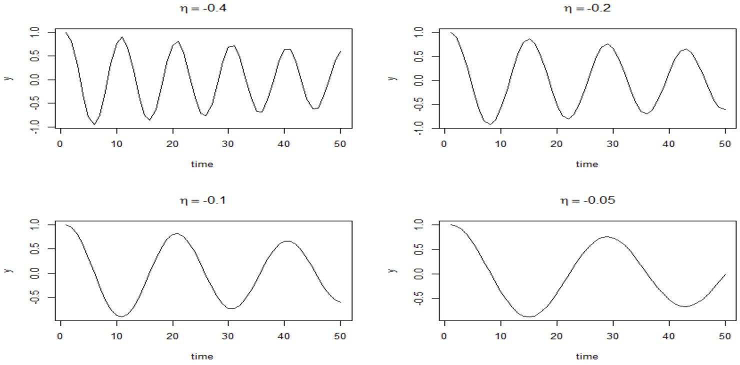
parameter	values
μ_η	-0.4, -0.2, -0.1, -0.05
μ_ζ	0, -0.02, -0.05, -0.1
μ_{bs}	0, 0.05, 0.1, 0.2

Table: Embedding dimension and τ for each value of μ_η

μ_η	d	τ
-0.4	4	1
-0.2	6	1
-0.1	8	1
-0.05	6	2

Simulation Study

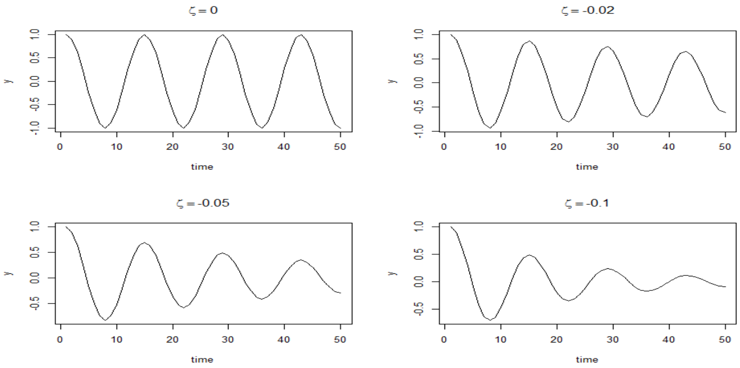
Figure: Oscillating time series with varying values of η



Note. All were simulated with $\zeta = -0.02$

Simulation Study

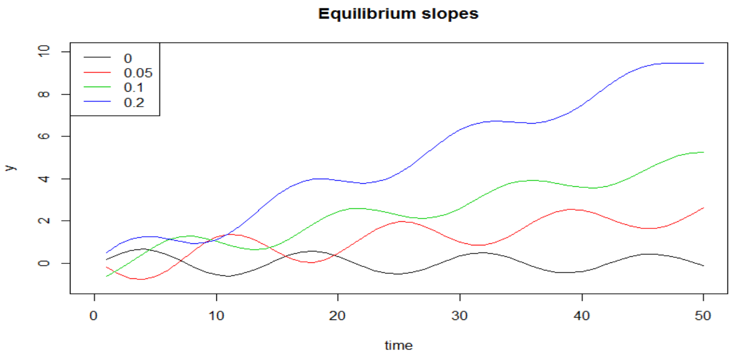
Figure: Oscillating time series with varying values of ζ



Note. All were simulated with $\eta = -0.2$

Simulation Study

Figure: Oscillating time series with varying values of b_s



Note. All were simulated with $\eta = -0.2$ and $\zeta = -0.02$

Simulation Study

- Bayesian estimation characteristics
  - General Bayesian software JAGS (Plummer, 2003), with its interface R package R2jags (Su & Yajima, 2015).
  - Convergence of the Markov chains will be determined using the potential scale reduction factor (PSRF, or R-hat), convergence when R-hat is lower than 1.10 (Brooks & Gelman, 1998; Gelman & Rubin, 1992).
  - Chains = 3
  - Kept iterations = 5000 for each chain
- Evaluation of the simulation, BMNLDE was compared with the closest frequentist model (OpenMx).
  - Convergence
  - $Bias = \theta - \hat{\theta}$ at the subject and sample level.
  - Mean Square Error (MSE), $MSE = Bias^2 + Var(\hat{\theta})$
  - 95% Credible Interval coverage for sample parameters (can not be calculated for OpenMx)

