Using multi-membership multilevel VAM model to evaluate teacher and school effects

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Background

- Value-added modeling (VAM) has been implemented to estimate school and teacher effects nationwide in order to take into account important factors that have impact on the school and teacher effect

- Multilevel modeling (MLM) is one of the modeling methods that has often been implemented in VAMs
  - Allows to capture important factors to be controlled to estimates accurate teacher and school effects (Bryk & Raudenbush, 1987; Laird & Ware, 1982; Raudenbush & Bryk, 2002)
  - Allows to count nesting structure of data which is typically seen in educational settings (e.g., Students are nested within teachers and schools)

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Background cont.

- Using simple MLM, it is hard to apply to some specific data structures that are commonly seen in educational settings (e.g., co-teaching)

- MLM can only deal with purely hierarchical data structure (e.g., one lower level is nested within only one higher level unit)

- If this issue is not adequately taken into account, VAM is likely to be inaccurate in terms of the estimate of the effectiveness of teachers and schools(Beretvas, 2010; Chung and Beretvas, 2012)

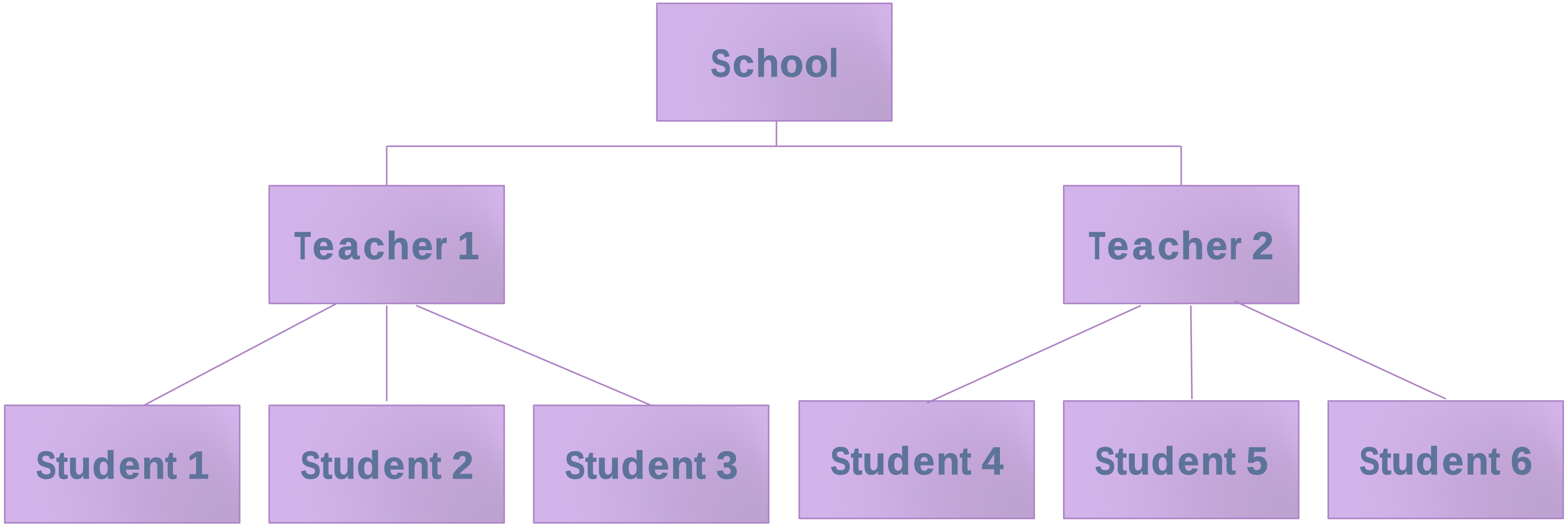
- Multi-membership multilevel modeling (MMML) allows the lower levels to be nested simultaneously within one or more upper levels.

Purpose

- Due to the complexity of the statistical technique, multi-membership multilevel (MMML) has not been utilized in developing VAMs with practical student learning data by the school districts.
- To illustrate how to implement MMML to develop VAMs in educational settings that students are nested within one or more teachers.

