Bayesian Structural Equation Models with Small Samples: A Systematic Review

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M3 conference - May 24, 2017

Quotes

- Rupp et al., 2004: “Bayesian parameter estimation is more appropriate than ML estimation for smaller sample sizes (...).”

- Kruschke et al., 2012: “Bayesian methods can be used regardless of the overall sample size or relative sample sizes across conditions or groups.”

Goal

Is it valid to use Bayesian instead of Maximum Likelihood estimation for SEM when the sample size is small?

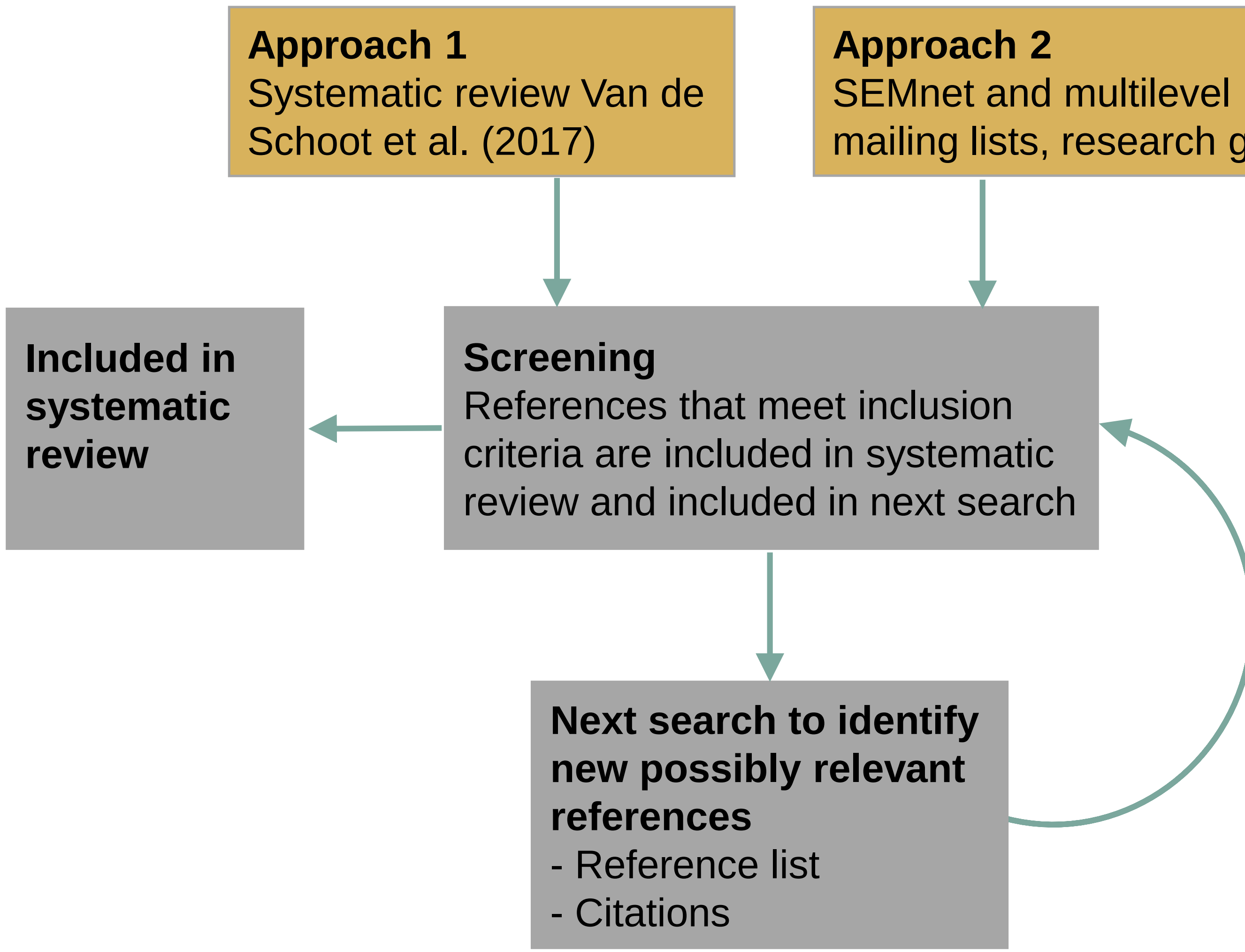
- Systematic literature review

Methods – Inclusion Criteria

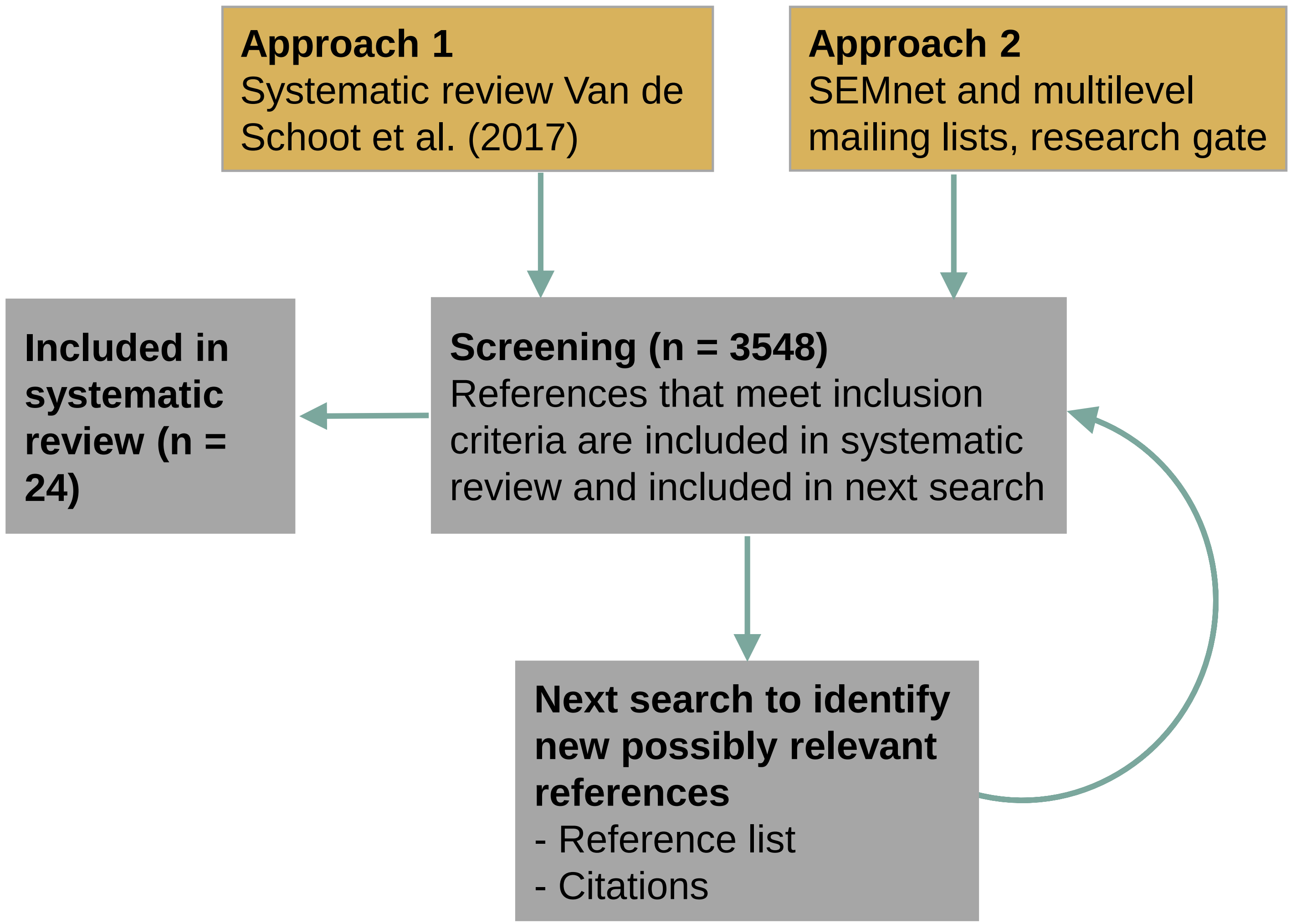
- Simulation study
- Bayesian parameter estimation vs Maximum Likelihood
- Small sample sizes
- Structural Equation Models

