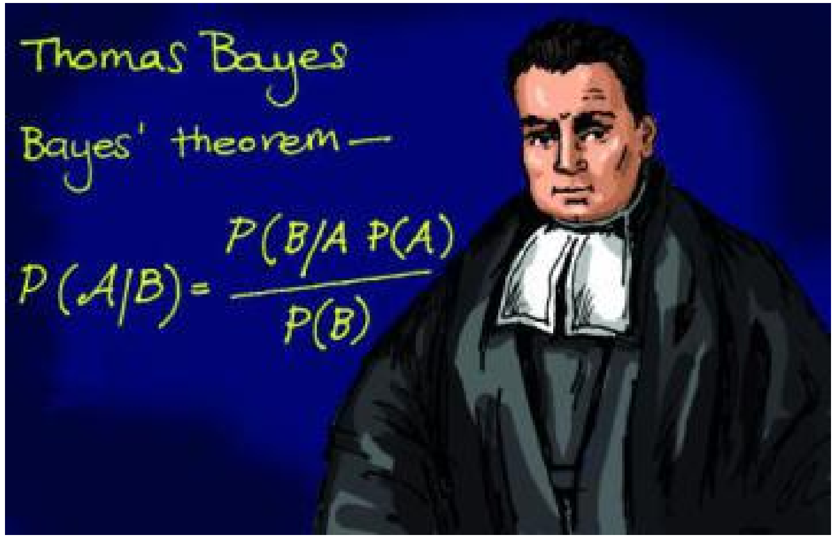
$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Bayesian Testing in SCD Research:

A FORWARD PARADIGM SHIFT



Why trust statistical inferences?

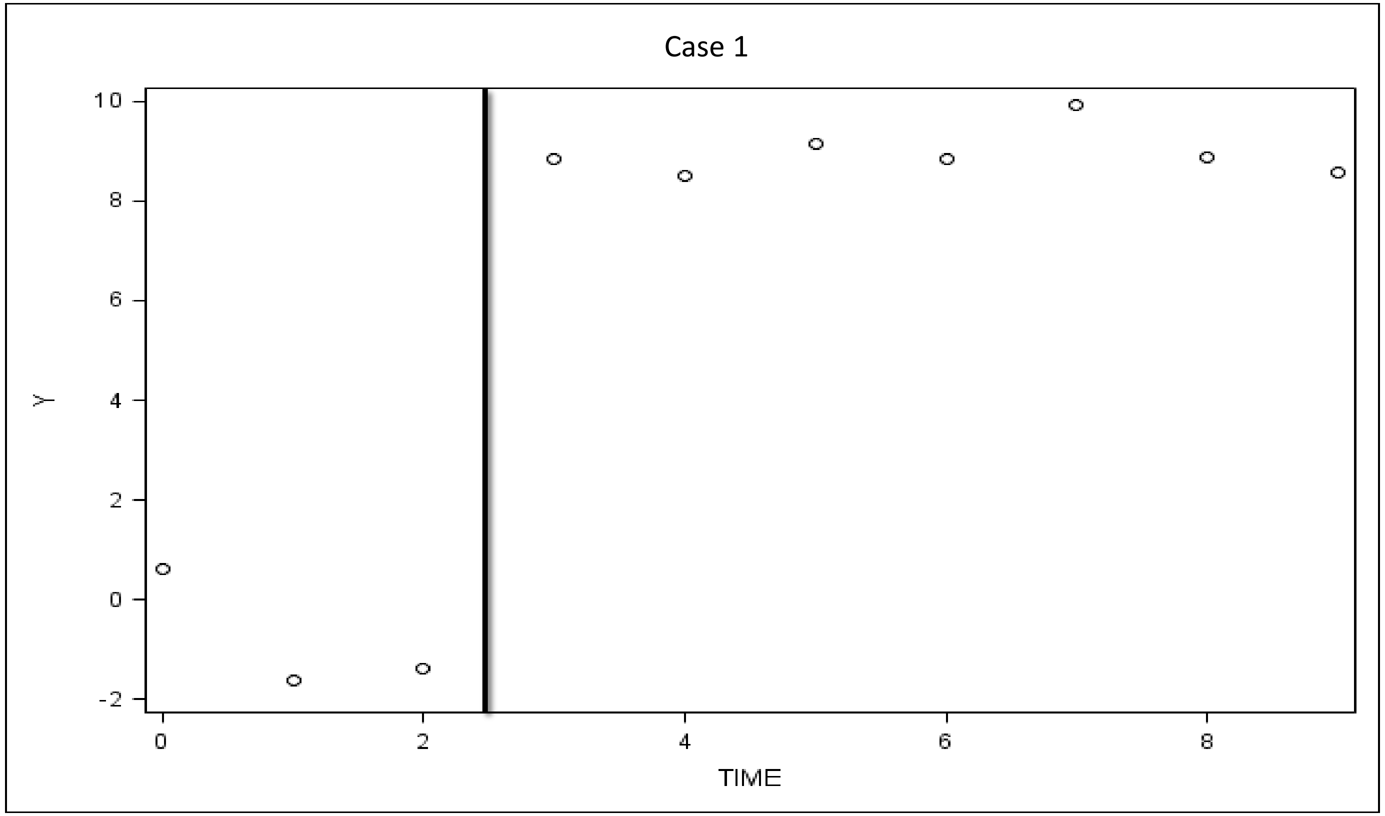
Tyler Hicks, Ph.D.
Jason Travers, Ph.D.
Leslie Bross, M.S.

