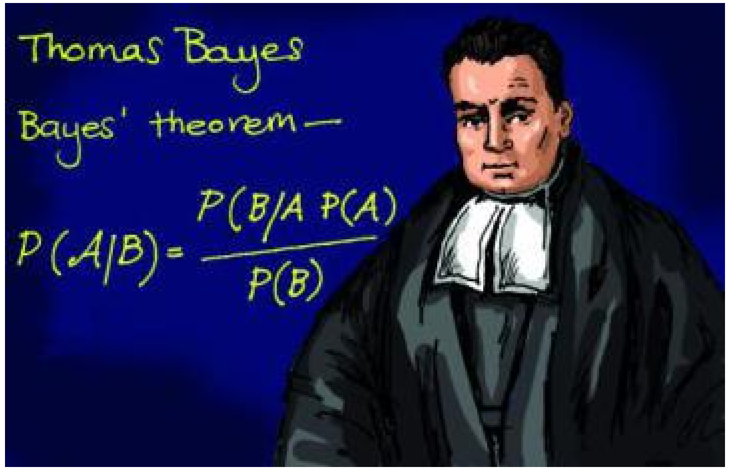
$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Design-Comparable Effect Sizes

ABAYESIAN APPROACH TO SINGLE-CASE RESEARCH



How do you synthesize SCD and Group Research?

Tyler Hicks, Ph.D.
Eun Sook Kim, Ph.D.
Seang Hwane Joo, Ph.D.
Jeong Hoon Choi, Ph.D.
John Ferron, Ph.D.
Jeff Kromrey, Ph.D.

University of Kansas
University of South Florida

$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$

A design-comparable *d*-Type Effect Size

Synthesizing SCD and Group Research

$$\delta_{AB} = \frac{E\left(Y_{iB}(A)\right) - E\left(Y_{iB}(N)\right)}{\sqrt{Var\left(Y_{iB}(N)\right)}}$$

where $E(Y_{ib}(\mathrm{A}))$ denotes the expected outcome at time B for the i^{th} individual given treatment began at time A, $E(Y_{ib}(\mathrm{B}))$ denotes the expected outcome at time B for that same individual absent treatment, and $\sqrt{Var(Y_{iB}(N))}$ denotes the expected deviation in outcomes from $Y_{ib}(\mathrm{N})$ on the outcome scale.

Larry Hedges
Statistician

Source of Citation:

Pustejovsky, J.E, Hedges, L.V., & Shadish, W.R. (2014). Design-Comparable Effect Sizes in Multiple Baseline Designs: A General Modeling Framework. *Journal of Educational and Behavioral Statistics. 39.* 368-393.