- Peer-reviewed articles
- Field: social sciences

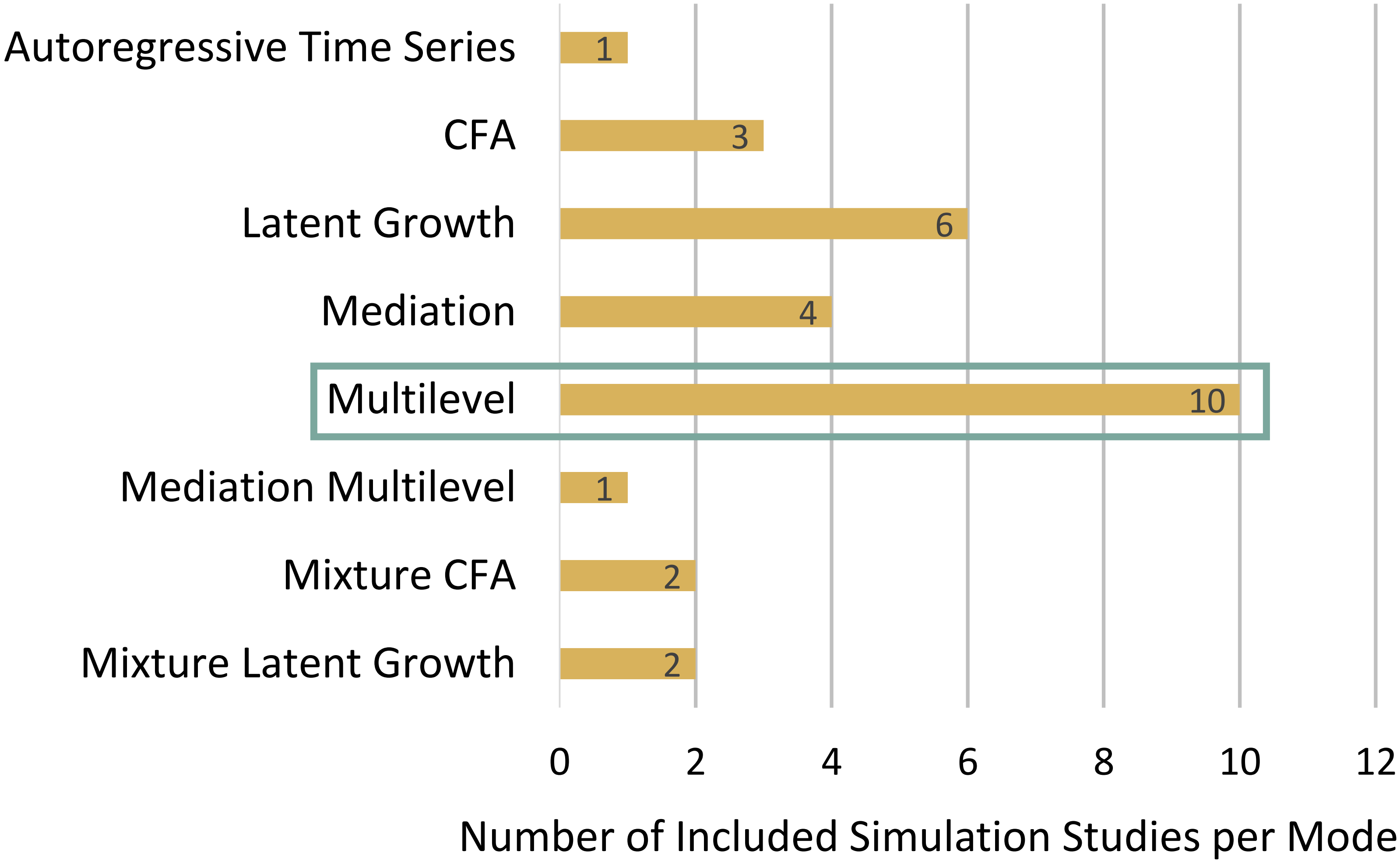
Methods – Searches



Methods – Searches



Results



Autoregressive Time Series
CFA
Latent Growth
Mediation
Multilevel
Mediation Multilevel
Mixture CFA
Mixture Latent Growth
1
3
6
4
10
1
2
2
0
2
4
6
8
10
12
Number of Included Simulation Studies per Model