Simple VAM using multilevel modeling

- A student nested within a teacher within a school



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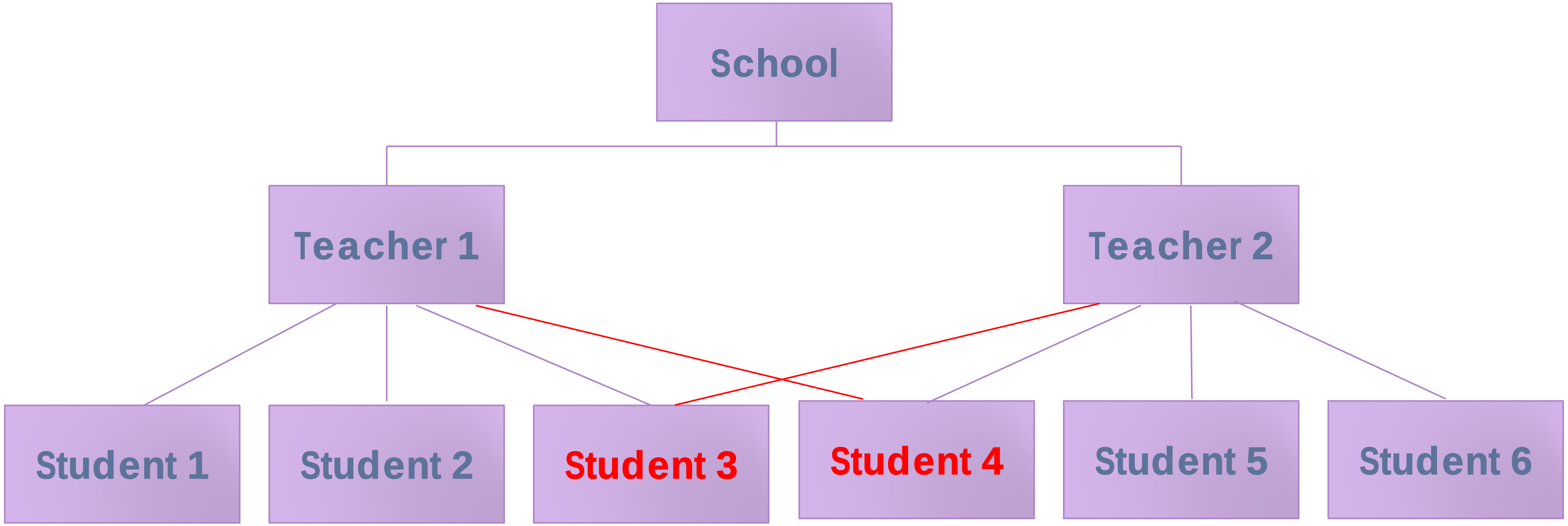
Example for simple multilevel modeling

- Jayden and Lia are attending the school A. Jayden is in Mr. Thomas's class and Lia is Mr. Hall's class.
- Carter and Jeremy are attending the school B. Both Carter and Jeremy are in Ms. Karol's class.

Student	School	Teacher
Jayden	School A	Mr. Thomas
Lia	School A	Mr. Hall
Carter	School B	Ms. Karol
Jeremy	School B	Ms. Karol

Issue of nesting more than one upper levels

- A student nested within two or more teachers within a school



Example for MMML modeling

- Jayden and Lia are attending the school A. Jayden is in Mr. Thomas's class and Lia is Mr. Hall's class. However, Lia sees a ESE teacher, Ms. Brown once a week to improve her reading skill.
- Carter and Jeremy are attending the school B. Both Carter and Jeremy are in Ms. Karol's class. However, Jeremy sees a ESE teacher, Ms. White three days a week to get help for his learning disability.

Student	School	1st Teacher	ESE teacher	ESE teacher
Jayden	School A	Mr. Thomas		
Lia	School A	Mr. Hall	Ms. Brown	
Carter	School B	Ms. Karol		
Jeremy	School B	Ms. Karol		Ms. White

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Introduction of MMML model

- Lowest units are nested within more than one upper unit
  - E.g., Students nested within more than one teachers
    Patients nested within more than one doctors
- Allow to take into account weights for different units
  - E.g., Students working with a primary teacher and a ESE teacher
- Sum of weights equals to one
  - E.g., Primary teacher(.7) and ESE teacher (.3)
- Upper level covariate(e.g., Class size of each teacher) need to be weighted based on the teacher weight
  - E.g., Student working with a teacher 1(.7) and a teacher 2 (.3)
    Class size of teacher 1(15), Class size of teacher 2 (13)
    Weighted class size for the student is (15*.7)+(13*.3)=14.4