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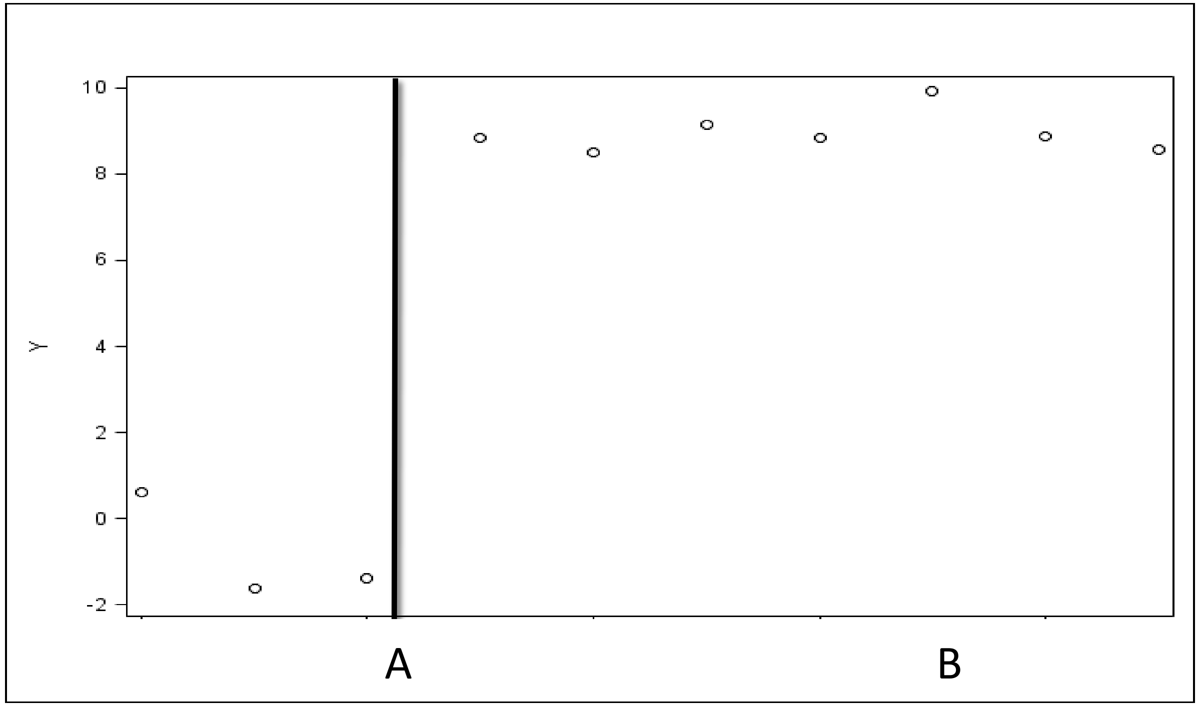

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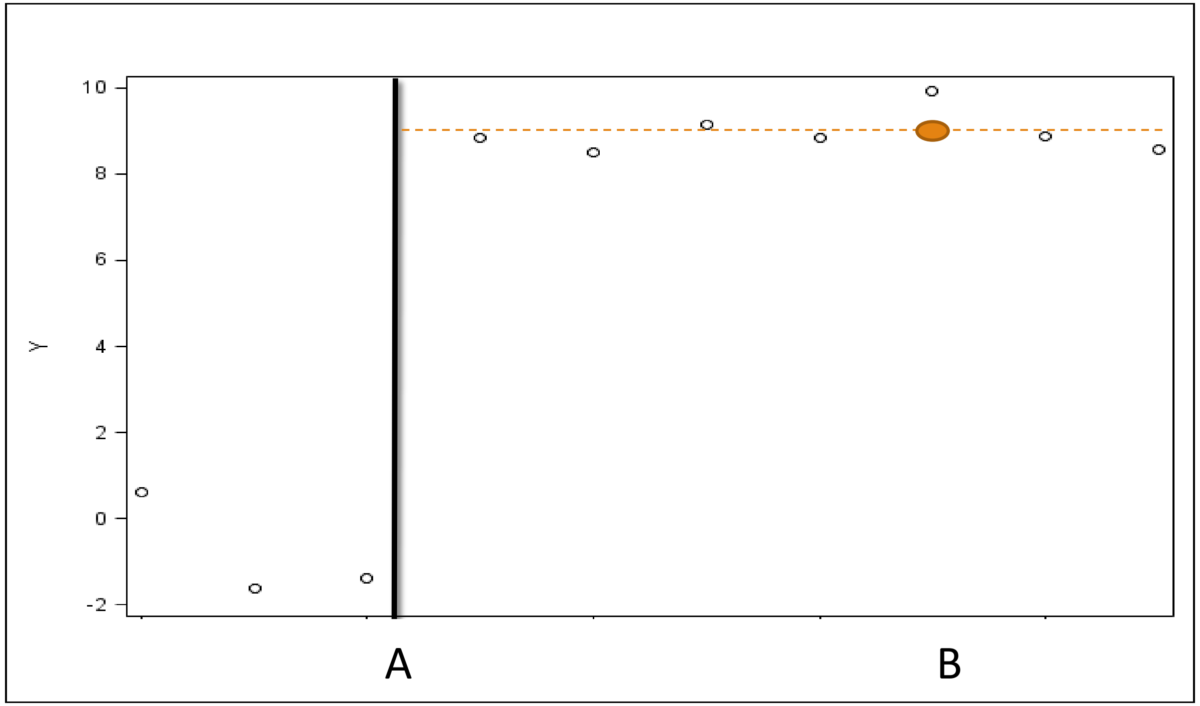

