Dynamical Systems Analysis in the Context of Statistical Methods and Research Design

Steven M. Boker

Department of Psychology
University of Virginia

Modern Modeling Methods

University of Connecticut
May 23, 2017

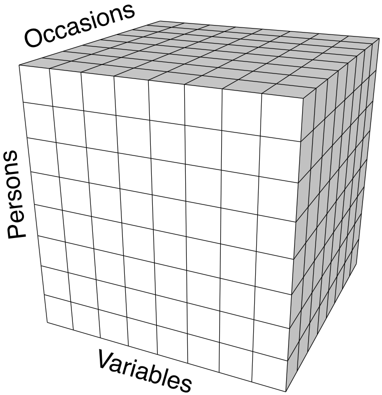
Three Types of Questions

Example: Cognitive Aging

- Policy makers ask about population statistics.
  1. Is the incidence of dementia increasing?
  2. What is the average age for first need of long term care?
- Clinicians ask about a person.
  1. What is the likelihood this person has dementia?
  2. Does this person need long term care?
- Educators ask about learning and development.
  1. How effective is an intensive cognitive training program?
  2. Are there age related differences in cortical plasticity?
- As methodologists we want to answer all these questions.

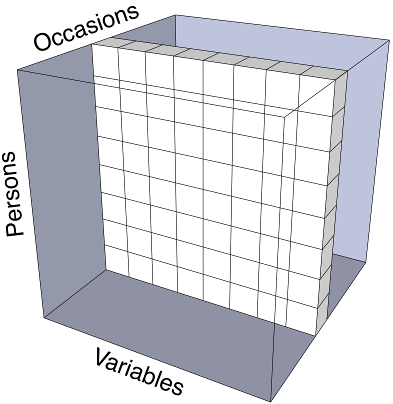
Cattell's Data Box

(Cattell, 1966)



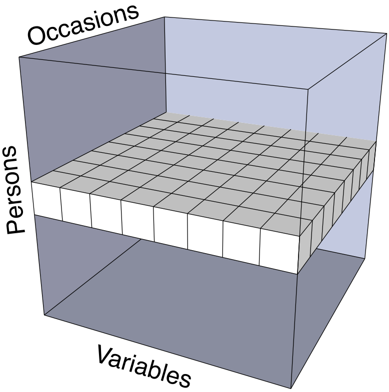
Data can be organized as *persons* × *variables* × *occasions*.

The Policy Maker's Questions



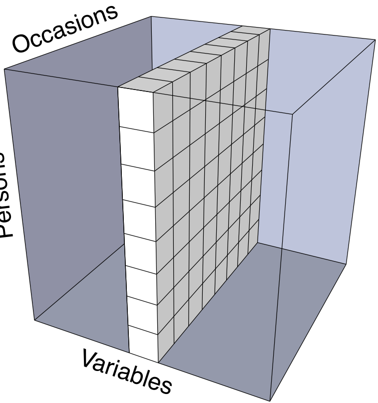
The policy maker requires *persons* × *variables* data and models.

The Clinician's Questions



The clinician requires *variables* × *occasions* data and models.

The Educator's Questions



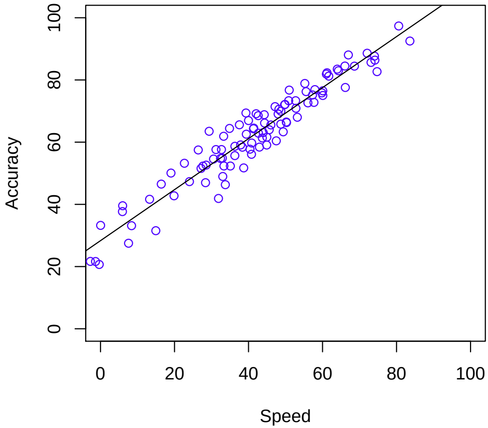
The educator requires *persons* $\times$ *occasions* data and models.

Integrating the Three Dimensions of the Data Box

- For any slice of the data box, consider the empty parts of the data box as a missing values problem.
  - Selecting on a particular person, variable, or occasion is not "missing at random" unless persons, variables, and occasions are all representative.
- Can we substitute one slice of the data box for another?
  - Molenaar (2004) concludes "no" by using an ergodicity argument.
- To illustrate the problem consider typing speed and accuracy.

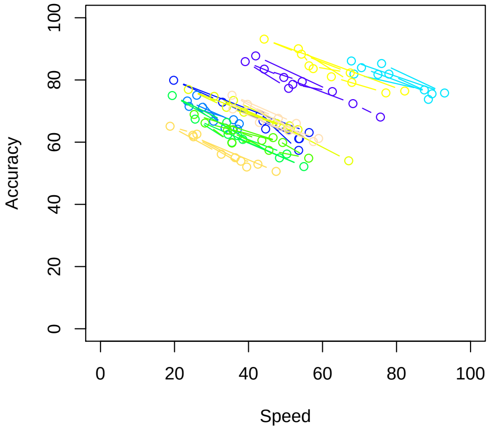
Typing Speed vs Accuracy – The Policy Maker

"Is typing speed and accuracy related in the population?"



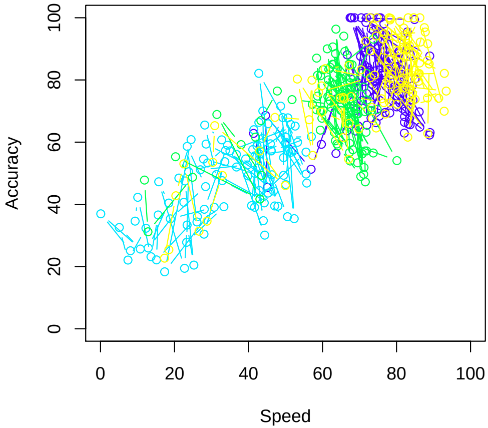
Typing Speed vs Accuracy – The Clinician

"If I ask a client to focus on typing faster, will that lead to greater accuracy?"



Typing Speed vs Accuracy – The Educator

Is it true that one cannot train a person to simultaneously type faster and more accurately?

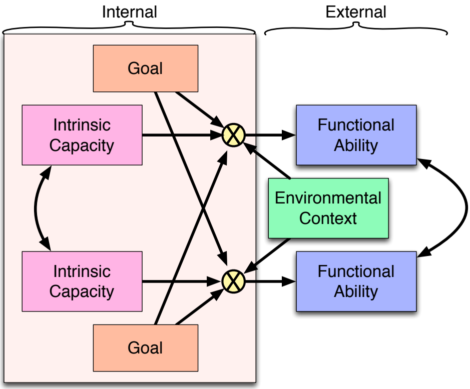


Time Dependence is not so Easy

- The data box metaphor breaks down on the occasions dimension.
- Time scale can matter.
  - Short term regulation may show a different time dependence than long term plasticity.
  - Contexts can change with time, leading to change in time dependence at multiple time scales.
  - All of this may be subject to feedback.