Weights for each teacher

Student	School	Weight of Mr. Thomas	Weight of Mr. Hall	Weight of Ms. Karol	Weight of Ms. Brown	Weight of Ms. White
Jayden	A	1	0	0	0	0
Lia	A	0	0.8	0	0.2	0
Carter	B	0	0	1	0	0
Jeremy	B	0	0	0.5	0	0.5

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Weights for teacher covariate

Student	School	Weight of Mr. Thomas	Weight of Mr. Hall	Weight of Ms. Karol	Weight of Ms. Brown	Weight of Ms. White	Class size of Mr. Thomas	Class size of Mr. Hall	Class size of Ms. Karol	Class size of Ms. Brown	Class size of Ms. White	Weighted class size
Jayden	A	1	0	0	0	0	**16**	13	15	12	11	16
Lia	A	0	0.8	0	0.2	0	16	**13**	15	**12**	11	(0.8*13)+(0.2*12)= **12.8**
Carter	B	0	0	1	0	0	16	13	**15**	12	11	15
Jeremy	B	0	0	0.5	0	0.5	16	13	**15**	12	**11**	(0.5*15)+(0.5*11)= **13**

Example for VAM model using MMML

- Value-added model to obtain teacher effect using student Achievement test
- 3 level model (School, Teacher, Student)
- Sample size
  - Number of students: 4689
  - Number of teachers: 69
  - Number of schools: 21
- Three covariates used
  - Student level: Prior achievement test, Attendance rate
  - Teacher level: Class size
- SAS proc mixed procedure

Raw Data

- Localid=student id
- Staff_id=teacher id
- School=School id
- Rawscore=dependent variable
- Covariates
  - Size=class size
  - Prior1_score=prior achievement score
  - Attn_rate=absence rate
- Postid=test id

	localid	staff_id	school	RawScore	size	prior1_score	attn_rate	postid
354	223901	21	1	20	17.413978495	192	4.4444444	1
355	223934	35	5	11	16.818181818	181	2.7932961	1
356	223936	42	6	8	13.397727273	180	7.2222222	1
357	223936	60	6	8	18.48	180	7.2222222	1
358	223937	3	5	21	19.166666667	190	6.6666667	1
359	223938	3	5	18	19.166666667	198	1.6666667	1
360	223970	8	17	16	18.747425997	199	1.6666667	1
361	223974	3	5	15	19.166666667	186	2.2222222	1
362	223975	8	17	8	18.747425997	196	5	1
363	223980	30	15	20	17.894047619	208	2.7777778	1
364	223982	34	18	14	17.176587302	175	5.5555556	1
365	223996	62	16	16	17.980487805	158	11.6666667	1
366	224000	14	13	18	18.472619048	171	2.2222222	1
367	224007	10	8	17	16.81547619	208	17.2222222	1
368	224032	34	18	14	17.176587302	193	3.8888889	1
369	224036	62	16	17	17.980487805	231	0	1
370	224042	57	3	19	18.711365211	197	3.8888889	1
371	224056	15	18	17	17.092105263	204	0.5555556	1
372	224057	23	3	11	15.383333333	146	2.7777778	1
373	224062	23	3	19	15.383333333	189	3.3333333	1
374	224064	50	20	14	17.682926829	182	10	1
375	224086	7	17	11	18.760869565	185	6.1111111	1
376	224097	23	3	17	15.383333333	187	2.2222222	1
377	224115	30	15	18	17.894047619	186	0.5555556	1
378	224132	14	13	22	18.472619048	215	5	1
379	224140	16	21	11	17.498217469	151	2.7777778	1
380	224154	28	4	16	17.803571429	191	7.2222222	1
381	224154	54	4	16	8.84375	191	7.2222222	1
382	224166	55	9	12	17.275487589	203	4.4444444	1

Step 1: Teacher weight

- p1-p69: Weight variables of assigned teachers for each student, and the number of weight variables is determined by the number of teachers of that test.