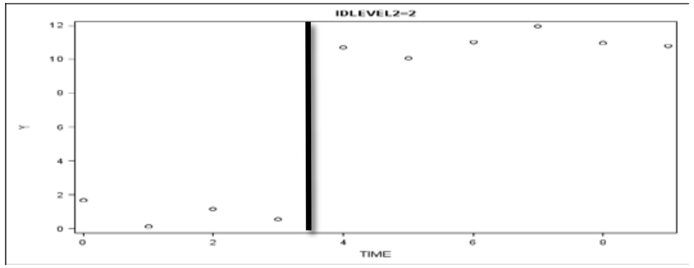
University of Kansas

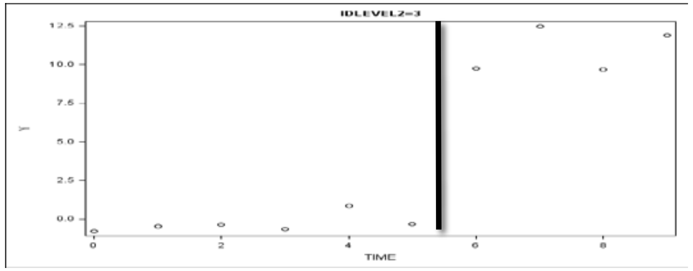
$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

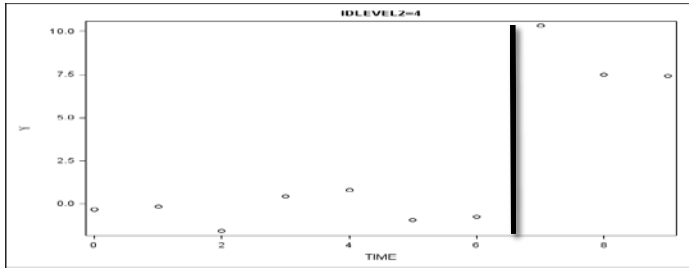
Statistics in SCD research

Supplementing visual analysis of graphs with statistics









Resource for further information:

Fisher, W.W. & Lerman, D.C. (2014). It has been said that, “There are three degrees of falsehoods: Lies, damn lies, and statistics”. *Journal of School Psychology, 52,* 243-248.