Intrinsic Capacity and Functional Ability

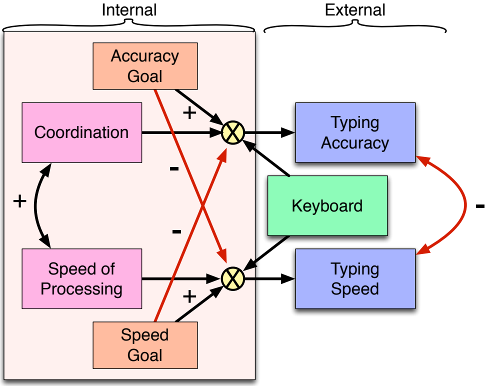
WHO Working Group on Metrics and Research Standards for Healthy Ageing



A model for time dependence and interaction with the environment.

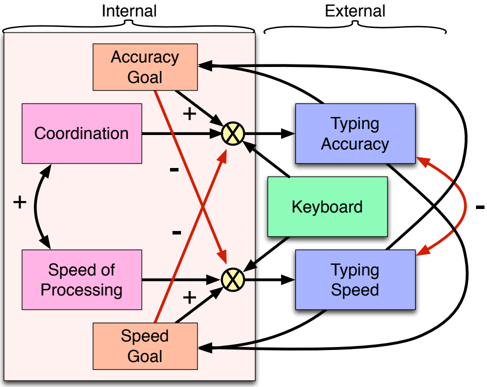
Typing Speed vs Accuracy

Instantiating Intrinsic Capacity and Functional Ability



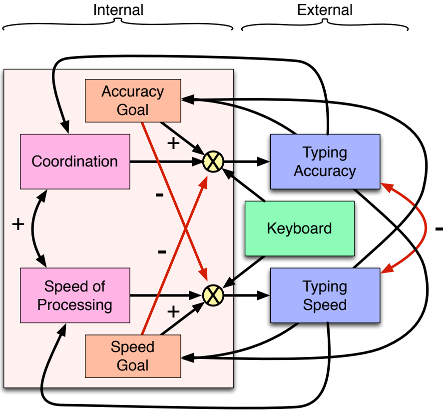
Positive between persons effects and negative within person effects.

Feedback of Performance on Goals



Perceptions can affect goals.

Feedback of Long Term Practice



Long term practice can lead to plasticity.

Dynamical Systems Analysis

- Dynamical systems analysis is a way to organize thinking about time dependency and feedback.
- Many have contributed to this field(e.g., Arnold, 1974; Bergstrom, 1966; Hotelling, 1927; Kalman, 1960; Oud & Jansen, 2000; Riley, Balasubramaniam, Mitra, & Turvey, 1998; Turvey, 1990)
- Here are five questions to think about.
  1. How does an individual regulate about an equilibrium, i.e., what are the dynamics?
  2. Are there individual differences in dynamics and equilibrium values?
  3. Are there age-related changes in equilibria?
  4. Are there age-related changes in dynamics?
  5. Are there age-related changes in complexity?

Modeling Complex Change

- In order to understand time dependency we need models for change.
- Differential equations explicitly model change.
- We will look at two simple systems to illustrate this.
  1. A first order linear system (exponential decay).
  2. A second order linear system (damped linear oscillator).

Resilience as Linear Damping

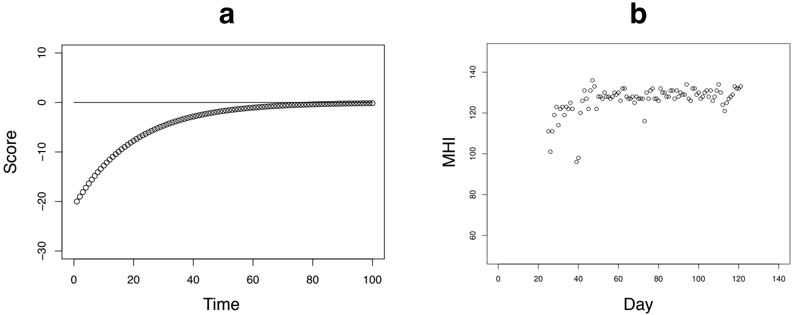
Example from (Boker, Montpetit, Hunter, & Bergeman, 2010)

- What would be necessary conditions for a regulatory system to bring the individuals back to equilibrium?
  1. Be able to detect the displacement from equilibrium.
  2. Be able to react in such a way that change in the individual's state led to a decrease in the displacement.
- A simple–minded model for resilience: the change in state with respect to time is a linear function of the displacement from equilibrium.

$$\dot{y}(t) = \zeta y(t)$$

- If $\zeta < 0$, "The farther I am from equilibrium, the faster I want to return to equilibrium."

Resilience as Linear Damping



(a) First order linear differential equation model of resilience.
(b) Mental Health Inventory scores from 90 days self–report from a recent widow, where loss of spouse occurred on day=0 (data from Bisconti, Bergeman, & Boker, 2004).

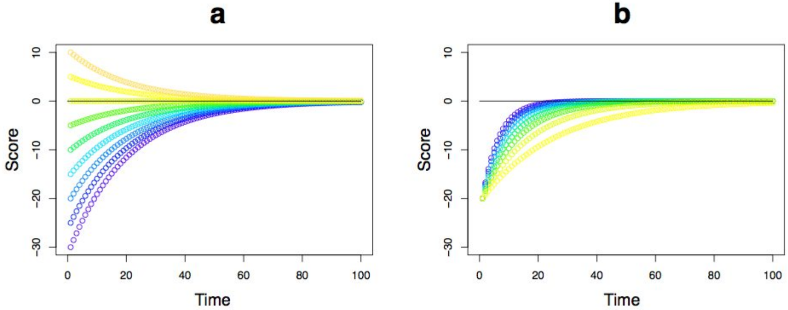
Resilience as Linear Damping

- The initial condition is not specified in this equation.

$$\dot{y}(t) = \zeta y(t)$$

- The equation specifies the shape of the bowl, but not where the marble starts.
- Thus, this is a model of *how* someone regulates but not a model of *what* the context is.

Resilience as Linear Damping



(a) The effect of individual differences in initial conditions.
(b) The effect of individual differences in regulation, i.e., $\dot{y}(t) = \zeta_i y(t)$ for person i.

Resilience as a Perfect Zero–Length Spring

- Elasticity is a property of an object that relates how the object changes shape, i.e., deforms, under stress.
- The deformation is a displacement from equilibrium and the stress is a force.
- In the "elastic regime" of an object, there is a linear relationship between the force exerted on the object and the displacement from equilibrium.
- A spring that has zero length (i.e., stretches for any displacement) is modeled by Hooke's Law.

$$F(t) = \eta x(t)$$

where $F(t)$ is the force exerted by the spring at time t, η is a constant called the *stiffness parameter* and $x(t)$ is the displacement from equilibrium.

Resilience as a Perfect Zero–Length Spring

- Force is related to acceleration through Newton's second law of motion, $F = ma$, force is equal to the mass of an object times its acceleration.
- Thus we can re–express Hooke's Law as

$$\begin{aligned} F(t) &= \eta x(t) \\ ma &= \eta x(t) \\ m\ddot{x}(t) &= \eta x(t) \\ \ddot{x}(t) &= \frac{\eta}{m} x(t) \end{aligned}$$

where m is mass and $\ddot{x}(t)$ is acceleration, the second derivative of $x(t)$ with respect to time.