	localid	school	RawScore	size	prior1_score	attn_rate	postid	p1	p2	p28	p42	p43	p54	p55	p60
1	223936	6	8	13.397727273	180	7.2222222	1	0	0	0	0.5	0	0	0	0.5
2	223937	5	21	19.166666667	190	6.6666667	1	0	0	0	0	0	0	0	0
3	223938	5	18	19.166666667	198	1.6666667	1	0	0	0	0	0	0	0	0
4	223970	17	16	18.747425997	199	1.6666667	1	0	0	0	0	0	0	0	0
5	223974	5	15	19.166666667	186	2.2222222	1	0	0	0	0	0	0	0	0
6	223975	17	8	18.747425997	196	5	1	0	0	0	0	0	0	0	0
7	223980	15	20	17.894047619	208	2.7777778	1	0	0	0	0	0	0	0	0
8	223982	18	14	17.176587302	175	5.5555556	1	0	0	0	0	0	0	0	0
9	223996	16	16	17.980487805	158	11.6666667	1	0	0	0	0	0	0	0	0
10	224000	13	18	18.472619048	171	2.2222222	1	0	0	0	0	0	0	0	0
11	224007	8	17	16.81547619	208	17.2222222	1	0	0	0	0	0	0	0	0
12	224032	18	14	17.176587302	193	3.8888889	1	0	0	0	0	0	0	0	0
13	224036	16	17	17.980487805	231	0	1	0	0	0	0	0	0	0	0
14	224042	3	19	18.711365211	197	3.8888889	1	0	0	0	0	0	0	0	0
15	224056	18	17	17.092105263	204	0.5555556	1	0	0	0	0	0	0	0	0
16	224057	3	11	15.383333333	146	2.7777778	1	0	0	0	0	0	0	0	0
17	224062	3	19	15.383333333	189	3.3333333	1	0	0	0	0	0	0	0	0
18	224064	20	14	17.682926829	182	10	1	0	0	0	0	0	0	0	0
19	224086	17	11	18.760869565	185	6.1111111	1	0	0	0	0	0	0	0	0
20	224097	3	17	15.383333333	187	2.2222222	1	0	0	0	0	0	0	0	0
21	224115	15	18	17.894047619	186	0.5555556	1	0	0	0	0	0	0	0	0
22	224132	13	22	18.472619048	215	5	1	0	0	0	0	0	0	0	0
23	224140	21	11	17.498217469	151	2.7777778	1	0	0	0	0	0	0	0	0
24	224154	4	16	17.803571429	191	7.2222222	1	0	0	0.5	0	0	0.5	0	0

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Step 2: weighted teacher covariate

- size1-size69: Class size for each teacher

Weighted class size=
(13.398*0.5)+(18.48*0.5)

	localid	school	RawScore	prior1_score	attn_rate	p28	p42	p54	p60	size28	size42	size54	size60	cons	size_final
1	223936	6	8	180	7.2222222	0	0.5	0	0.5	17.803571429	13.397727273	8.84375	18.48	1	15.938863636
2	223937	5	21	190	6.6666667	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	19.166666667
3	223938	5	18	198	1.6666667	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	19.166666667
4	223970	17	16	199	1.6666667	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	18.747425997
5	223974	5	15	186	2.2222222	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	19.166666667
6	223975	17	8	196	5	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	18.747425997
7	223980	15	20	208	2.7777778	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	17.894047619
8	223982	18	14	175	5.5555556	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	17.176587302
9	223996	16	16	158	11.6666667	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	17.980487805
10	224000	13	18	171	2.2222222	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	18.472619048
11	224007	8	17	208	17.2222222	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	16.81547619
12	224032	18	14	193	3.8888889	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	17.176587302
13	224036	16	17	231	0	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	17.980487805
14	224042	3	19	197	3.8888889	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	18.711365211
15	224056	18	17	204	0.5555556	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	17.092105263
16	224057	3	11	146	2.7777778	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	15.383333333
17	224062	3	19	189	3.3333333	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	15.383333333
18	224064	20	14	182	10	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	17.682926829
19	224086	17	11	185	6.1111111	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	18.760869565
20	224097	3	17	187	2.2222222	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	15.383333333
21	224115	15	18	186	0.5555556	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	17.894047619
22	224132	13	22	215	5	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	18.472619048
23	224140	21	11	151	2.7777778	0	0	0	0	17.803571429	13.397727273	8.84375	18.48	1	17.498217469
24	224154	4	16	191	7.2222222	0.5	0	0.5	0	17.803571429	13.397727273	8.84375	18.48	1	13.323660714

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Step 2: SAS code for weighted teacher covariate

```
data y3;
array size_teacher(*) size1-size&n;
do staff_id=1 to &n;
set y2;
size_teacher(staff_id)=size;
end;
drop size staff_id;
run;
```

```
data vam;
set y4;
cons = 1;
array ps(*) p1-p&n;
array si(*) size1-size&n;
size_final = 0;
do j = 1 to &n;
size_final = size_final + ps(j)*si(j);
end;
```