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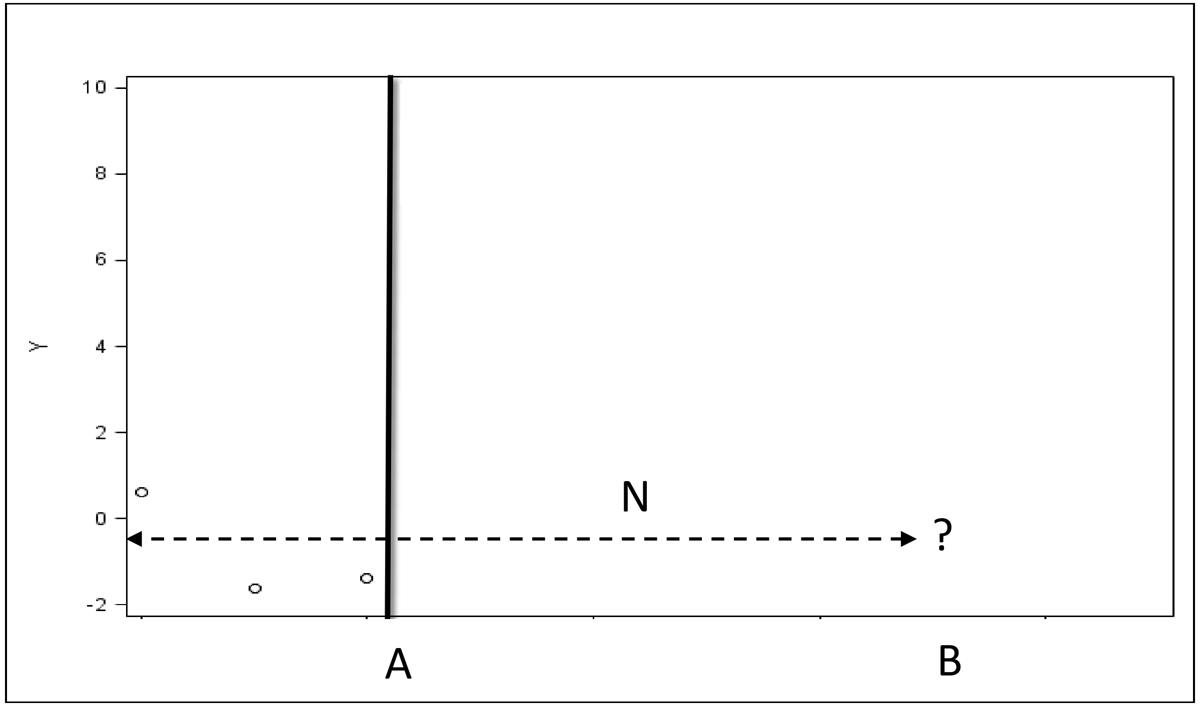

Note. A+B=N

$$\delta_{AB} = \frac{E\left(Y_{iB}(A)\right) - E\left(Y_{iB}(N)\right)}{\sqrt{Var\left(Y_{iB}(N)\right)}}$$