$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$

A General Modeling Framework

Single-Case Research with Multiple Baseline Designs

Level 1:

$$Y_i = \beta_{0i} + \beta_{1i} * Phase_{ti} + e_{ti} \qquad e_{ti} = \phi e_{(t-1)i} + u_t \qquad u_t \sim \text{N}(0, \sigma^2)$$

Level 2:

$$\beta_{0i} = \gamma_{00} + \eta_{0i} \qquad \eta_{0i}, \eta_{1i} \sim BVN(\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$\beta_{1i} = \gamma_{10} + \eta_{1i} \qquad \boldsymbol{\mu} = \begin{vmatrix} \mathbf{0} \\ \mathbf{0} \end{vmatrix}, \boldsymbol{\Sigma} = \begin{vmatrix} \boldsymbol{\tau}_{00}^2 & \boldsymbol{\tau}_{01}^2 \\ \boldsymbol{\tau}_{10}^2 & \boldsymbol{\tau}_{11}^2 \end{vmatrix}$$

Source of Citation:

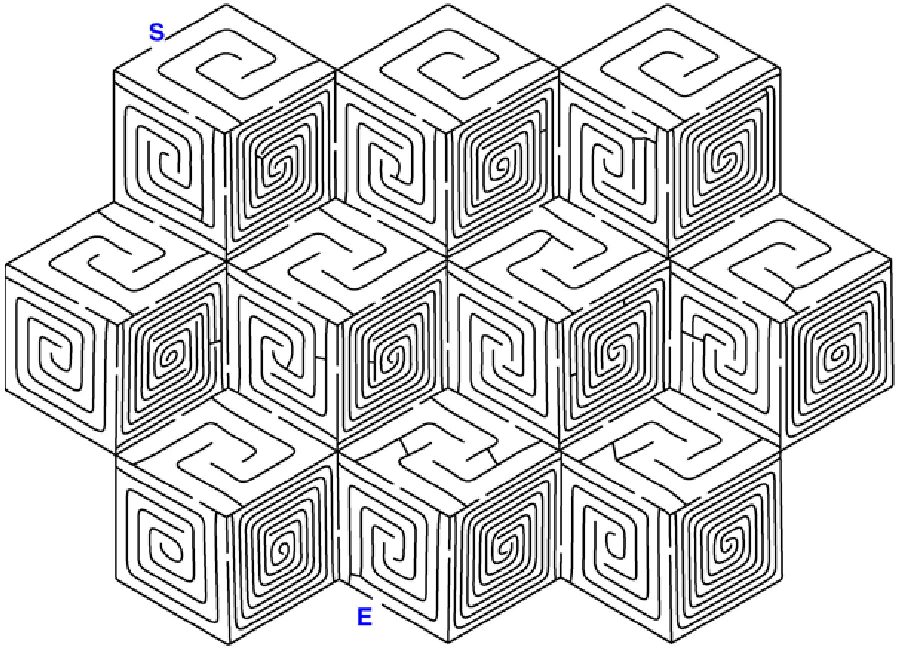
Beak, E.K. & Ferron, J.M. (2013). Multilevel models for multi-baseline data: Modeling across participant variation in autocorrelation and residual variance. *Behavior Research Methods. 45.* 65-74.

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Modeling data in SCD Research

The need for a principled test to navigate the maze of modeling configurations

- Add Interaction Effects?
- Posit Autocorrelation?
- Specify Error Structure?
- Make Slopes Random?
- Compute effect?



Source of Citation:

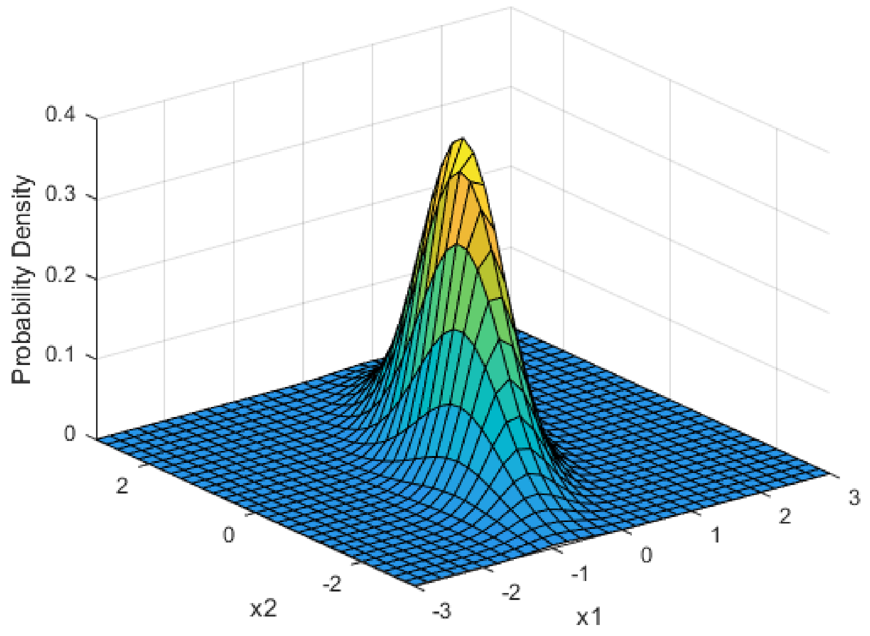
Beak, E.K, Moeyaert, M., Petit-Bois, M., Beretvas, S.N., Noortgate, & Ferron, J.M. (2014). The use of multilevel analysis for integrating single-case experimental design results within a study and across studies. *Neuropyschological Rehabilitation. 24.* 590-606.