Step 3: SAS code for MMML modeling

```
proc mixed data=vam covtest;
class school cons;
model rawscore = attn_rate prior1_score size_final / solution
DDFM=bw;
random intercept / sub= school;
random p1-p&n / s sub= cons (school) type=toeplitz(1);
run;
```

Step 4: Output and result

The Mixed Procedure

Model Information

Data Set	WORK.G1
Dependent Variable	rawscore
Covariance Structures	Variance Components, Banded Toeplitz
Subject Effects	school, cons(school)
Estimation Method	REML
Residual Variance Method	Profile
Fixed Effects SE Method	Model-Based
Degrees of Freedom Method	Between-Within

Dimensions

Covariance Parameters	3
Columns in X	4
Columns in Z per Subject	70
Subjects	21
Max Obs per Subject	318

Number of Observations

Number of Observations Read	4689
Number of Observations Used	4689
Number of Observations Not Used	0

Fixed and variance components

Covariance Parameter Estimates

Cov Parm	Subject	Estimate	Standard Error	Z Value	Pr > Z
Intercept	school	0.08294	0.1040	0.80	0.2127
Variance	cons(school)	0.6345	0.1477	4.29	<.0001
Residual		5.2335	0.1090	48.02	<.0001

Fit Statistics

-2 Res Log Likelihood	21247.0
AIC (Smaller is Better)	21253.0
AICC (Smaller is Better)	21253.0
BIC (Smaller is Better)	21256.1

Solution for Fixed Effects

Effect	Estimate	Standard Error	DF	t Value	Pr > |t|
Intercept	0.2899	0.8314	20	0.35	0.7309
attn_rate	-0.01131	0.008065	4665	-1.40	0.1610
size_final	-0.00484	0.04302	4665	-0.11	0.9105
prior1_score	0.08414	0.001750	4665	48.08	<.0001

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Empirical Bayes estimates

Solution for Random Effects

School and teacher effect

SE of School and teacher effect

School effect

Teacher effect

Effect	school	cons	Estimate	Std Err Pred
Intercept	1		0.1080	0.2504
p1	1	1	0	0.7966
p2	1	1	0	0.7966
p3	1	1	0	0.7966
p4	1	1	0.6162	0.3654
p5	1	1	0.4191	0.3744
p6	1	1	0	0.7966
p63	1	1	0	0.7966
p64	1	1	0	0.7966
p65	1	1	0	0.7966
p66	1	1	0	0.7966
p67	1	1	0	0.7966
p68	1	1	0	0.7966
p69	1	1	0	0.7966
Intercept	2		0.01997	0.2503
p1	2	1	0	0.7966
p2	2	1	0	0.7966
p3	2	1	0	0.7966
p4	2	1	0	0.7966
p5	2	1	0	0.7966
p6	2	1	0.2776	0.3910
p7	2	1	0	0.7966
p8	2	1	0	0.7966
p9	2	1	0	0.7966
p10	2	1	0	0.7966
p11	2	1	0	0.7966
p12	2	1	0	0.7966
p13	2	1	0	0.7966

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Step 5: Teacher VAM score and SE

- Teacher VAM score =

EB estimate of the teacher effect + .5* EB estimate of the School component

- Teacher VAM score SE =

$$\sqrt{\begin{array}{c} Var(\text{EB estimate of the teacher effect}) \\ +.25 * var(\text{EB estimate of the School component}) \\ +Cov(\text{EB estimate of the teacher effect, EB estimate of the School component}) \end{array}}$$

Teacher VAM score and SE

Solution for Random Effects

Effect	school	cons	Estimate	Std Err Pred
Intercept	1		0.1080	0.2504
p1	1	1	0	0.7966
p2	1	1	0	0.7966
p3	1	1	0	0.7966
p4	1	1	0.6162	0.3654
p5	1	1	0.4191	0.3744
p6	1	1	0	0.7966
p63	1	1	0	0.7966
p64	1	1	0	0.7966
p65	1	1	0	0.7966
p66	1	1	0	0.7966
p67	1	1	0	0.7966
p68	1	1	0	0.7966
p69	1	1	0	0.7966

