Application of Generalized Structured Component Analysis to Item Response Theory

Modern Modeling Methods (5/25/2016)
Overview

• Introduction to generalized structured component analysis (GSCA)
  – As a structural equation model

• GSCA
  – Model specification, estimation, & evaluation

• Application
  – Item response theory in educational research

• Research topics
Intro to GSCA

• Structural equation modeling (SEM)
  – SEM has been used for the analysis of interdependencies among observed variables and underlying constructs, often called latent variables

• Components in SEM (using LISREL Model)
  – Measurement model
  – Structural model
Components in SEM (LISREL Model)

\[ \gamma = B\gamma + \zeta \]

\[
\begin{bmatrix}
\gamma_1 \\
\gamma_2 \\
\zeta_1 \\
\zeta_2
\end{bmatrix}
= \begin{bmatrix}
0 & 0 & \gamma_1 \\
b & 0 & \gamma_2
\end{bmatrix}
+ \begin{bmatrix}
\zeta_1 \\
\zeta_2
\end{bmatrix}
\]

**Measurement Model**

\[ z = C\gamma + \varepsilon \]

\[
\begin{bmatrix}
z_1 \\
z_2 \\
z_3 \\
z_4
\end{bmatrix}
= \begin{bmatrix}
c_1 & 0 \\
c_2 & 0 \\
0 & c_3 \\
0 & c_4
\end{bmatrix}
\begin{bmatrix}
\gamma_1 \\
\gamma_2
\end{bmatrix}
+ \begin{bmatrix}
\varepsilon_1 \\
\varepsilon_2 \\
\varepsilon_3 \\
\varepsilon_4
\end{bmatrix}
\]
Intro to GSCA

• To approaches to SEM

- **Factor-based approach**
  - Latents ≈ Factors
  - Covariance Structure Analysis (Jöreskog, 1970)

- **Component-based approach**
  - Latents ≈ Components
  - PLS Path Modeling (Wold, 1982)
  - **GSCA** (Hwang & Takane, 2004)
## Intro to GSCA

### Similarities and dissimilarities

<table>
<thead>
<tr>
<th>Model Specification</th>
<th>Factor-based</th>
<th>Component-based</th>
</tr>
</thead>
<tbody>
<tr>
<td>Latent variables</td>
<td>CSA</td>
<td>PLS</td>
</tr>
<tr>
<td>Factors (Random)</td>
<td>Components (Fixed)</td>
<td></td>
</tr>
<tr>
<td>Parameters</td>
<td>Loadings, Path coefficients, Error variances, Factor variances and/or means</td>
<td>Loadings, Path coefficients, Component weights</td>
</tr>
<tr>
<td>Input data</td>
<td>Covariance/Correlation</td>
<td>Individual-level raw data</td>
</tr>
<tr>
<td>Estimation method</td>
<td>ML (mainly)</td>
<td>Least squares</td>
</tr>
<tr>
<td>Global optimization function</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>
Intro to GSCA

• GSCA (Hwang & Takane, 2004)
  – Utilizes least square estimation method
  – Computes a composite component score using weights

• GSCA consists of three models
  – A measurement model: \( z = C'\gamma + \epsilon \)
  – A structural model: \( \gamma = B'\gamma + \zeta \)
  – A weighted relation model: \( \gamma = W'z \)
A Weighted Relation Model in GSCA

\[ \begin{align*}
\gamma_1 &= z_1 w_1 + z_2 w_2 \\
\gamma_2 &= z_3 w_3 + z_4 w_4 \\
\gamma &= Wz
\end{align*} \]
GSCA

- Model specification
  - GSCA consists of three models
    - A measurement model: \( z = C'\gamma + \epsilon \)
    - A structural model: \( \gamma = B'\gamma + \zeta \)
    - A weighted relation model: \( \gamma = W'z \)
GSCA

• Features in GSCA
  – No model identification problems and improper solutions
  – No rigid distributional assumptions
  – Stable parameter estimates even in small samples
GSCA