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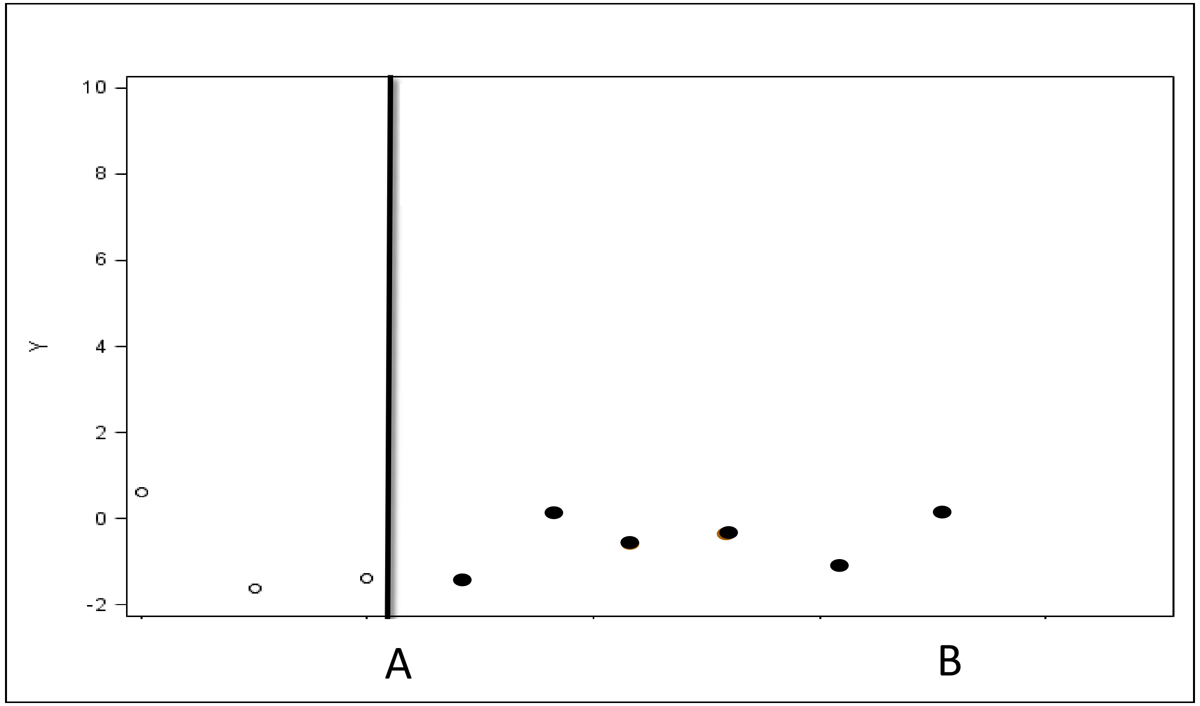