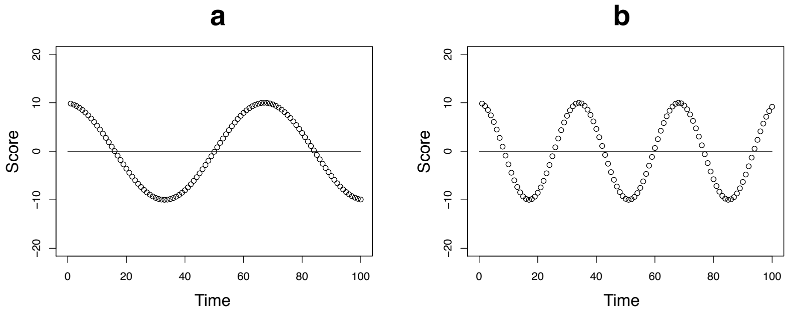
Resilience as a Perfect Zero–Length Spring

- If we assume that the mass of a human regulatory system is equal to one — we argue that the scale for mass is arbitrary in the context of human resilience — then Hooke's Law becomes

$$\ddot{x}(t) = \eta x(t)$$

  where $x(t)$ is the displacement from equilibrium at time t, η is a constant less than zero called the *stiffness parameter* in the context of elasticity, and $\ddot{x}(t)$ is the second derivative with respect to time at time t.
- If $\eta < 0$ then we might say "The farther I am from equilibrium, the more I want to accelerate back towards equilibrium."

Resilience as a Perfect Zero–Length Spring



(a) Slow oscillations result from a spring with less stiffness.
(b) Faster oscillations result from greater spring stiffness.

Resilience as a Zero–Length Spring with Damping

- A real spring, or rubber ball, or other physical system that exhibits elasticity tends to dissipate stress into heat.
- The elasticity of the system is not perfect, it is damped by friction or a mechanism such as viscosity — a rubber ball tends to eventually stop bouncing.
- Consider an automobile's springs and shock absorbers.
  1. A sports car with very stiff springs and very high damping shock absorbersdoes not sway relative to the road but you feel every little bump.
  2. An old pickup truck with worn springs and shock absorbers bounces up and down for a long time after a bump, but small bumps are taken up by the soft springs.
  3. A luxury sedan has springs that are matched with the shock absorbers and the mass of the body of the car so that most bumps in the road pass unnoticed but also does not bounce much.

Resilience as a Zero–Length Spring with Damping

- Suppose $y(t) = \dot{x}(t)$, so the damping equation becomes

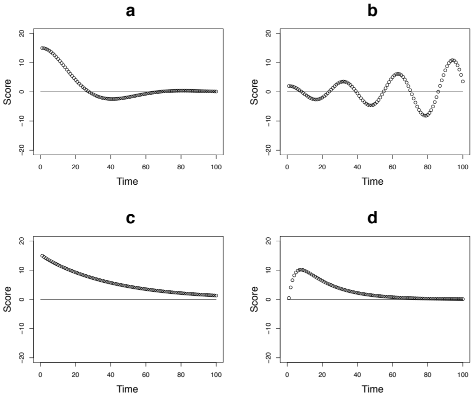
$$\begin{aligned} \dot{y}(t) &= \zeta y(t) \\ \ddot{x}(t) &= \zeta \dot{x}(t) \end{aligned}$$

- We can now combine the two previous models as a linear combination

$$\ddot{x}(t) = \zeta \dot{x}(t) + \eta x(t)$$

- When $\zeta < 0$ and $\eta < 0$, “The farther I am from equilibrium, the more I want to turn and go back towards equilibrium, but at the same time the faster I go, the more I want to slow down.”

Resilience as a Zero–Length Spring with Damping



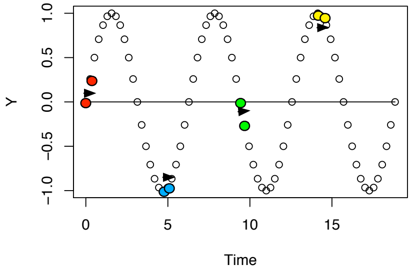
(a) A slow frequency with high damping may only cross the equilibrium once.
(b) A positive ζ coefficient leads to amplification of oscillations.
(c) An overdamped system does not cross the equilibrium at all.
(d) The same system as (c), but with different initial conditions.

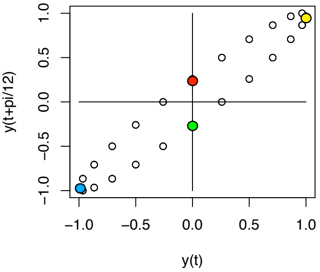
Discrete Time versus Continuous Time

- Autoregressive and cross-lag models use discrete time modeling.
- We have been talking about continuous time modeling.
- Why not just use auto- and cross-regression?
- Discrete time modeling confounds sampling interval and time dependence in the model parameters (Oud & Jansen, 2000; Oud, 2007; Singer, 1993).