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Two Statistical Paradigms for Testing

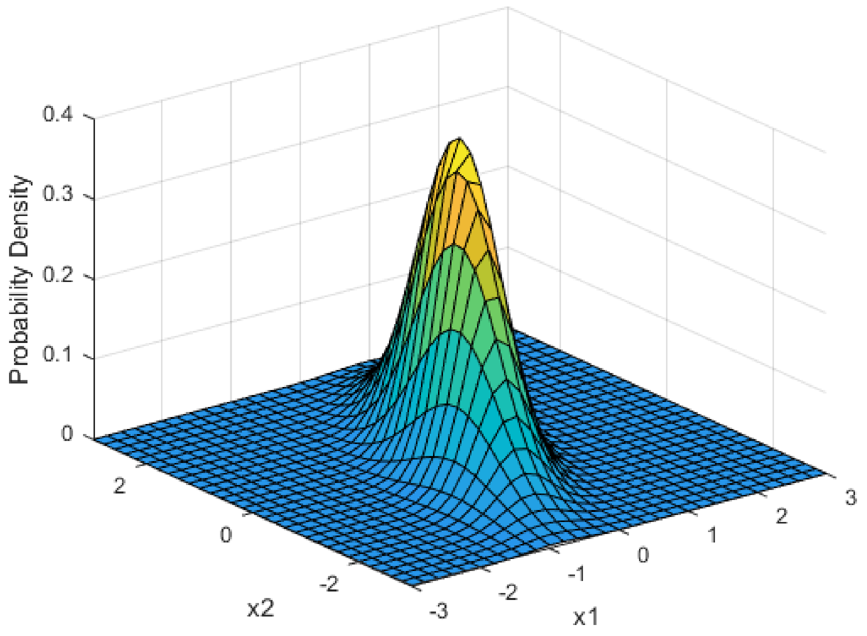
What warrant do we have for trusting estimates?

BAYESIAN LOGIC



Posterior Distribution

FREQUENTIST LOGIC



Sampling Distribution

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Sources of Epistemic Warrant

A philosophical analysis of the justification for a 95% Confidence Interval

Harold Jeffreys

We are 95% confident this interval encloses the truth because of inductive logic

Bayes' rule

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

We are 95% confident this interval encloses the truth because our formula captures it 95% of the time.

Jerzy Neyman

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Frequentist Paradigm

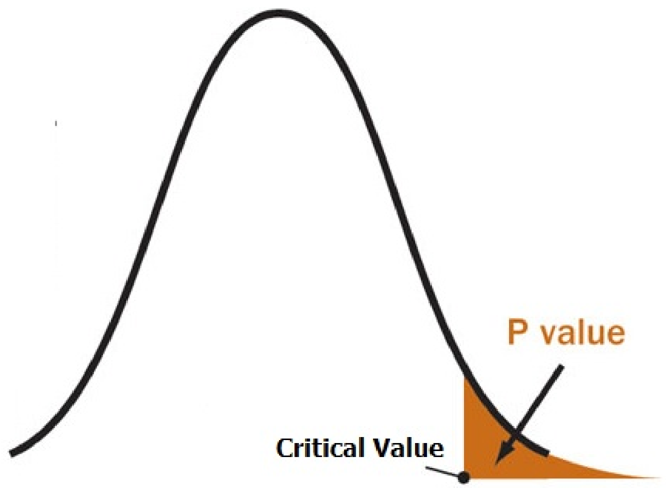
Eliminate null hypotheses from the competition with *p*-values

Common Approaches to testing

- Parametric Methods
- Bootstrapping Methods
- Non-Parametric Methods

Note. All of these approaches have practical limitations for statistical testing in SCD research.