Results – Sample Size

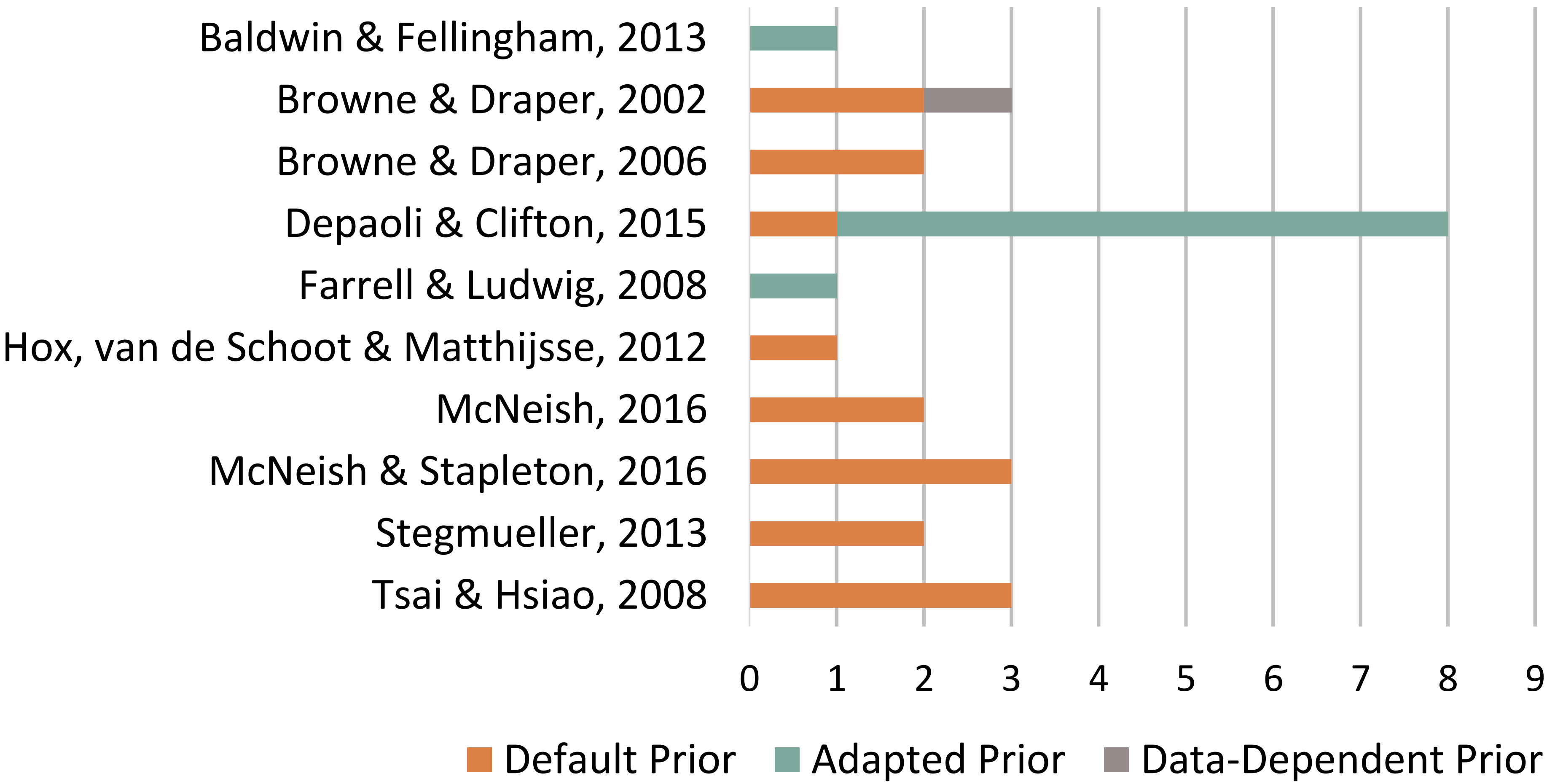
Studies	Number of clusters	Cluster size
Baldwin & Fellingham, 2013	**8, 16**	**5, 15**
Browne & Draper, 2002	<u>12</u>, 48	(un)balanced, mean = 18
Browne & Draper, 2006	<u>6</u>, <u>12</u>, 24, 48	(un)balanced, mean = 18
Depaoli & Clifton, 2015	**40**, 50, 100, 200	**5**, 10, 20
Farrell & Ludwig, 2008	(i) 20; (ii) 5; (iii) 80	(i) **20**, 80, 500; (ii) 500; (iii) **20**
Hox, van de Schoot & Matthijsse, 2012	**10, 15, 20**	1755
McNeish, 2016	**8, 10, 14**	7-14
McNeish & Stapleton, 2016	**4, 8, 10, 14**	7-14, 17-34
Stegmueller, 2013	**5, 10, 15, 20, 25, 30**	500
Tsai & Hsiao, 2008	**15**	**6**