Time–Delay Embedded State Space and Lag

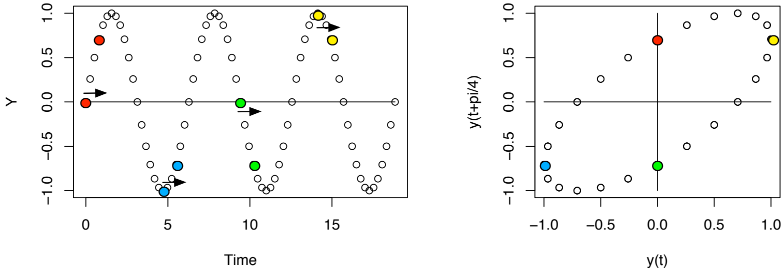
$r(y_{(t)}, y_{(t+\pi 12)}) = .97$





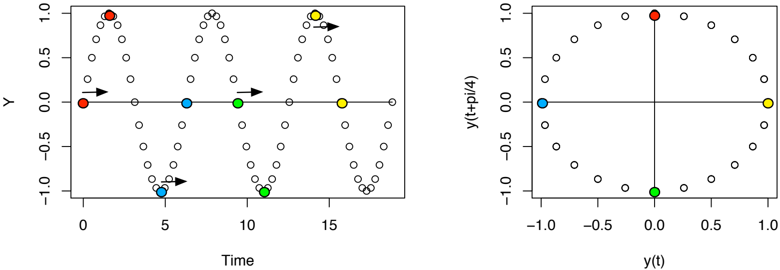
Time–Delay Embedded State Space and Lag

$r(y_{(t)}, y_{(t+\pi 4)}) = .7$



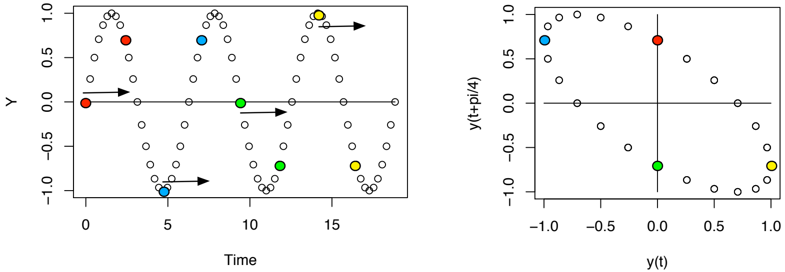
Time–Delay Embedded State Space and Lag

$r(y_{(t)}, y_{(t+\pi 2)}) = 0$



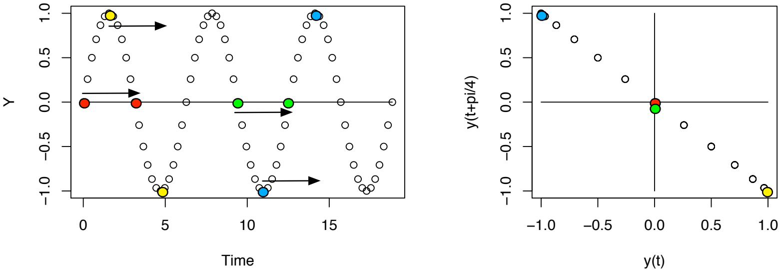
Time–Delay Embedded State Space and Lag

$r(y_{(t)}, y_{(t+3\pi4)}) = -.7$



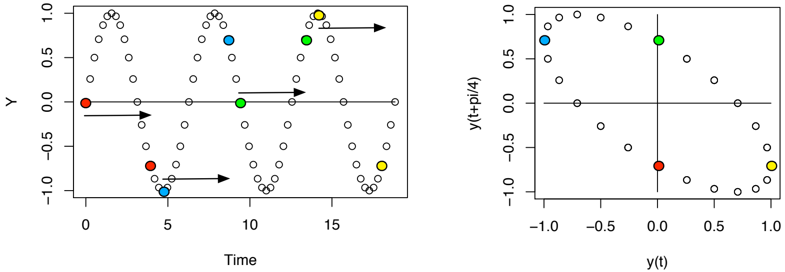
Time–Delay Embedded State Space and Lag

$r(y_{(t)}, y_{(t+\pi)}) = -1.0$



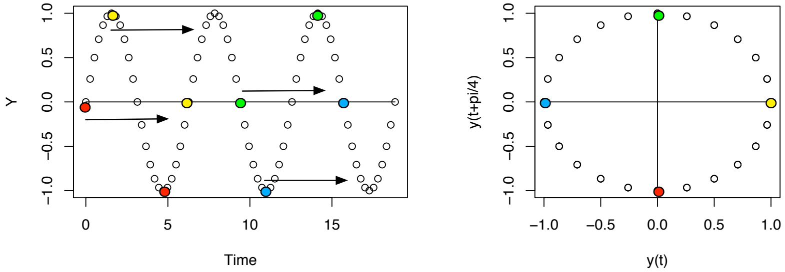
Time–Delay Embedded State Space and Lag

$r(y_{(t)}, y_{(t+3\pi/2)}) = -.7$



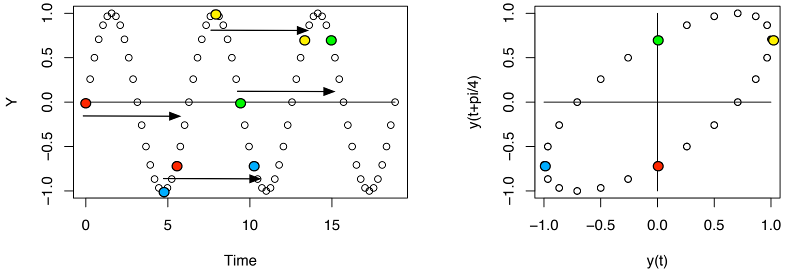
Time–Delay Embedded State Space and Lag

$r(y_{(t)}, y_{(t+3\pi/2)}) = 0.00$



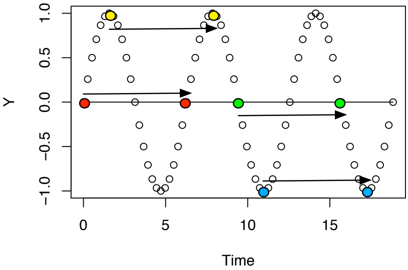
Time–Delay Embedded State Space and Lag

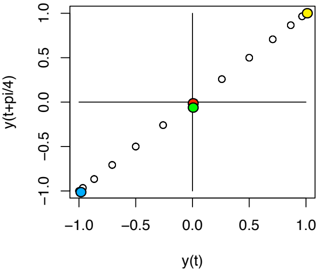
$r(y_{(t)}, y_{(t+7\pi/4)}) = 0.7$

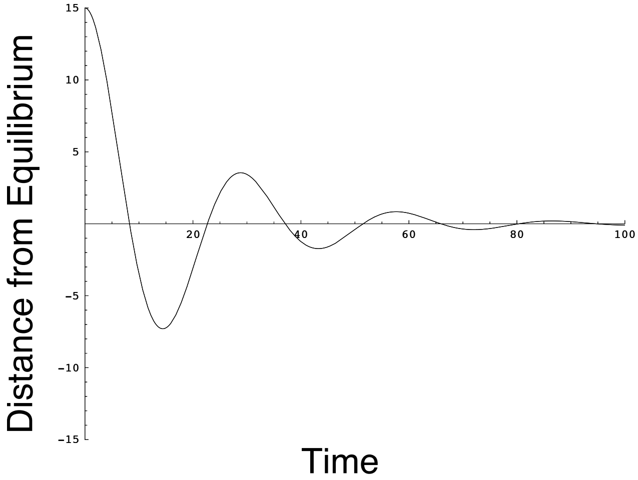


Time–Delay Embedded State Space and Lag

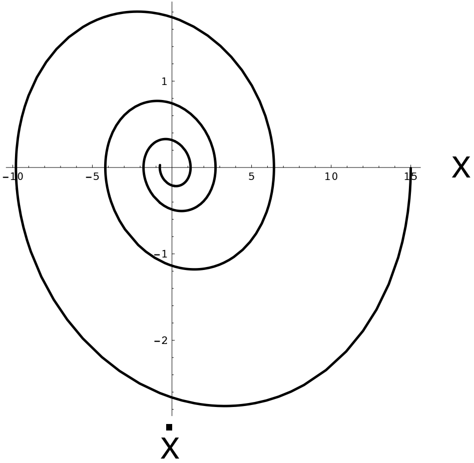
$r(y_{(t)}, y_{(t+2\pi)}) = 1.0$





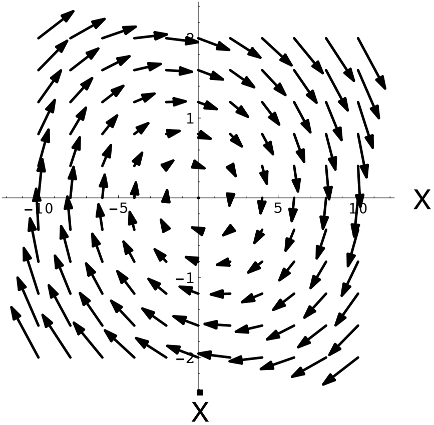


Time series plot of the damped zero length spring



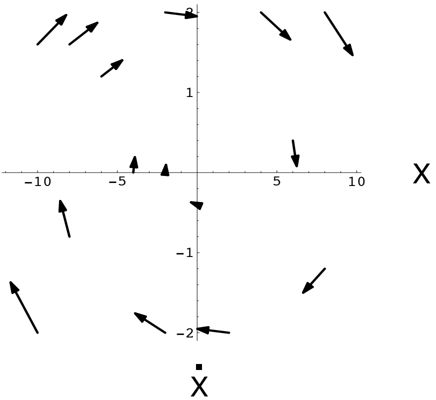
State space plot of the damped zero length spring

Equilibria and Basins of Attraction



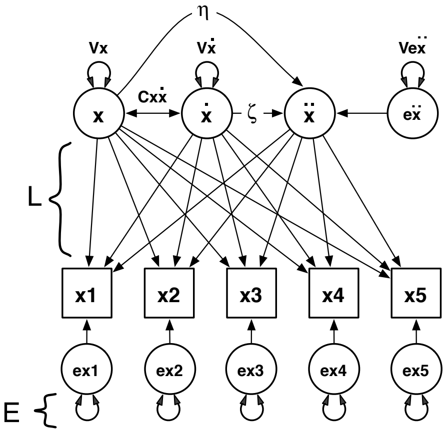
Vector field plot of the damped zero length spring.