Results: Convergence

- BMNLDE: 100% (proper convergence without outliers)
- OpenMx: 92.8% (proper convergence without outliers)
  - Had to exclude the correlation between zeroth and first derivative.
  - Sensitive to starting values.

Results: Bias

Table: Average Bias between the two estimation methods

parameter	BMNLDE	OpenMx	cohen d	Δ
b_{ji}	0.053	0.065	0.070	-0.012
b_{js}	0.004	0.004	0.093	0.000
η_j	-0.024	-0.027	0.163	0.003
ζ_j	0.024	0.026	0.085	-0.003
μ_{bi}	0.049	0.063	0.040	-0.014
μ_{bs}	0.005	0.004	0.013	0.001
μ_η	-0.024	-0.027	0.185	0.003
μ_ζ	0.024	0.027	0.085	-0.003
ρ_b	0.057	NA	NA	NA
ρ_x	-0.007	NA	NA	NA
σ_{bi}	0.161	0.171	0.037	-0.010
σ_{bs}	0.030	-0.003	1.139	0.033
σ_η	-0.042	-0.009	1.002	-0.033
σ_ζ	-0.038	0.049	3.982	-0.087

Results: Bias

Table: η_p^2 effect of varying conditions on bias

	BMNLDE			OpenMx		
parameter	μ_η	μ_ζ	$\mu_\eta * \mu_\zeta$	μ_η	μ_ζ	$\mu_\eta * \mu_\zeta$
b_{ji}	0.334	0.001	0.003	0.364	0.002	0.003
b_{js}	0.647	0.005	0.005	0.635	0.004	0.005
η_j	0.527	0.017	0.006	0.460	0.015	0.005
ζ_j	0.679	0.357	0.166	0.788	0.007	0.226
μ_{bi}	0.070	0.001	0.007	0.076	0.001	0.007
μ_{bs}	0.022	0.000	0.008	0.020	0.000	0.009
μ_η	0.526	0.016	0.006	0.467	0.012	0.004
μ_ζ	0.563	0.258	0.104	0.698	0.004	0.147
ρ_b	0.400	0.002	0.003	NA	NA	NA
ρ_x	0.297	0.408	0.088	NA	NA	NA
σ_{bi}	0.397	0.002	0.007	0.402	0.001	0.007
σ_{bs}	0.000	0.002	0.004	0.001	0.003	0.003
σ_η	0.704	0.002	0.003	0.919	0.010	0.005
σ_ζ	0.314	0.025	0.032	0.233	0.048	0.021

Results: Bias

Table: Average Bias for BMNLDE and OpenMx across μ_η and μ_ζ

μ_η	population	-0.40	-0.20	-0.10	-0.05	-0.40	-0.20	-0.10	-0.05
		BMNLDE				OpenMx			
$\mu_\zeta = 0$									
ζ_j	≈ 0	0.000	0.002	0.020	0.031	0.000	0.006	0.043	0.055
μ_ζ	0	-0.001	-0.001	0.023	0.033	0.000	0.002	0.046	0.056
ρ_x	0	0.000	0.001	0.020	0.022	NA	NA	NA	NA
$\mu_\zeta = -0.02$									
ζ_j	≈ -0.02	0.003	0.006	0.023	0.038	-0.005	0.008	0.042	0.059
μ_ζ	-0.02	0.003	0.009	0.023	0.038	-0.005	0.010	0.043	0.060
ρ_x	0	-0.009	-0.005	0.005	0.015	NA	NA	NA	NA
$\mu_\zeta = -0.05$									
ζ_j	≈ -0.05	0.006	0.011	0.031	0.055	-0.011	0.005	0.046	0.073
μ_ζ	-0.05	0.006	0.013	0.030	0.051	-0.010	0.008	0.045	0.069
ρ_x	0	-0.021	-0.020	-0.014	0.011	NA	NA	NA	NA
$\mu_\zeta = -0.10$									
ζ_j	≈ -0.10	0.009	0.020	0.044	0.079	-0.027	0.002	0.051	0.092
μ_ζ	-0.10	0.010	0.020	0.044	0.078	-0.025	0.002	0.052	0.092
ρ_x	0	-0.044	-0.050	-0.041	0.003	NA	NA	NA	NA