FREQUENTIST *P*-VALUE



Would randomization turn up data more inconsistent with the null hypothesis than observed data?

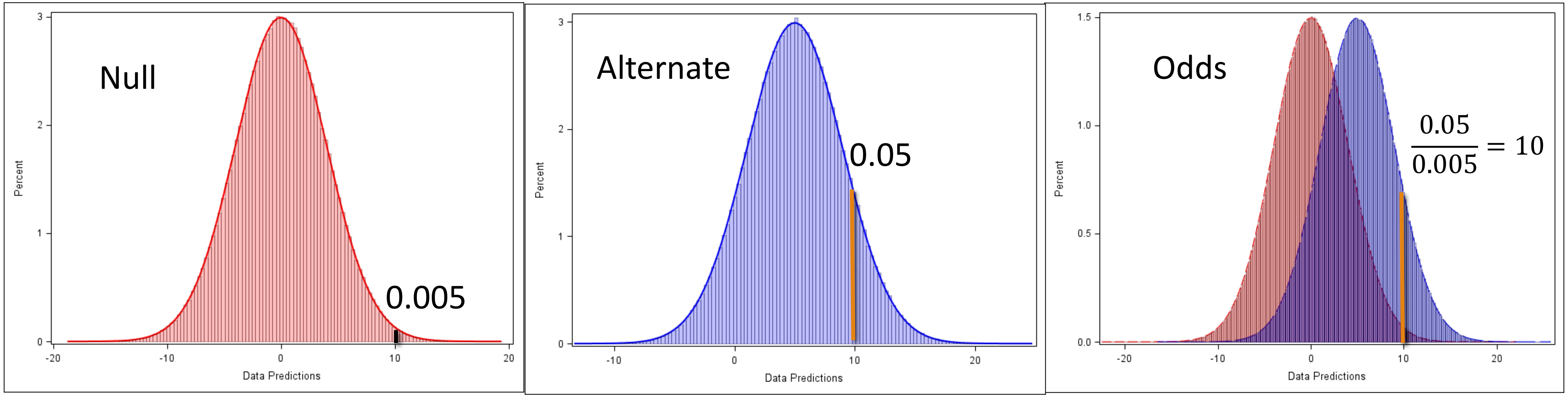
$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Bayesian Paradigm

Conceptual Equation:

Posterior odds= Prior odds*Bayes' factor

Model selection focuses on Bayes' Factor Analysis



Note. We induce the alternate makes the data 10 times likelier than the null.

Source for further information:

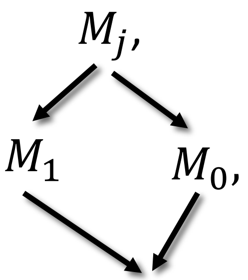
Rouder, J.N., Speckman, P.L., Sun, D., & Morey, R.D. (2009). Bayesian *t* tests for accepting and rejecting the null hypothesis. *Psychonomic Bulletin and Review. 16.* 225-237.