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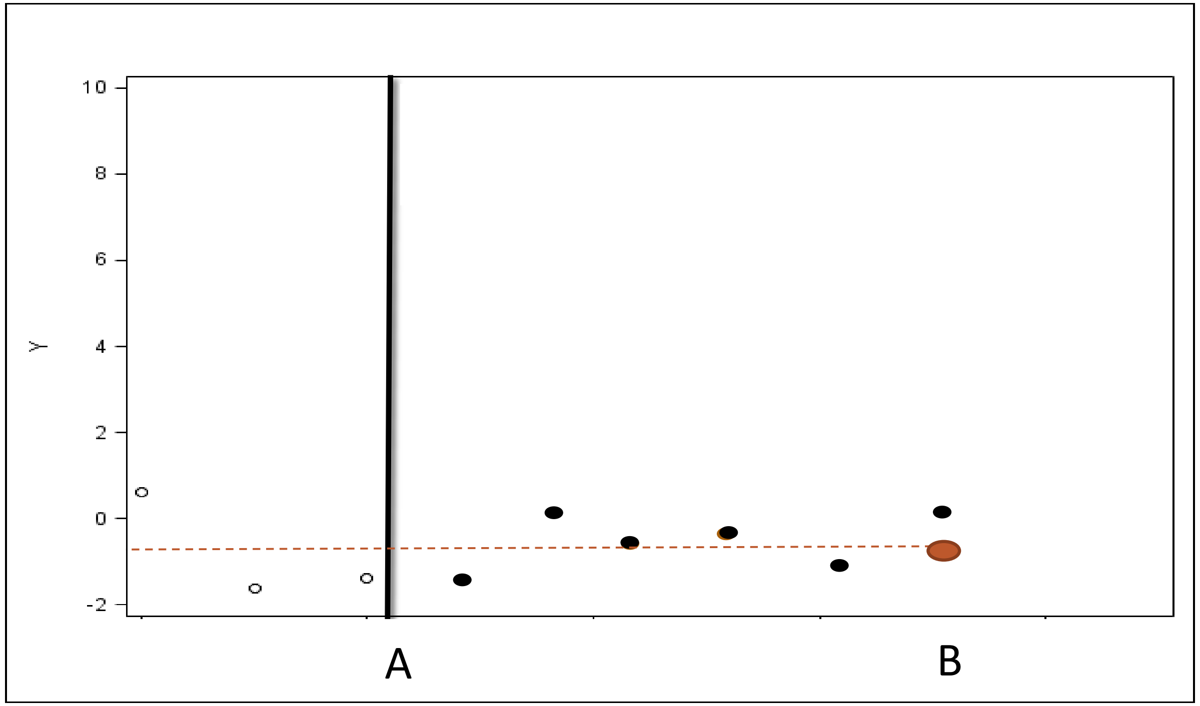

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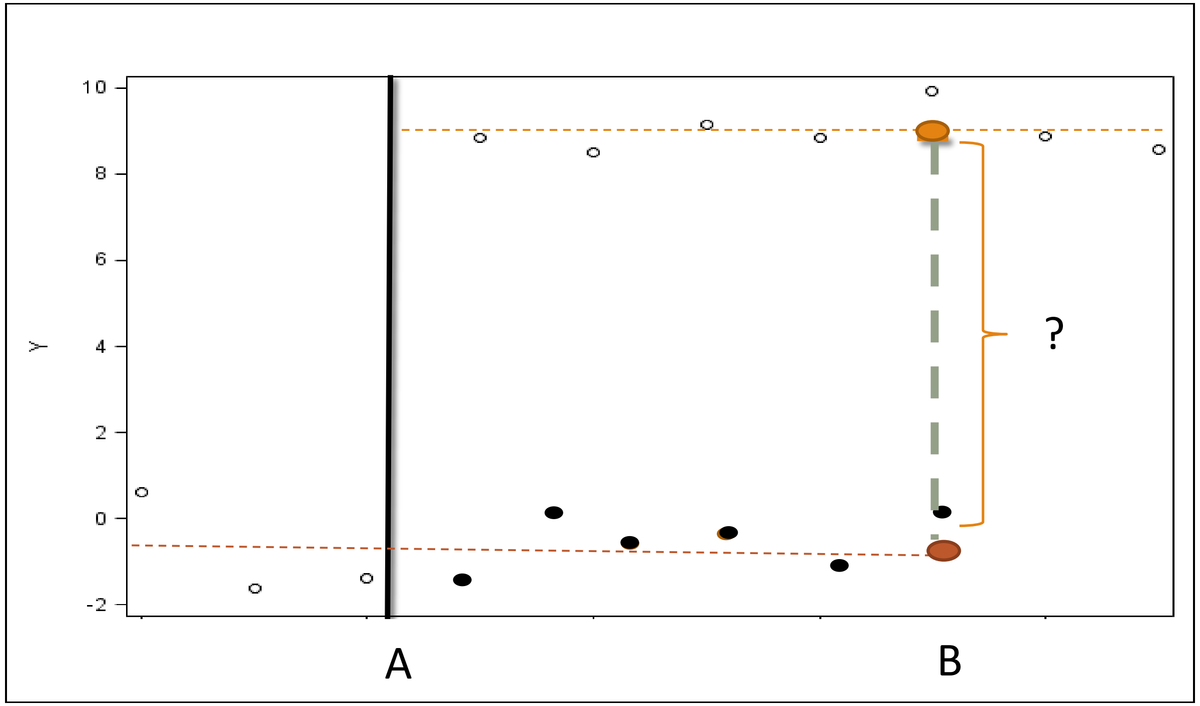