Results: Bias

- BMNLDE: $\mu_{bs} = 0.004$, $\sigma_{bs} = 0.029$
- OpenMx: $\mu_{bs} = 0.004$, $\sigma_{bs} = -0.003$

Table: Average Bias for BMNLDE and OpenMx across μ_η values

μ_η	population	-0.40	-0.20	-0.10	-0.05	-0.40	-0.20	-0.10	-0.05
		BMNLDE				OpenMx			
b_{ji}	≈ 0	0.037	0.019	-0.020	0.176	0.037	0.020	-0.008	0.193
b_{js}	$\approx 0, 0.05, 0.1, 0.2$	0.000	0.001	0.006	0.010	0.000	0.001	0.005	0.009
η_j	$\approx -0.40, -0.20, -0.10, -0.05$	-0.014	-0.010	-0.033	-0.038	-0.023	-0.010	-0.035	-0.040
μ_{bi}	0	0.043	0.017	-0.050	0.188	0.043	0.019	-0.041	0.205
μ_η	$-0.40, -0.20, -0.10, -0.05$	-0.015	-0.009	-0.032	-0.038	-0.024	-0.009	-0.036	-0.040
ρ_b	0.3	0.021	0.002	-0.007	0.212	NA	NA	NA	NA
σ_{bi}	2	0.038	0.039	0.099	0.467	0.050	0.047	0.100	0.468
σ_η	0.1	-0.031	-0.027	-0.041	-0.068	0.052	-0.007	-0.029	-0.056
σ_ζ	0.1	-0.035	-0.055	-0.028	-0.035	0.061	0.035	0.064	0.041

Results: Bias

Table: Average $z - score$ of the estimated parameters from the population means

parameter	σ	BMNLDE	OpenMx
μ_{bi}	2	0.025	0.032
μ_{bs}	0.3	0.015	0.013
μ_{η}	0.1	-0.236	-0.268
μ_{ζ}	0.1	0.237	0.265

Results: 95% Credible Interval coverage

Table: 95% Credible Interval coverage for BMNLDE across μ_η and μ_ζ

μ_η	-0.40	-0.20	-0.10	-0.05
$\mu_\zeta = 0$				
μ_ζ	100	99.25	79.00	43.00
ρ_x	100	100	98.00	98.00
$\mu_\zeta = -0.02$				
μ_ζ	100	95.75	78.75	24.50
ρ_x	100	100	100	100
$\mu_\zeta = -0.05$				
μ_ζ	100	96.75	62.50	5.50
ρ_x	100	98.00	99.00	100
$\mu_\zeta = -0.10$				
μ_ζ	100	94.25	30.25	0
ρ_x	99.00	76.75	85.50	99.00

Results: 95% Credible Interval coverage

- $\mu_{bs} = 96.41$, $\sigma_{bs} = 90.22$

Table: 95% Credible Interval coverage for BMNLDE across μ_η values

μ_η	-0.40	-0.20	-0.10	-0.05
μ_{bi}	92.12	93.69	92.94	85.25
μ_η	97.75	90.06	6.94	0
ρ_b	86.56	85.00	84.56	29.12
σ_{bi}	93.88	93.81	93.94	46.56
σ_η	81.94	40.00	0.06	0
σ_ζ	99.81	13.88	69.25	52.00