• Advantages over ML-based SEM & SEM with PLS
  – Avoid improper solutions by replacing factors with linear composites of observed variables (Same as in partial least squares (PLS; Wold, 1966, 1973, 1982))
  – Address the global optimization problem (Mulaik, 1972), which is an additional feature that PLS does not have
GSCA

• Parameter estimates
  – Least Square Criterion
    \[ \Phi = \sum_{j=1}^{J} SS(E_j) = \sum_{j=1}^{J} \text{tr}(E_j'E_j), \text{ where } E_j = [E_{Mj}, E_{Sj}] \]

  – Alternating Least Square Algorithm (de Leeuw, Young, & Takane, 1976)

  – Bootstrap method (Efron, 1982)
GSCA

• Model evaluation

  – Overall model fit measures (using variances)

  • \( FIT = 1 - \frac{SS(ZV-ZWA)}{SS(ZV)} = \frac{1}{T} \sum_{t=1}^{T} R_t^2 \) (Henseler, 2012)
    
    – Indicates the proportion of the total variance explained by a given particular model specification (Similar as the R-squared)
    
    – Can be used in model comparison with Bootstrapping standard errors or confidence intervals of the difference in FIT

  • \( AFIT = 1 - (1 - FIT) \frac{d_0}{d_1} \) (Hwang et al., 2007)
    
    – Adjusted FIT (Similar as the adjusted R-squared)
GSCA

• Model evaluation
  – Overall model fit measures (using covariances)

  • $GFI = 1 - \frac{\text{trace}[(S-\hat{\Sigma})^2]}{\text{trace}(S^2)}$ (Jöreskog & Sörbom, 1986)
    – Cut-off = higher than 0.9 (McDonal & Ho, 2002)

  • $SRMR = \sqrt{2 \sum_{j=1}^{J} \sum_{q=1}^{J} \frac{[s_{jq}-\hat{\sigma}_{jq}]^2}{s_{jj}s_{qq}J(J+1)}}$ (Hwang, 2008)
    – Cut-off = less than 0.08 (Hu & Bentler, 1999)
GSCA

• Model evaluation
  – Local model fit measures
    • \( FIT_M = 1 - \frac{SS(Z-ZWC)}{SS(Z)} \)
    • \( FIT_S = 1 - \frac{SS(ZW-ZWB)}{SS(ZW)} \)
  – Composite reliability (Werts et al., 1974)
    • \( \rho_p = \frac{\left( \sum_{j=1}^{Jp} c_{pj} \right)^2}{\left( \sum_{j=1}^{Jp} c_{pj} \right)^2 + \sum_{j=1}^{Jp}(1-c_{pj}^2)} \)
GSCA

• Applicability of GSCA
  – Nonlinear GSCA (NL-GSCA; Hwang & Takane, 2010) for non-normal distribution in SEM
  – Fuzzy clusterwise GSCA for group-level heterogeneity such as mixture modeling, latent class/transition analysis, clustering, or classification in ML-based SEM (Hwang, DeSarbo, & Takane, 2007)
  – Longitudinal and time series data analysis (Jung et al., 2012)
Application

• Data
  – Math achievement data from Test of Early Mathematics Ability – 3 (TEMA-3)
    • Measure of math concepts, processes, and knowledge skills for children ages from 3 years to 8 years
  – Participants: 389 children from state-funded and/or Head Start pre-kindergarten classrooms
    • 182 boys (46.7%) at the beginning of data collection
    • Average age of 54.46 month (47 to 59 months, SD=3.47)
Bi-factor model for TEMA-3 (Ryoo, et al., 2015)

where ‘f1’ is representing Counting objects, ‘f2’ is Verbal counting, ‘f3’ is Numerical comparison, ‘f4’ is Set construction, ‘f5’ is Numeral literacy, ‘f6’ is Number facts, and ‘f7’ is Calculation.
Application