Sampling from an Attractor



A sample from the vector field can reconstruct the attractor.

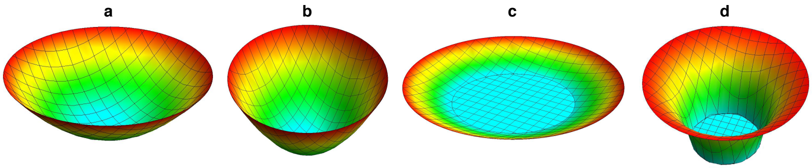
SEM Model to Estimate a Resilience Attractor



Dynamical Systems Analysis Questions (Redux)

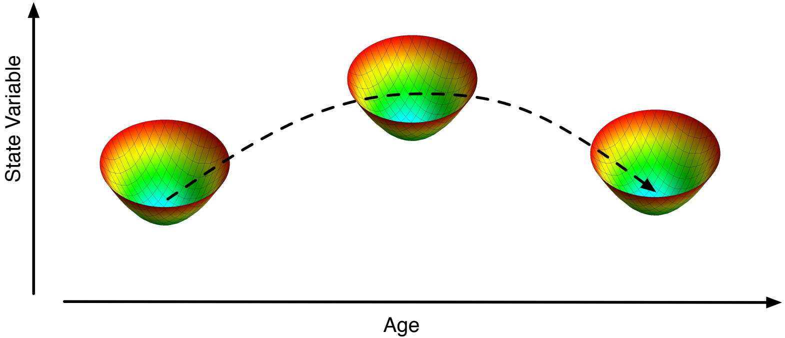
- How does each individual regulate about an equilibrium?
- Are there individual differences in
  - dynamics and equilibrium values?
  - changes in equilibria?
  - changes in dynamics?
  - changes in complexity?

Individual Differences in Attractors



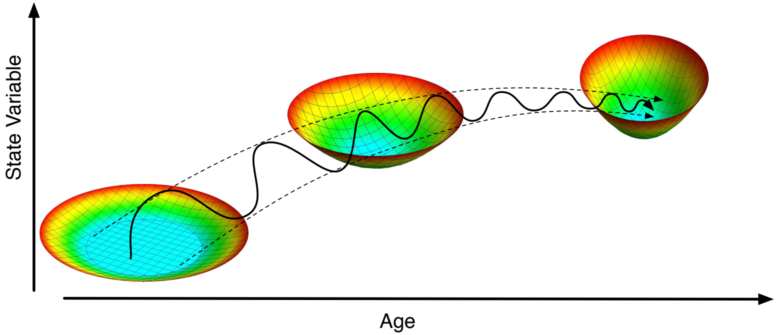
Individuals may differ in how they regulate

Age-Related Change in Equilibrium



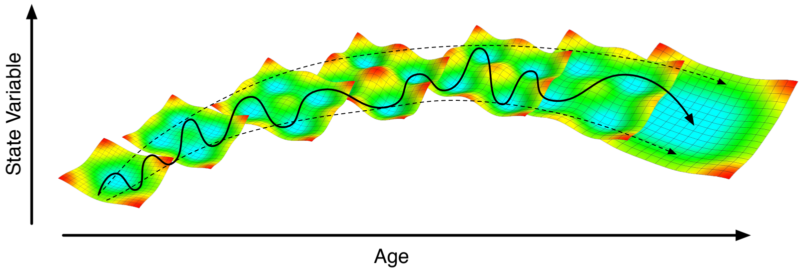
The equilibrium value for a state variable may change with age.

Age-Related Change in Dynamics



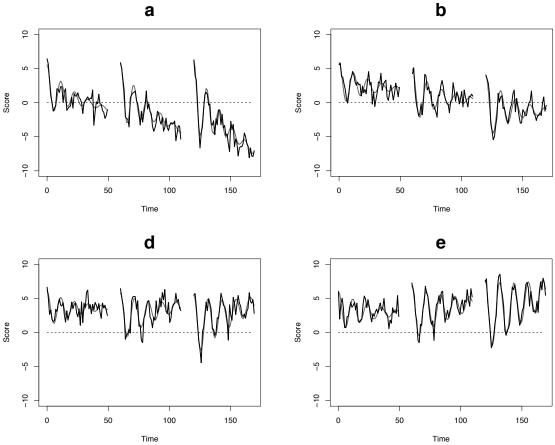
Age–related change in regulation visualized as a changing basin of attraction. Note that both decreased amplitude of fluctuations as well as higher speed fluctuations are associated with the narrowing of these attractor basins.

Age Related Change in Equilibrium Complexity



Age–related changes in complexity of selection represented as as developmental change in complexity of basins of attraction. In this hypothetical example, a single basin of attraction in childhood bifurcates into four basins of attraction which then collapse into a single basin of attraction in old age.

Multiscale Dynamics



Multiple bursts with longitudinal change in dynamics and both equilibrium intercept and slope.

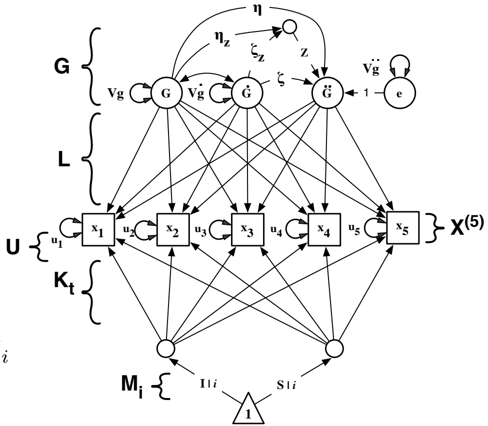
A Multi-Level Multi-Timescale Path Model

Combination of Latent Differential Equation and Latent Growth Curve

$$\ddot{g}_{ij} = \eta_i g_{ij} + \zeta_i \dot{g}_{ij} + e_{ij}$$
$$\eta_i = \eta + \eta_z z_i$$
$$\zeta_i = \zeta + \zeta_z z_i$$

$$\mathbf{X}_i^{(5)} = \mathbf{M}_i \mathbf{K}_t + \mathbf{G}\mathbf{L} + \mathbf{U}_i$$