Results: Mean Square Error

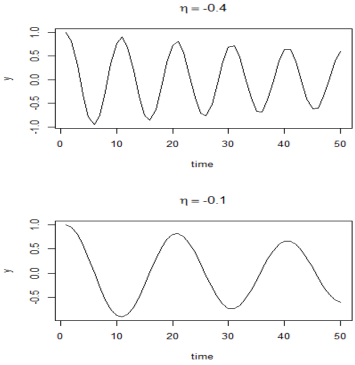
Table: Mean Square Error for the BMNLDE and OpenMx models

parameter	BMNLDE	OpenMx
μ_{bi}	0.125	0.126
μ_{bs}	0.002	0.002
μ_{η}	0.001	0.001
μ_{ζ}	0.001	0.002
ρ_{b}	0.024	NA
ρ_{x}	0.001	NA
σ_{bi}	0.107	0.108
σ_{bs}	0.002	0.001
σ_{η}	0.002	0.002
σ_{ζ}	0.002	0.003

Discussion

- The BMNLDE is a general method to estimate mixed-effects differential equations. Estimate subject and sample parameters. Includes short and long term trends.
- In general the model presents appropriately low bias.
- The condition that has greater effect on bias is μ_η, as it approaches 0 bias increases.
- The other condition that affects bias is μ_ζ, as it distances from 0 bias increases.
- The inferences from the model follows the same pattern, with parameters with problematically low CI coverage.
- Comparing BMNLDE to OpenMx, bias an MSE are equivalent across methods. BMNLDE has advantages over OpenMx
  - BMNLDE estimates subject and sample parameters.
  - OpenMx present convergence problems when the covariance between latent factors is inluded
  - OpenMx is sensitive to starting values.
  - BMNLDE is faster to reach a convergent solution.

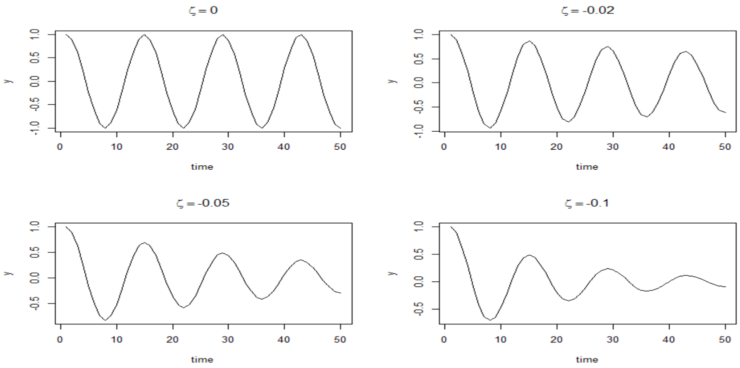
Figure: Oscillating time series with varying values of η



Note. All were simulated with $\zeta = -0.02$

ζ

Figure: Oscillating time series with varying values of ζ



Note. All were simulated with $\eta = -0.2$

Future Research

- Increase replications (will be included for publication)
- Low frequency models with longer time series (will be included for publication as study 2)
- Data density requires, subjects and time points
- Prior sensitivity
- Extend to more models, like couple equations
- Non-stationarity in η and ζ
- Is CI coverage problem related to data density or follows the known issues of using embedded matrix?

Sedentary Behavior in Older Adults

- The data consists of time series from the physical activity of older adults who wore an activPAL monitor (Grant, Dall, Mitchell, & Granat, 2008; Kim, Barry, & Kang, 2015).
- My interest is to describe the sedentary behavior (SB) of healthy and older adults with Alzheimer Dementia (AD), and identify differences between these groups.
- Physical activity is related to increased cognitive faculties and overall health outcomes for older adults, mitigating conditions such as obesity, high blood glucose levels, type 2 diabetes, and cardiovascular problems (Chastin & Granat, 2010).
- SB was characterize in function of their position, sitting and lying (Proper, Singh, van Mechelen, & Chinapaw, 2011).