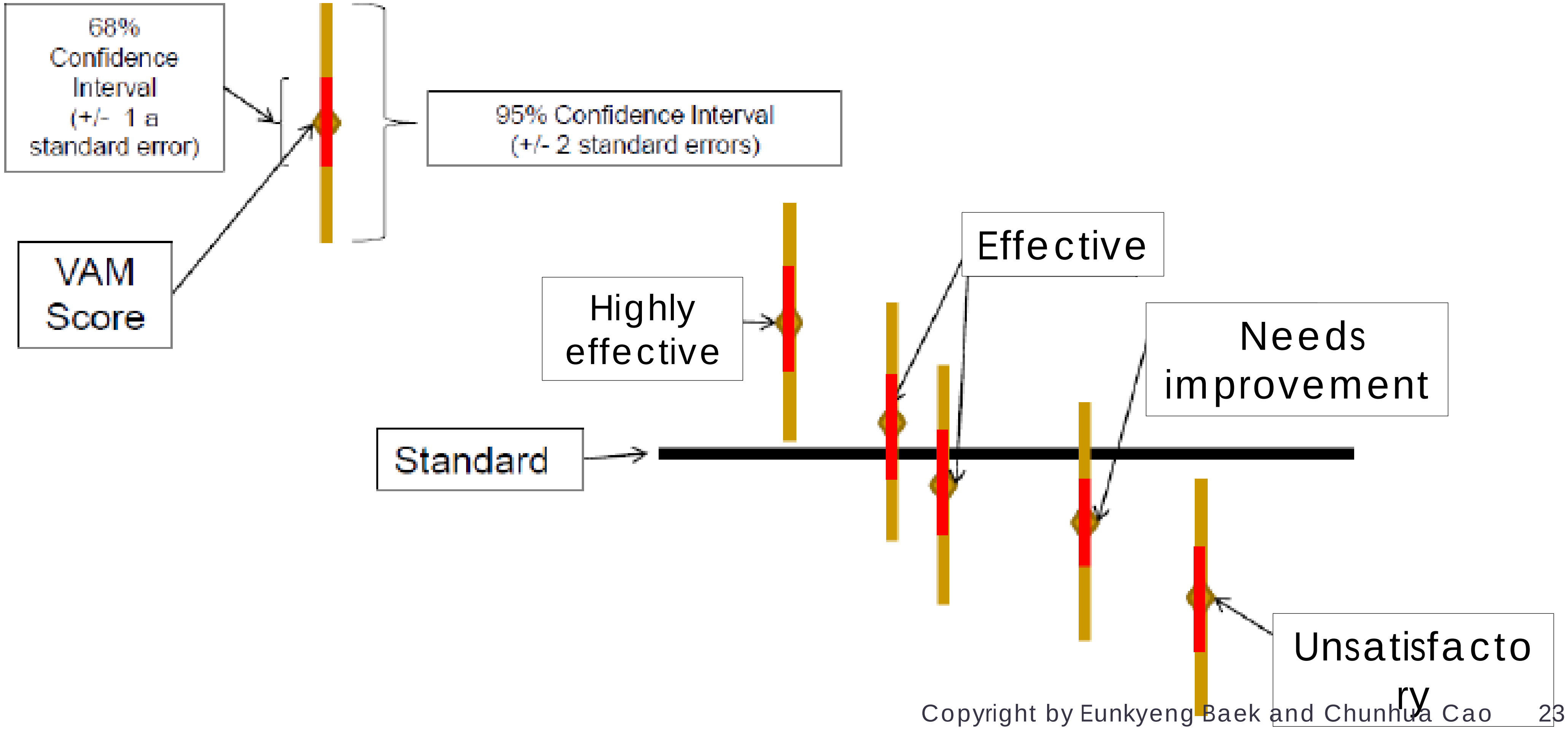
VAM score = 0.6162+(0.5*0.1080) = **0.6702**

SE = $\sqrt{(0.3654)^2 + .25(0.2504)^2 + cov(0.3654, 0.2504)}$

= **0.386**

Step 6: VAM classification

95% CI: VAM score+/- 1.96*SE
68% CI: VAM score+/-1*SE



VAM Classification

Category	Frequency	Percent	Cumulative Frequency	Cumulative Percent
Highly effective	9	13.04	9	13.04
Effective	41	59.42	50	72.46
Needs improvement	10	14.49	60	86.96
Unsatisfactory	9	13.04	69	100.00

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Thank you

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