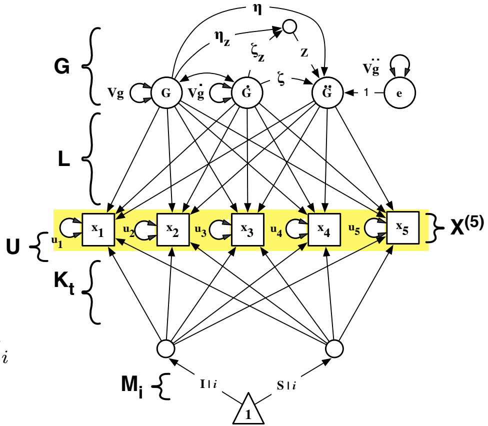
(Boker, Staples, & Hu, 2016)



A Multi-Level Multi-Timescale Path Model

Time Delay Embedded Data

$$\ddot{g}_{ij} = \eta_i g_{ij} + \zeta_i \dot{g}_{ij} + e_{ij}$$
$$\eta_i = \eta + \eta_z z_i$$
$$\zeta_i = \zeta + \zeta_z z_i$$



$$\mathbf{X}_i^{(5)} = \mathbf{M}_i \mathbf{K}_t + \mathbf{GL} + \mathbf{U}_i$$

(Boker et al., 2016)

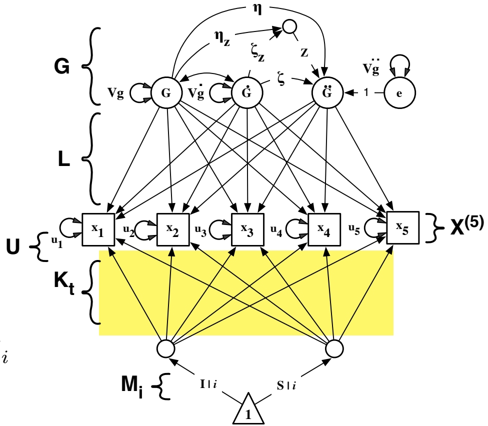
A Multi-Level Multi-Timescale Path Model

Growth Curve Loadings Calculated per Individual and per Elapsed Time

$$\begin{aligned} \ddot{g}_{ij} &= \eta_i g_{ij} + \zeta_i \dot{g}_{ij} + e_{ij} \\ \eta_i &= \eta + \eta_z z_i \\ \zeta_i &= \zeta + \zeta_z z_i \end{aligned}$$

$$\mathbf{X}_i^{(5)} = \mathbf{M}_i \mathbf{K}_t + \mathbf{G}\mathbf{L} + \mathbf{U}_i$$

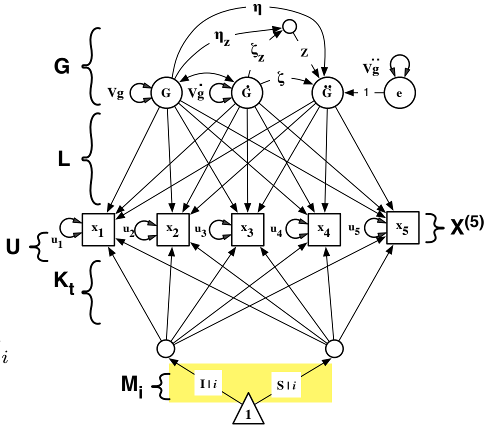
(Boker et al., 2016)



A Multi-Level Multi-Timescale Path Model

Equilibrium Multilevel Slope and Intercept

$$
\begin{aligned}
\ddot{g}_{ij} &= \eta_i g_{ij} + \zeta_i \dot{g}_{ij} + e_{ij} \\
\eta_i &= \eta + \eta_z z_i \\
\zeta_i &= \zeta + \zeta_z z_i
\end{aligned}
$$



$$
\mathbf{X}_i^{(5)} = \mathbf{M}_i \mathbf{K}_t + \mathbf{GL} + \mathbf{U}_i
$$

(Boker et al., 2016)

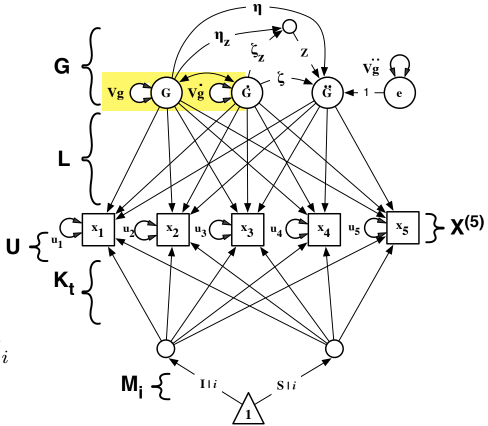
A Multi-Level Multi-Timescale Path Model

Variance of Displacement and First Derivative

$$\begin{aligned}\ddot{g}_{ij} &= \eta_i g_{ij} + \zeta_i \dot{g}_{ij} + e_{ij} \\ \eta_i &= \eta + \eta_z z_i \\ \zeta_i &= \zeta + \zeta_z z_i\end{aligned}$$

$$\mathbf{X}_i^{(5)} = \mathbf{M}_i \mathbf{K}_t + \mathbf{G}\mathbf{L} + \mathbf{U}_i$$

(Boker et al., 2016)

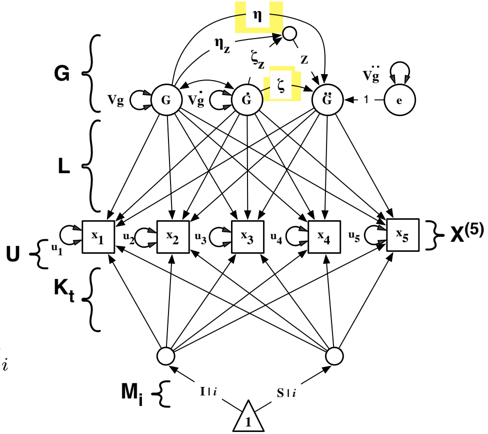


A Multi-Level Multi-Timescale Path Model

Fixed Effects for Stiffness and Damping Parameters

$$\begin{aligned} \ddot{g}_{ij} &= \eta_i g_{ij} + \zeta_i \dot{g}_{ij} + e_{ij} \\ \eta_i &= \eta + \eta_z z_i \\ \zeta_i &= \zeta + \zeta_z z_i \end{aligned}$$

$$\mathbf{X}_i^{(5)} = \mathbf{M}_i \mathbf{K}_t + \mathbf{G}\mathbf{L} + \mathbf{U}_i$$



(Boker et al., 2016)

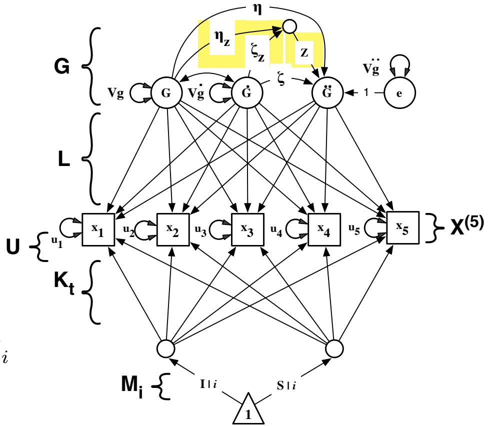
A Multi-Level Multi-Timescale Path Model