Sedentary Behavior in Older Adults

- The measurement of SB and physical activity in general has improved due to the use objective measures such as the activPAL and the ActiGraph (Kang & Rowe, 2015; Kim et al., 2015). But: How do researchers handle this continuous measure of physical activity?
- Commonly used as cross-sectional data because the repeated measures are summarized in total numbers, such as the percentage of time spent sedentary (Chastin & Granat, 2010).
- SB is expected to follow an oscillating form over time (Rowlands et al., 2015), and it is expected for people to vary their SB around an average (equilibrium) SB.
- Where the BMNLDE would describe this behavior estimating the DLO.

Data Description

- The monitor was worn by 91 participants for a minimum of 5 days.
- Participants maintained a sleep diary to validate the activPAL data.
- Six subjects experienced monitor malfunctions.
- Wear-time validation and excluded time not worn and sleep time.
- Subjects who had less than 10 hours of wear-time validated data per day had that day excluded.
- The final sample consisted of 37 subjects diagnosed with AD and 48 healthy older adults.
- For the analysis we will sum up the SB time every 10 minutes, summing the seconds of standing time for each 10 minute epoch.
- The first 5 valid days for each subject were included in the analysis.

Data Description

Table: Description of the sample

	Healthy	AD	Total
Sample size	48	37	85
% of males	33.3	64.9	47.1
% of white	95.8	86.5	91.8
Age (SD)	73.3 (6.8)	73.3 (7.7)	73.3 (7.2)
Education (SD)	17.1 (3.4)	15.7 (2.8)	16.5 (3.2)
BMI (SD)	26.6 (4.2)	26.5 (4.6)	26.5 (4.4)
VO2 max (SD)	1.6 (0.4)	1.6 (0.5)	1.6 (0.5)
Number of valid 10 minute epochs (range)	455.1 (394 - 503)	440.0 (325 - 505)	448.6 (325 - 505)

Analysis

- To define the embedding dimension, we followed the method proposed by Hu et al. (2014) to estimate the model with different d.
- First model, we estimated the BMNLDE for the whole sample.
- Second model, we estimated the BMNLDE with group (i.e., healthy and AD) mean differences for the four random parameters (μ_η, μ_ζ, μ_{bi}, μ_{bs}).
- We calculated posterior distribution of the mean group differences.

Model Priors

- Follows the same parameterization as the simulation
- Only difference in the following priors

$$\mu_{bi} \sim N(217, 100000) \tag{23}$$

$$\mu_{bs} \sim N(0, 1000) \tag{24}$$

Results: Embedding Dimension

- Estimated with detrended time series in OpenMx, embedding dimension from 4 to 11 (sensitive to starting values).
- η stabilized at $d = 7$ ($\eta = -0.12$), were η change lower that 0.01.
- Following the Nyquist limit for $d = 7$ and 10 minute epochs, means that we expect to find oscillations of at least 2.3 hours.

Results: Overall Sample

- Estimated with and without the correlation between zeroth (x) and first ($\dot{x}$) derivatives.
- Comparing the DIC for these two models, the first model shows better fit ($DIC_{dif} = 1398.8$)
- Parameters were equivalent, and correlation was 0 ($\rho x = 0.000002, 95\% CI = -0.014, 0.013$)

Results: Overall Sample

Table: Overall sample BMNLDE results

parameter	mean	2.5%	97.5%
μ_η	-0.137	-0.163	-0.111
μ_ζ	-0.001	-0.028	0.026
σ_η	0.121	0.103	0.141
σ_ζ	0.118	0.101	0.137
μ_{bi}	220.154	200.791	240.326
μ_{bs}	-0.002	-0.007	0.004
σ_{bi}	93.546	79.947	109.069
σ_{bs}	0.025	0.021	0.029
ρ_b	-0.508	-0.655	-0.334
σ_x	164.363	162.841	165.840
$\sigma_{\dot{x}}$	41.840	41.406	42.271
$\sigma_{\ddot{x}}$	25.553	24.864	25.569
σ_ϵ	145.532	144.992	146.064
λ	2.828	2.594	3.144