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

MCMC Simulation

Circumventing integral calculus to compute Bayes' Factor

- M_0 and M_1 denote null and alternate models
  - M_0: *μ=0.25*
  - M_1: *μ=0.75*
  - $P(\theta) \sim Beta(a = \mu * 12, b = (1 - \mu) * 12)$ *Note. Subjective prior*
- M_j denotes a model index variable
  - Outputs two values of 1 (alternate model) and 0 (null model)
  - $P(M_j)$~Bern(0.5) *Note.* Neutral Prior



If $M_j = 1$ then μ=0.25

If $M_j = 0$ then μ=0.75

R ~ Binomial(N,θ), Data: R=6, N=9

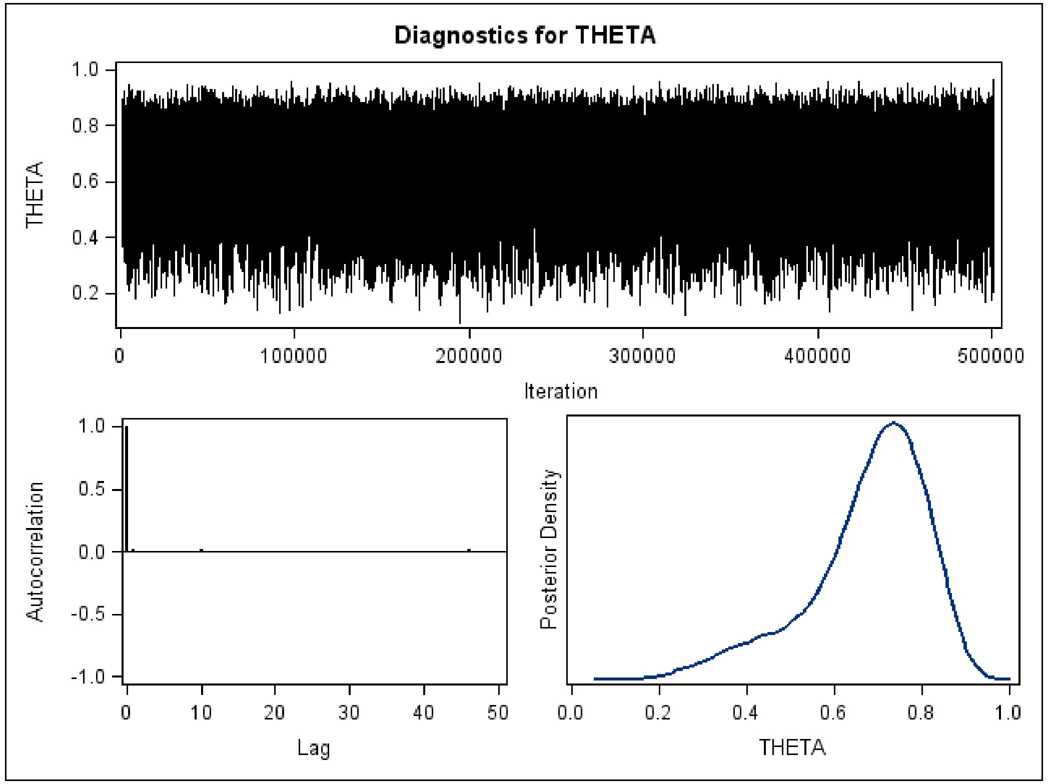
```
PROC MCMC DATA=ZERO NBI=1000 NMC=5000000
THIN=10 MONITOR=(Mj);

        PARM THETA Mj;

        HYPERPRIOR Mj ~ BINARY(0.5);
        IF Mj=0 THEN MU=0.25; *Null;
        IF Mj=1 THEN MU=0.75; *Alternate;

        PRIOR THETA~BETA(MU*12, (1-MU)*12);
        *12 was an arbitrary constant;

        MODEL R ~ BINARY(N,THETA);
        ODS OUTPUT PostSumINT=BayesFactor;
RUN;
```

