Effects of Measurement Error on Autoregressive Parameters in Time Series Analysis

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BACKGROUND
In psychology, interest in investigating intradividual processes through individual time series analysis has grown in the last two decades. However, despite the fact that research has shown that failure to account for measurement error leads to biased estimates of autoregressive parameters (e.g., Schenker & Gentleman, 2001; Shuurman, Houtveen, & Hamaker, 2015), the majority of these analyses do not explicitly incorporate measurement error. This is problematic because the variables typically used in the social sciences are difficult or impossible to measure directly, meaning that measurement error is omnipresent in these fields.

METHOD
Monte Carlo Simulation
• Data were generated from a lag-1 DFA model with 1 latent factor (DFA[1,1]) using the MARSS package in R (R Core Team, 2016)
• 500 data sets generated per condition
• The DFA[1,1] was fit to the generated data and lag-1 AR (AR[1]) and lag-1 AR + WN (AR[1] + WN) models were fit to composites of the indicators from the DFA[1,1] model
• The following parameters were varied across conditions:
  • Factor loading/univariate variance ratio
  • .5/.75, .7/.51, and .9/.19
  • Number of indicator variables
  • 3, 5, and 7
  • Magnitude of AR parameter
  • .2, .5, and .8
  • Number of time points
  • 50, 100, and 500

RESULTS
Evaluating Model Performance
The performance of each of the models was assessed by examining the relative bias and the 95% coverage rates for the AR parameters from the DFA[1,1], AR[1], and AR[1] + WN models. Relative bias was computed as (\(\hat{\phi} - \phi\))/\(\phi\), where \(\hat{\phi}\) is the estimated value of the AR parameter and \(\phi\) is the true parameter. Coverage rates were formulated by examining the proportion of times the true value of the AR parameter occurred within the 95% confidence intervals of the maximum likelihood estimated values in each of the simulated conditions.

Model Convergence
A number of the models failed to converge across some of the simulation conditions. For the DFA[1,1] and AR[1] + WN models, convergence greatly improved as the number of time points and the AR parameter increased, and as the amount of unique variance decreased. The AR[1] model converged 100% of the time for all of the simulation conditions examined. Results are reported for the converged solutions only.

Relative Bias
• When the true AR parameter is high and the time series is sufficiently long (e.g., \(T > 100\)), the AR[1] + WN model performs as well, or nearly as well, as the DFA[1,1] model at recovering the true AR parameter. Both the amount of relative bias and the 95% coverage rates for the estimates of the AR parameter seemed to improve with increasing length of time series and when there was a smaller proportion of unique variance. The AR[1] model tended to severely underestimate the AR parameter in most of the simulation conditions.

DISCUSSION
When the true AR parameter is high and the time series is sufficiently long (e.g., \(T > 100\)), the AR[1] + WN model performs as well, or nearly as well, as the DFA[1,1] model at recovering the true AR parameter. Both the amount of relative bias and the 95% coverage rates for the estimates of the AR parameter seemed to improve with increasing length of time series and when there was a smaller proportion of unique variance. The AR[1] model tended to severely underestimate the AR parameter in most of the simulation conditions.

CONCLUSIONS
Overall, these findings suggest that in the absence of multiple indicators, the AR[1] + WN model may be a reasonable model for incorporating measurement error in time series analysis as long as convergence criteria are met. The AR[1] model, on the other hand, tends to lead to attenuation of the AR estimate in the presence of measurement error.

CONTACT
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