Results: Multiple group model

Table: Multiple-group BMNLDE results

parameter	mean	2.5%	97.5%	mean	2.5%	97.5%
	AD group			Healthy group		
μ_η	-0.144	-0.201	-0.087	-0.132	-0.176	-0.089
μ_ζ	-0.001	-0.058	0.058	0.000	-0.046	0.045
σ_η	0.177	0.141	0.221	0.153	0.125	0.188
σ_ζ	0.173	0.138	0.218	0.153	0.125	0.188
μ_{bi}	211.301	184.029	238.987	226.614	198.228	255.478
μ_{bs}	-0.006	-0.013	0.002	0.001	-0.006	0.009
σ_{bi}	84.676	66.991	106.859	100.222	81.936	122.802
σ_{bs}	0.023	0.018	0.029	0.027	0.022	0.033
ρ_b	-0.546	-0.743	-0.288	-0.504	-0.688	-0.273
σ_x	162.085	159.815	164.351	166.075	164.079	168.038
$\sigma_{\dot{x}}$	41.593	40.939	442.246	42.008	41.430	42.575
$\sigma_{\ddot{x}}$	24.640	24.061	25.211	25.660	25.194	26.140
σ_ϵ	146.229	145.411	147.038	145.022	144.356	145.714
λ	2.758	2.339	3.571	2.880	2.498	3.518

Table: Group differences for the random parameters

parameter	mean	2.5%	97.5%
μ_η	-0.012	-0.084	0.060
μ_ζ	-0.001	-0.074	0.075
σ_η	0.024	-0.026	0.078
σ_ζ	0.020	-0.029	0.074
μ_{bi}	-15.313	-54.314	23.106
μ_{bs}	-0.007	-0.018	0.004
σ_{bi}	-15.625	-43.966	12.527
σ_{bs}	-0.004	-0.012	0.004

Results: Group Differences

Table: Cohen's *d* for the mean group differences of the random parameters

parameter	mean	2.5%	97.5%
μ_{η}	-0.074	-0.505	0.363
μ_{ζ}	-0.007	-0.452	0.461
μ_{bi}	-0.165	-0.584	0.243
μ_{bs}	-0.273	-0.699	0.150

Discussion

- BMNLDE was implemented to describe the oscillating sedentary behavior of older adults.
- Showing a simple extension from the simulation by estimating a multiple group model, and compare parameters of interest between groups.
- No differences between healthy and AD older adults.
- AD subjects would be classified with mild or early stage AD (Petersen, 2003).
- AD should be seen as an illness that progresses in a continuum between *normal* and *demented* (Selkoe & Schenk, 2003).
- The first diagnosis is guided by cognitive symptoms which in its early stages it is characterized by memory complaints, objective memory impairment, normal general cognition, and preserved activities of daily living (Petersen, 2003; Petersen et al., 2009).
- Future research; could focus in different group comparison (like gender), the inclusion of subject level covariates.

References I

Boker, S., Neale, M., & Rausch, J. (2004). Latent differential equation modeling with multivariate multi-occasion indicators. In *Recent developments on structural equation models* (pp. 151–174). Springer. Retrieved 2015-12-26, from http://link.springer.com/chapter/10.1007/978-1-4020-1958-6-9

Boker, S. M., & Nesselroade, J. R. (2002, January). A Method for Modeling the Intrinsic Dynamics of Intraindividual Variability: Recovering the Parameters of Simulated Oscillators in Multi-Wave Panel Data. *Multivariate Behavioral Research*, *37*(1), 127–160. Retrieved 2015-04-05, from http://www.tandfonline.com/doi/abs/10.1207/S15327906MBR3701-06 doi: 10.1207/S15327906MBR3701-06

References II

Boker, S. M., Staples, A. D., & Hu, Y. (2016, April). Dynamics of Change and Change in Dynamics. *Journal for Person-Oriented Research*, *2*(1-2), 34–55. Retrieved 2016-06-28, from `http://www.person-research.ouradmin.se/articles/volume2_1_2/filer/8.pdf` doi: 10.17505/jpor.2016.05

Brooks, S., & Gelman, A. (1998). General methods for monitoring convergence of iterative simulations. *Journal of Computational and Graphical Statistics*, *7*(4), 434–455. doi: 10.1080/10618600.1998.10474787