Bold = defined as a small sample size by the authors of the original paper.
<u>Underlined</u> = not defined by the authors of original paper, defined by authors of current study.

Results – Priors

- Default prior = general prior, ‘naive’ use of Bayes
- Adapted prior = specific prior information included
- Data-dependent prior = partly based on Maximum Likelihood estimate

Results – Priors



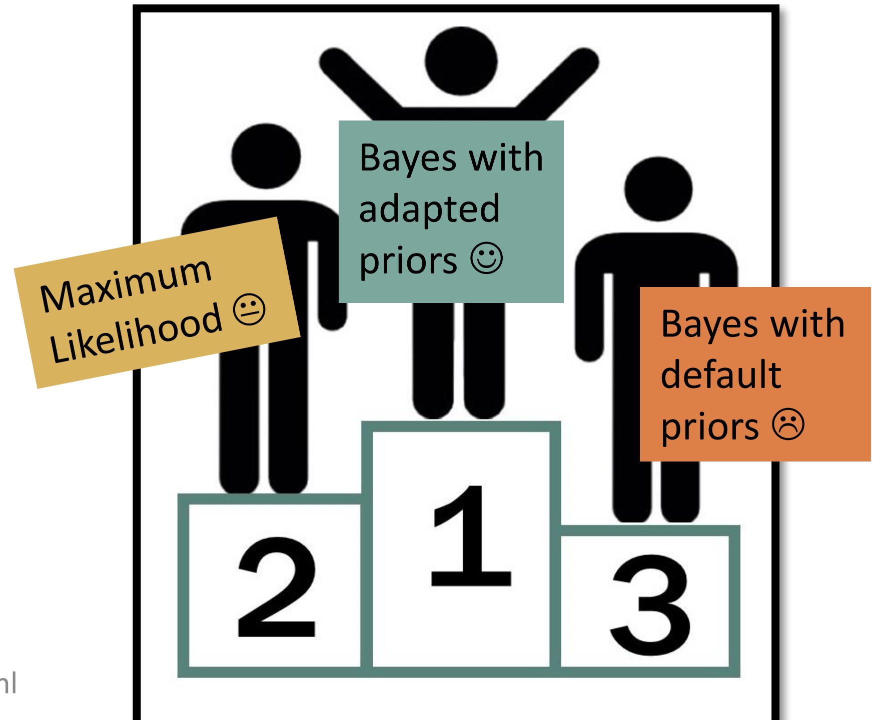
Results – Bayes vs ML

n = 3 studies investigated adapted priors

n = 1: no clear difference between ML and Bayes

n = 2: Bayes adapted > ML

and ML > Bayes default



Results – Bayes vs ML

n = 3 studies investigated adapted priors

n = 1: no clear difference between ML and Bayes