• Data
  – Longitudinal study for three years
    • Fall and Spring of Pre-K, Spring of K, and Spring of 1st grade
  – Two sub-datasets were used in this study
    • Data1: Whole group of 294 at Spring of 1st grade
    • Data2: Its subgroup of 50% randomly selected children (N=147)
Bi-factor model for TEMA-3 (Ryoo, et al., 2015)
- Verbal counting factor (f2) -

where ‘f1’ is representing Counting objects, ‘f2’ is Verbal counting, ‘f3’ is Numerical comparison, ‘f4’ is Set construction, ‘f5’ is Numeral literacy, ‘f6’ is Number facts, and ‘f7’ is calculation.
Application

• To item response theory (IRT)
  – The two parameter logistic (2PL) model
    \[
    \pi_i = \frac{1}{1 + \exp(-\beta_{1i}(z - \beta_{2i}))}
    \]
  
  • Difficulty parameter \((\beta_{2i})\)
  • Discrimination parameter \((\beta_{1i})\)
  – 2PL model also produces examinee’s ability score \((\theta_j)\) where \(j\) denotes jth examinee
Application

• Ideas behind this application
  – Maximum likelihood estimate (MLE) used in IRT provides unbiased estimates when (1) sample data are large and (2) multivariate normality assumption are met
  – What if we have small sample data for ML-based IRT or if multivariate normality assumption are not met
    • Biased estimates and not efficient estimates
Application

• Alternatives
  – Small sample issue: non-parametric IRT
  – Multivariate normality issue: Bayesian IRT
  – But, we still have question about generalizing to big data (e.g., fMRI brain-imaging data (Jung, et al., 2012))

• GSCA accounts for both small sample and computer intensity for big data
Application

• GSCA accounts for both small sample and computer intensity for big data
  – Least square estimate (LS) provides unbiased point estimates regardless to distributional assumptions
    • Bootstrap estimates for interval estimates and/or hypothesis testing
  – LS estimation is efficient and computationally faster for both small and large samples
Application

• GSCA accounts for both small sample and computer intensity for big data
  – (Known) LS cannot be used for estimation for other distributional assumptions like binomial and multinomial – Really?
  • Not really (Hwang & Takane, 2010)
Application

• Nonlinear GSCA (Hwang & Takane, 2010)
  – Applying GSCA to qualitative data such as nominal and categorical data
  – How? Resolve the linearity issue afflicting LS methods by applying the optimal scaling method (Kruskal, 1964a,b; McDonald, 2000; Young, 1981)
Application

• Component-based IRT (CB-IRT)
  – Application of nonlinear GSCA to IRT
  – Estimation procedure
    • Phase one
      – Updating model parameters including loadings and weights
    • Phase two (optimal scaling phase)
      – Step 1: Updating the model prediction $\hat{s}_j$ corresponding to $s_j$ for fixed parameters from Phase one
      – Step 2: Obtaining the optimally transformed data $s_j$ such that it is as close to $\hat{s}_j$ as possible in the LS sense
Bi-factor model for TEMA-3 (Ryoo, et al., 2015) - Verbal counting factor (f2) -

where ‘f1’ is representing Counting objects, ‘f2’ is Verbal counting, ‘f3’ is Numerical comparison, ‘f4’ is Set construction, ‘f5’ is Numeral literacy, ‘f6’ is Number facts, and ‘f7’ is calculation.
Application

• Result of discrimination in 2PL model
Application

• Result of discrimination in CB-IRT model
Application

• Results
  – Estimates in CB-IRT over different sample sizes are relatively closer (Right)
  – 2PL provides relatively more consistent SEs (Left)
Application

• Ongoing research
  – Interpretable composite scores comparable to ability in ML-based IRT
  – Proper model comparison tools that can be used in differential item functioning, equating, and linking in IRT literature
Research topics in GSCA

• Model evaluation
  – Confirmatory Tetrad Analysis (CTA) for model comparison (Bollen & Ting, 1993)

• Application GSCA to longitudinal/multilevel data analysis
  – Multilevel latent class/transition analysis
  – Dynamic SEM
Reference