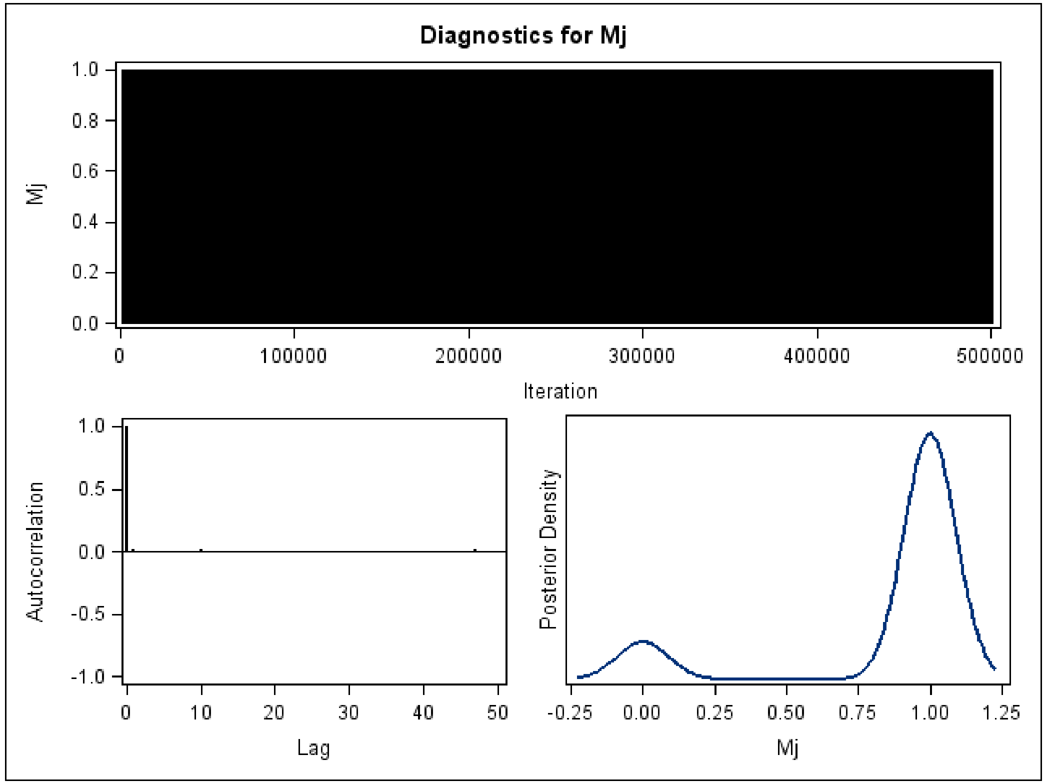
Note. I borrowed the above example from John Kruschke's chapter on the same topic. Kruschke , J. K. (2011) *Doing Bayesian data analysis: A tutorial with R and BUGS* [Chapter 12]. New York, NY*:* Elsevier.

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Output

PROC MCMC results





$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

MCMC Simulation
Circumventing integral calculus

```
PROC MCMC DATA=ZERO NBI=1000 NMC=5000000
THIN=10 MONITOR=(Mj);

       PARM THETA Mj;

       HYPERPRIOR Mj ~ BINARY(0.5);
       IF Mj=0 THEN MU=0.25; *Null;
       IF Mj=1 THEN MU=0.75; *Alternate;

       PRIOR THETA~BETA(MU*12, (1-MU)*12);
       *12 was an arbitrary constant;

       MODEL R ~ BINARY(N,THETA);
       ODS OUTPUT PostSumINT=BayesFactor;
RUN;
```

```
DATA BayesFactor; SET BayesFactor;
      A_BF=Mean/(1-MEAN);
      N_BF=(1-MEAN)/MEAN; RUN;

PROC PRINT DATA=BayesFactor;
      RUN;
```

Parameter	Mean	A_BF
Mj	0.8661	6.46558

Approximate Bayes' Factor, alternate over null.

$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$

An alternate idea for Bayesian testing

PROC MCMC code

```
PROC MCMC DATA=ZERO NBI=1000 NMC=500000
THIN=10;

 PARM THETA;

 MU=0.75; *Alternate;

 PRIOR THETA~BETA(MU*12, (1-MU)*12);

 MODEL R ~ BINOMIAL(N, THETA);

 PREDDIST NSIM=50000 OUTPRED=ALTERNATE;

 RUN;
```

New line requests data predictions, $p(\hat{y}|y, m_1)$

```
PROC MCMC DATA=ZERO NBI=1000 NMC=500000
THIN=10;

 PARM THETA;

 MU=0.25; *Null;

 PRIOR THETA~BETA(MU*12, (1-MU)*12);

 MODEL R ~ BINOMIAL(N, THETA);

 PREDDIST NSIM=50000 OUTPRED=NULL;

 RUN;
```