Second Level Predictor of Stiffness and Damping

$$
\begin{aligned}
\ddot{g}_{ij} &= \eta_i g_{ij} + \zeta_i \dot{g}_{ij} + e_{ij} \\
\eta_i &= \eta + \eta_z z_i \\
\zeta_i &= \zeta + \zeta_z z_i
\end{aligned}
$$

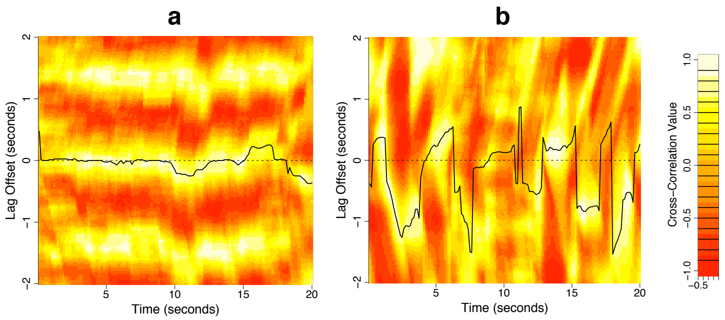
$$\mathbf{X}_i^{(5)} = \mathbf{M}_i \mathbf{K}_t + \mathbf{G}\mathbf{L} + \mathbf{U}_i$$

(Boker et al., 2016)



Nonstationarity

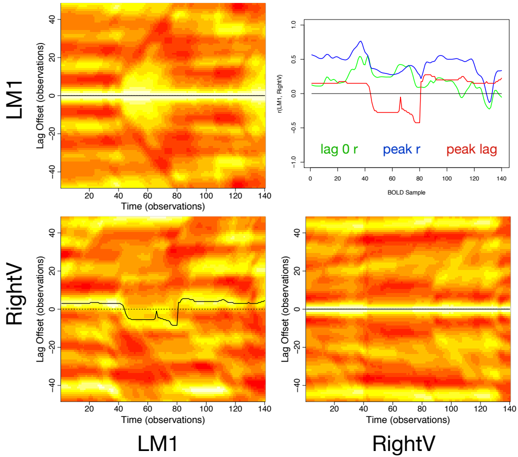
- When equal–N contiguous subsamples of a time–series give time dependent estimates of the statistical properties of the full time–series, it is said to be nonstationary.
- Consequences of nonstationarity.
  - Each sample of occasions *may not* be representative.
  - It *does* matter when you start sampling.
  - Including more occasions of measurement in your analysis *may not* be better since this can lead to less sensitive estimates of changes in statistical properties.
  - If you split occasions of measurement into two halves, differences in model parameter estimates may be due to a combination of nonstationarity and unreliability.
- Nonstationarity may be reliable and should be estimated.



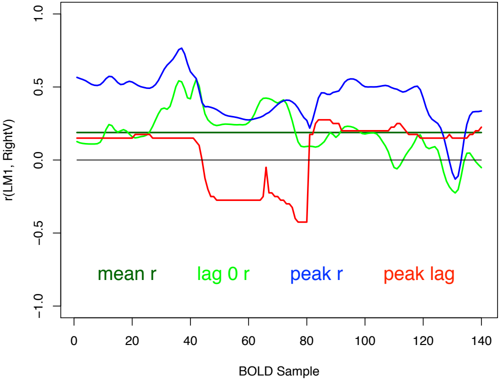
Density plots of WCC from (a) body velocities in dance, and (b) from head velocity in conversation.

1. Dance data exhibits stationarity.
2. Conversation data exhibits nonstationarity but still has a discernable pattern.

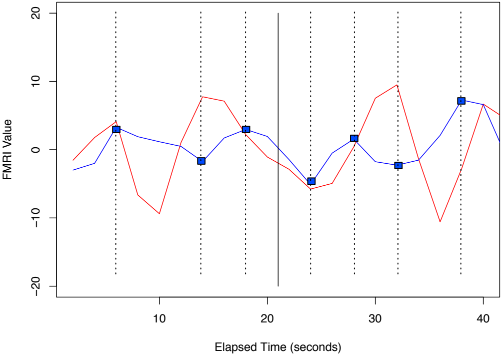
Nonstationarity Attenuates Unlagged Correlation



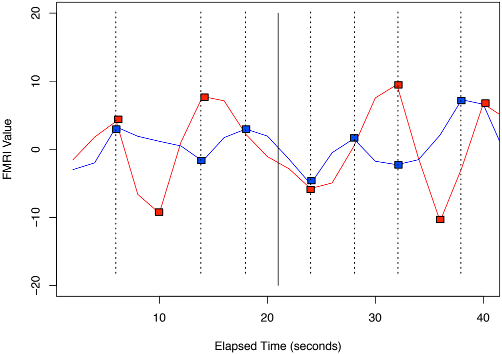
Nonstationarity Attenuates Unlagged Correlation



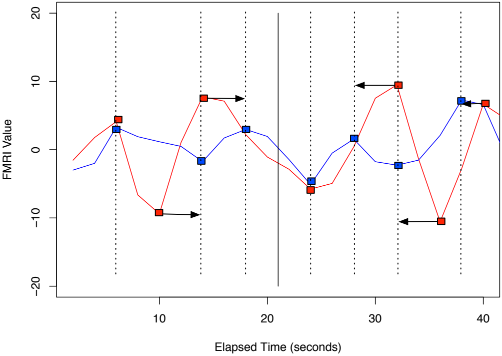
Latent Feature Decomposition



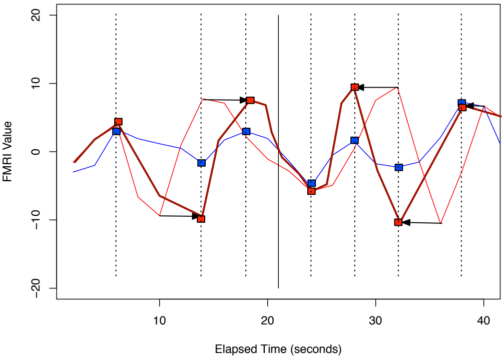
Latent Feature Decomposition



Latent Feature Decomposition



Latent Feature Decomposition

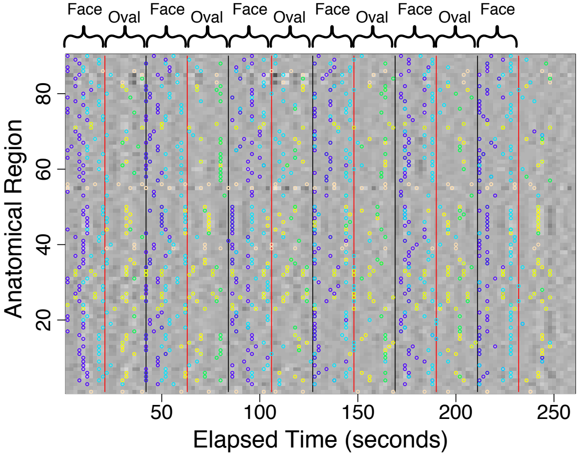


Latent Feature Decomposition

- Three types of data come back from the LFD Algorithm.
  1. Time Index of the peaks.
  2. Time Warp applied to each peak.
  3. Signal Value at each peak.
- We reduce the dimensionality of these data with PCA.
- We simplify the loading structure with an oblique rotation.
- The signals are then “un-warped” and areas with large loadings and large component scores are plotted.