Chastin, S., & Granat, M. (2010, January). Methods for objective measure, quantification and analysis of sedentary behaviour and inactivity. *Gait & Posture*, *31*(1), 82–86. Retrieved 2015-11-20, from `http://linkinghub.elsevier.com/retrieve/pii/S096663620900602X` doi: 10.1016/j.gaitpost.2009.09.002

References III

Deboeck, P. R., Boker, S. M., & Bergeman, C. S. (2008, December). Modeling Individual Damped Linear Oscillator Processes with Differential Equations: Using Surrogate Data Analysis to Estimate the Smoothing Parameter. *Multivariate Behavioral Research*, *43*(4), 497–523. Retrieved 2015-11-09, from http://www.tandfonline.com/doi/abs/10.1080/00273170802490616 doi: 10.1080/00273170802490616

Gelman, A., Carlin, J. B., Stern, H. S., Dunson, D. B., Vehtari, A., & Rubin, D. B. (2013). *Bayesian Data Analysis* (Third ed.). Boca Raton, FL: Chapman & Hall/CRC.

Gelman, A., & Rubin, D. (1992). Inference from iterative simulation using multiple sequences. *Statistical Science*, *7*(4), 457–472. doi: 10.1214/ss/1177011136

References IV

Grant, P. M., Dall, P. M., Mitchell, S. L., & Granat, M. H. (2008). Activity monitor accuracy in measuring step number and cadence in community-dwelling older adults. *Journal of aging and physical activity*, *16*(2), 201.

Hu, Y., Boker, S., Neale, M., & Klump, K. L. (2014). Coupled latent differential equation with moderators: Simulation and application. *Psychological Methods*, *19*(1), 56–71. Retrieved 2015-02-17, from http://doi.apa.org/getdoi.cfm?doi=10.1037/a0032476 doi: 10.1037/a0032476

Kang, M., & Rowe, D. A. (2015, July). Issues and Challenges in Sedentary Behavior Measurement. *Measurement in Physical Education and Exercise Science*, *19*(3), 105–115. Retrieved 2015-12-21, from http://www.tandfonline.com/doi/full/10.1080/1091367X.2015.1055566 doi: 10.1080/1091367X.2015.1055566

References V

Kim, Y., Barry, V. W., & Kang, M. (2015, July). Validation of the ActiGraph GT3x and activPAL Accelerometers for the Assessment of Sedentary Behavior. *Measurement in Physical Education and Exercise Science*, *19*(3), 125–137. Retrieved 2015-12-21, from http://www.tandfonline.com/doi/full/10.1080/1091367X.2015.1054390 doi: 10.1080/1091367X.2015.1054390

Maxwell, S. E., & Boker, S. M. (2007). Multilevel Models of Dynamical Systems. In S. M. Boker & M. J. Wenger (Eds.), *Data Analytic Techniques for Dynamical Systems.* Lawrence Erlbaum Associates, Inc.

McArdle, J. J., & Nesselroade, J. R. (2014). *Longitudinal Data Analysis Using Structural Equation Models*. Washington, DC: American Psychological Association.

References VI

Molenaar, P. C. (2004). A manifesto on psychology as idiographic science: Bringing the person back into scientific psychology, this time forever. *Measurement*, *2*(4), 201–218. Retrieved 2015-04-06, from http://www.tandfonline.com/doi/abs/10.1207/s15366359mea0204-1

Oravecz, Z., Tuerlinckx, F., & Vandekerckhove, J. (2011). A hierarchical latent stochastic differential equation model for affective dynamics. *Psychological Methods*, *16*(4), 468–490. Retrieved 2015-01-23, from http://doi.apa.org/getdoi.cfm?doi=10.1037/a0024375 doi: 10.1037/a0024375

Oud, J. H. L. (2007). Continuous Time Modeling of Reciprocal Relationships in the Cross-Lagged Panel Design. In S. M. Boker & M. J. Wenger (Eds.), *Data Analytic Techniques for Dynamical Systems.* Lawrence Erlbaum Associates, Inc.