n = 2: Bayes adapted > ML, and ML > Bayes default

n = 1 study investigated default and data-dependent priors

n = 1: no clear difference between ML and Bayes

n = 6 studies investigated only default priors

n = 2: ML > Bayes default

Results – Bayes vs ML

n = 3 studies investigated adapted priors

n = 1: no clear difference between ML a

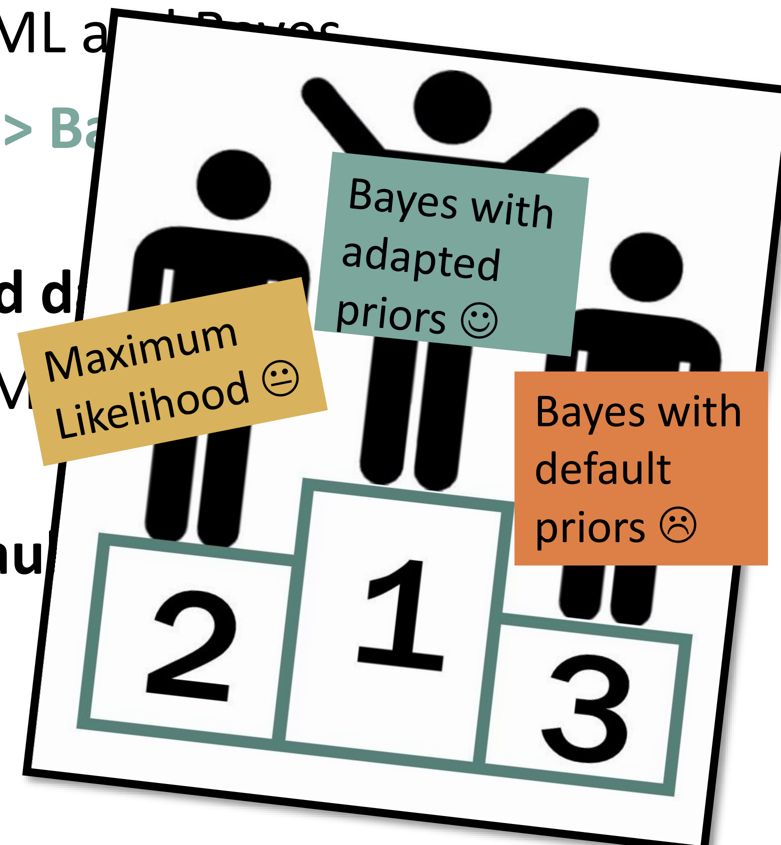
n = 2: Bayes adapted > ML, and **ML > Ba**

n = 1 study investigated default and d

n = 1: no clear difference between M

n = 6 studies investigated only defau

n = 2: **ML > Bayes default**



Results – Bayes vs ML

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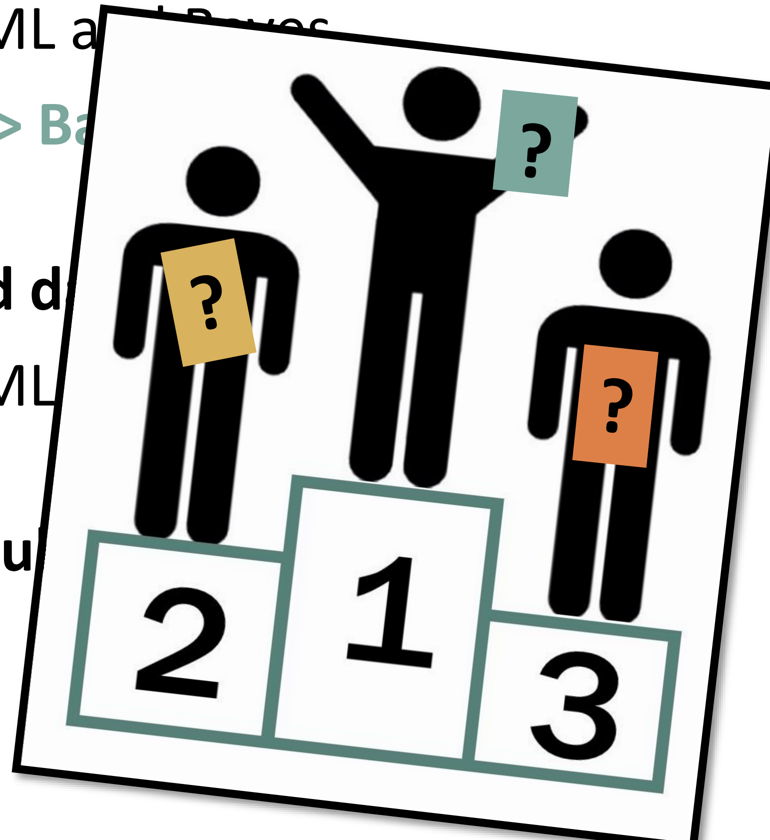
n = 2: **ML > Bayes default**