BOLD Signal Value Components

8 components plotting standardized loadings greater than 0.4



Latent Feature Decomposition

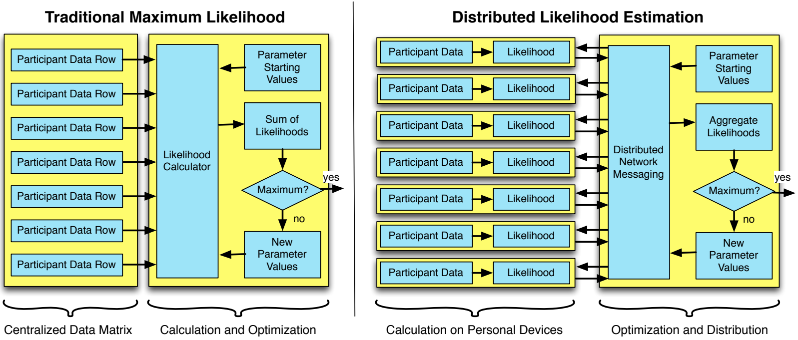
- Latent Feature Decomposition can help simplify nonstationary signals.
- LFD preserves desynchronization information.
- LFD does not attenuate correlations due to nonstationary lags.
- LFD can be applied to any continuous valued multivariate nonstationary time series
  - Motion capture from accelerometers or GPS.
  - EEG signals.
  - Physiological time series.
  - Daily hormone assays.
  - Intensive longitudinal behavioral questionnaires.

Rethinking Traditional Experimental Designs

See (Boker et al., 2015)

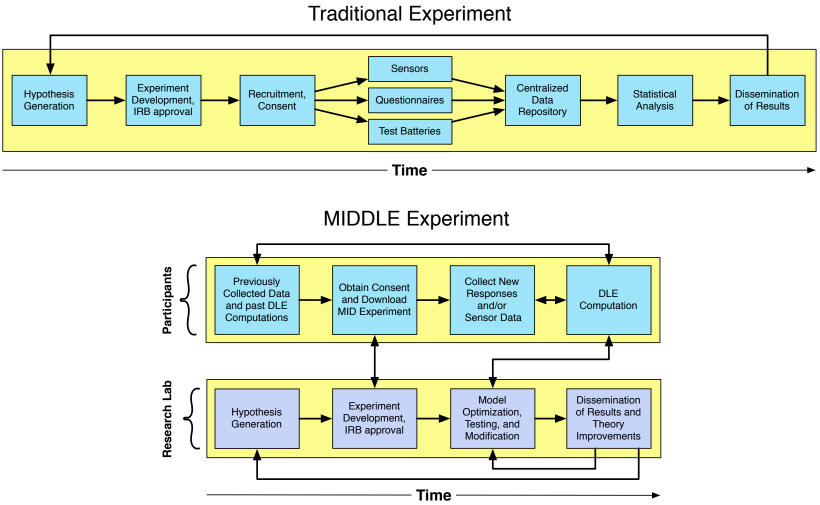
- Goals associated with traditional design are in conflict.
  1. Data sharing between experiments versus data ownership.
  2. Data linking versus data privacy.
  3. Longitudinal linking is practically impossible.
- The OpenMx team decided to re–think the traditional method.
- We asked: "Could participants maintain sole possession of their own data while experiments were run and statistical models estimated?"

Distributed Likelihood Estimation



- Traditional maximum likelihood uses a centralized data matrix.
- Distributed likelihood estimation calculates the likelihood of data stored on each personal device and *only returns the likelihood* to the centralized optimizer.

Traditional vs. MIDDLE Experiment



Advantages of MIDDLE

- Privacy
  - Participants maintain control and ownership of private data.
  - Since data are not revealed, participants may be more truthful.
- Dissemination of Results to Participants
  - Participants have results on their device and compare the results for the population to their own data and personalized model.
- Data Sharing and Linking‘
  - Data linking and sharing is automatic if a participant consents.
- Scalability
  - Likelihood calculation is distributed to where the data are stored.
- Just–In–Time Estimation
  - Statistical models are estimated while data are collected.
- Optimal Power
  - Stopping criterion can be a predetermined statistical power.

Thank You

Arnold, L. (1974). *Stochastic differential equations*. New York: Wiley.

Bergstrom, A. R. (1966). Nonrecursive models as discrete approximations to systems of stochastic differential equations. *Econometrica*, *34*, 173–182.

Bisconti, T. L., Bergeman, C. S., & Boker, S. M. (2004). Emotion regulation in recently bereaved widows: A dynamical systems approach. *Journal of Gerontology: Psychological Sciences*, *59*(4), 158–167.

Boker, S. M., Brick, T. R., Pritikin, J. N., Wang, Y., Oertzen, T. v., Brown, D., et al. (2015). Maintained individual data distributed likelihood estimation. *Multivariate Behavioral Research*, *50*, 726–740. (doi: 10.1080/00273171.2015.1094387, PMCID: PMC4804354)

Boker, S. M., Montpetit, M. A., Hunter, M. D., & Bergeman, C. S. (2010). Modeling resilience with differential equations. In P. Molenaar & K. Newell (Eds.), *Learning and development: Individual pathways of change* (pp. 183–206). Washington, DC: American Psychological Association.

Boker, S. M., Staples, A., & Hu, Y. (2016). Dynamics of change and changes in dynamics. *Journal of Person-Oriented Research*, *2*(1–2), 34–55.

Cattell, R. B. (1966). Psychological theory and scientific method. In R. B. Cattell (Ed.), *The handbook of multivariate experimental psychology* (pp. 1–18). New York: Rand McNally.

Hotelling, H. (1927). Differential equations subject to error, and population estimates. *Journal of the American Statistical Association*, *22*(159), 283–314.

Kalman, R. E. (1960). A new approach to linear filtering and prediction problems. *Transactions of the ASME Journal of Basic Engineering*, *82*, 35–45.

Molenaar, P. C. M. (2004). A manifesto on psychology as idiographic science: Bringing the person back into scientific psychology, this time forever. *Measurement: Interdisciplinary Research and Perspectives*, *2*(4), 201–218.

Oud, J. H. L. (2007). Continuous time modeling of reciprocal relationships in the cross-lagged panel design. In S. M. Boker & M. J. Wenger (Eds.), *Data analytic techniques for dynamical systems* (pp. 87–130). Mahwah, NJ: Lawrence Erlbaum Associates.

Oud, J. H. L., & Jansen, R. A. R. G. (2000). Continuous time state space modeling of panel data by means of SEM. *Psychometrika*, *65*(2), 199–215.

Riley, M. A., Balasubramaniam, R., Mitra, S., & Turvey, M. T. (1998). Visual influences on center of pressure dynamics in upright posture. *Ecological Psychology*, *10*(2), 65–91.

Singer, H. (1993). Continuous-time dynamical systems with sampled data, errors of measurement and unobserved components. *Journal of Time Series Analysis*, *14*, 527–545.

Turvey, M. T. (1990). Coordination. *American Psychologist*, *45*(8), 938–953.