$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$

An alternate idea for Bayesian testing

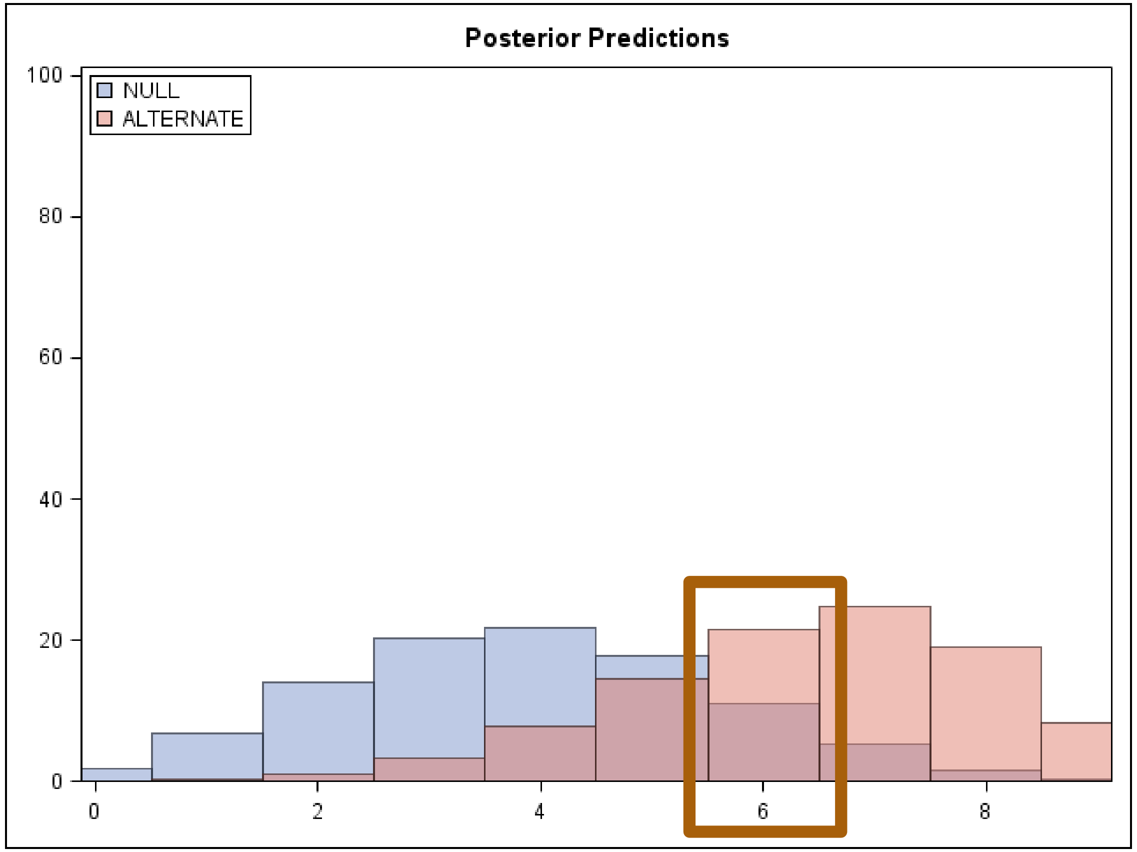
PROC MCMC results

$p(\hat{y}|y, m_0)$ =0.1097

$p(\hat{y}|y, m_1) = 0.2139$

R=1.950

We would more confidently predict the observed data using the alternate than the null. This evidence the alternate is more consistent with obtained data.



Discussion

A Few Talking Points

- Is there a place for statistical modeling in SCD research project?
  - Formalization could add quantitative precision to inferences
  - Intervention-builders could isolate factors to amplify desirable effects
- What is the warrant for our parameter estimates
  - Bayesian paradigm (Bayes' rule)
  - Frequentist paradigm (Asymptotic considerations)
- The need for direction and guidance
  - We demonstrated a general approach to approximating Bayes' factor modeling in PROC MCMC
  - Bayes' rule warrants Bayes' factor analysis in SCD research and, thereby, we do no need to appeal to asymptotic considerations to analyze data from SCD research to defense estimates.

Thanks

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