n = 4: **Bayes default > ML**



Results – Bayes vs ML

With a small sample size, performance of Bayes with default priors is worse than ML!

- High bias in variance components
- Default prior ≠ noninformative prior when the sample size is small!

McNeish (2016): “With small samples, the idea of noninformative priors is more myth than reality.”

Results – Bayes vs ML

- **Latent Growth Model:** Variance of latent slope is highly biased (McNeish, 2016)

- **Mixture Model**: Prior on the class proportions seems to be really important! (Depaoli, 2012; Depaoli, 2013)

- **CFA:** Large differences in performance of 3 default priors, especially with small samples (Van Erp , Mulder, Obserski, submitted)

Conclusion

Is it valid to use Bayesian instead of Maximum Likelihood estimation for SEM when the sample size is small?

- Bayesian estimation can have advantages
- Never naively use default priors

when the sample size is small!

Choose your priors carefully!

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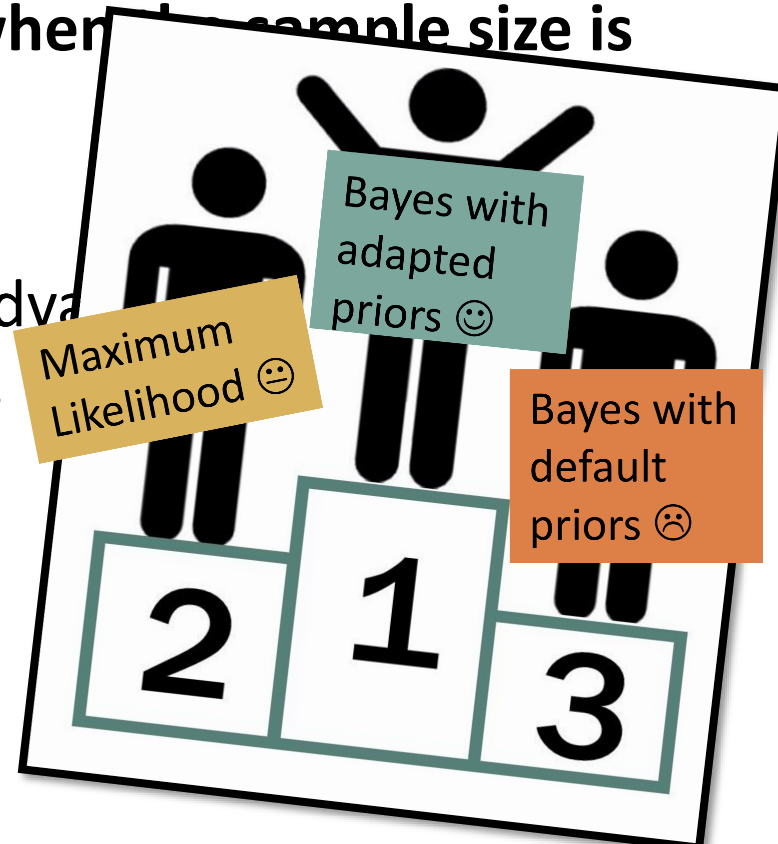

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References Other Models

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CFA
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