References VII

Petersen, R. C. (2003, August). Mild cognitive impairment clinical trials. *Nature Reviews Drug Discovery*, *2*(8), 646–653. Retrieved 2016-05-02, from http://www.nature.com/doifinder/10.1038/nrd1155 doi: 10.1038/nrd1155

Petersen, R. C., Roberts, R. O., Knopman, D. S., Boeve, B. F., Geda, Y. E., Ivnik, R. J., ... Jack, C. R. (2009, December). Mild Cognitive Impairment: Ten Years Later. *Archives of Neurology*, *66*(12). Retrieved 2016-05-02, from http://archneur.jamanetwork.com/article.aspx?doi=10.1001/archneurol.2009.266 doi: 10.1001/archneurol.2009.266

Plummer, M. (2003). *Jags: A program for analysis of bayesian graphical models using gibbs sampling.*

References VIII

Proper, K. I., Singh, A. S., van Mechelen, W., & Chinapaw, M. J. (2011, February). Sedentary Behaviors and Health Outcomes Among Adults. *American Journal of Preventive Medicine*, *40*(2), 174–182. Retrieved 2015-11-20, from http://linkinghub.elsevier.com/retrieve/pii/S0749379710006082 doi: 10.1016/j.amepre.2010.10.015

Rowlands, A. V., Gomersall, S. R., Tudor-Locke, C., Bassett, D. R., Kang, M., Fraysse, F., ... Olds, T. S. (2015, March). Introducing novel approaches for examining the variability of individuals' physical activity. *Journal of Sports Sciences*, *33*(5), 457–466. Retrieved 2015-11-20, from http://www.tandfonline.com/doi/abs/10.1080/02640414.2014.951067 doi: 10.1080/02640414.2014.951067

References IX

Selkoe, D. J., & Schenk, D. (2003, April). A LZHEIMER'S D ISEASE : Molecular Understanding Predicts Amyloid-Based Therapeutics. *Annual Review of Pharmacology and Toxicology*, *43*(1), 545–584. Retrieved 2016-05-03, from `http://www.annualreviews.org/doi/abs/10.1146/annurev.pharmtox.43.100901.140248` doi: 10.1146/annurev.pharmtox.43.100901.140248

Song, X.-Y., & Lee, S.-Y. (2012). *Bayesian and Advanced Bayesian Structural Equation Modeling: With Applications in the Medical and Behavioral Sciences*. West Sussex, United Kindom: JohnWiley & Son, Ltd.

Su, Y.-S., & Yajima, M. (2015). R2jags: Using r to run 'jags' [Computer software manual]. Retrieved from `https://CRAN.R-project.org/package=R2jags` (R package version 0.5-7)

References X

Tornoe, C. W., Agerso, H., Jonsson, E., Madsen, H., & Nielsen, H. A. (2004, October). Non-linear mixed-effects pharmacokinetic/pharmacodynamic modelling in NLME using differential equations. *Computer Methods and Programs in Biomedicine*, *76*(1), 31–40. Retrieved 2015-01-23, from `http://linkinghub.elsevier.com/retrieve/pii/S0169260704000136` doi: 10.1016/j.cmpb.2004.01.001

Voelkle, M. C., Oud, J. H. L., Davidov, E., & Schmidt, P. (2012). An SEM approach to continuous time modeling of panel data: Relating authoritarianism and anomia. *Psychological Methods*, *17*(2), 176–192. Retrieved 2015-01-23, from `http://doi.apa.org/getdoi.cfm?doi=10.1037/a0027543` doi: 10.1037/a0027543

References XI

Wang, L., Cao, J., Ramsay, J. O., Burger, D. M., Laporte, C. J. L., & Rockstroh, J. K. (2014, January). Estimating mixed-effects differential equation models. *Statistics and Computing*, *24*(1), 111–121. Retrieved 2015-01-23, from `http://link.springer.com/10.1007/s11222-012-9357-1` doi: 10.1007/s11222-012-9357-1