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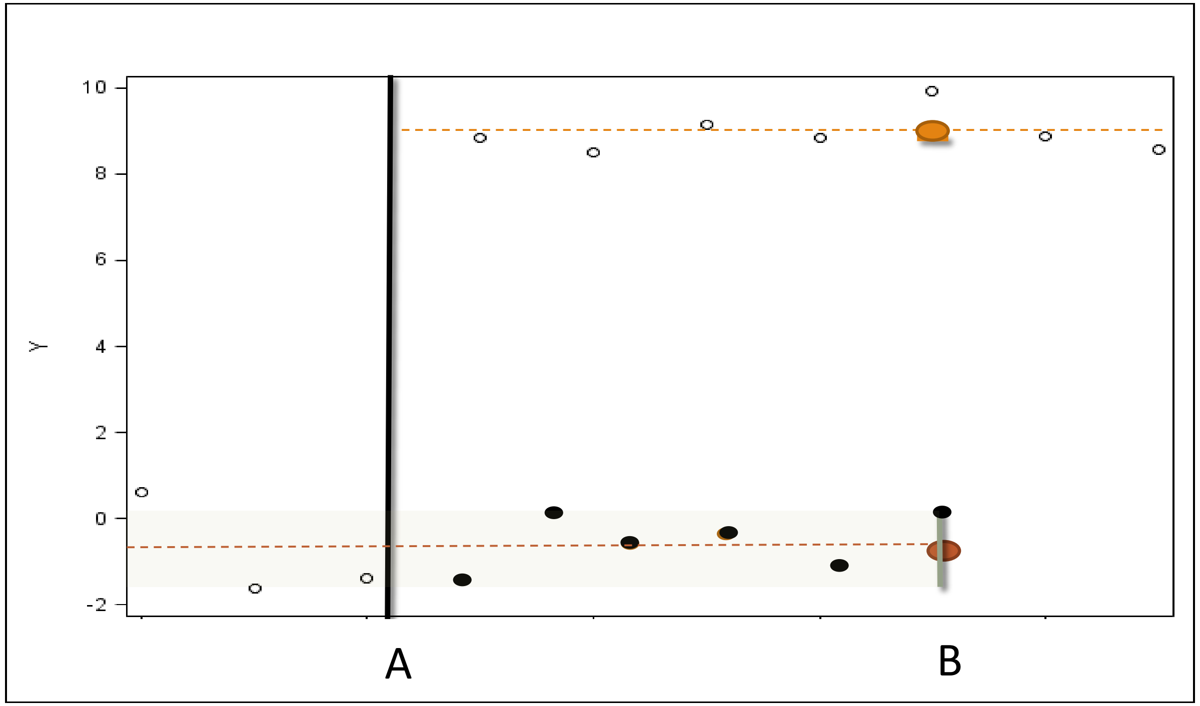

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A General Modeling Framework

Single-Case Research with Multiple Baseline Designs

Level 1:

$$Y_{ti} = \beta_{0i} + \beta_{1i} * Phase_{ti} + e_{ti} \qquad e_{ti} = \phi e_{(t-1)i} + u_t \qquad u_{ti} \sim \mathrm{N}(0, \sigma^2)$$

Level 2:

$$\beta_{0i} = \gamma_{00} + \eta_{0i} \qquad \eta_{0i} \sim N(0, \tau^2)$$

$$\beta_{1i} = \gamma_{10}$$

Source of Citation:

Beak, E.K. & Ferron, J.M. (2013). Multilevel models for multi-baseline data: Modeling across participant variation in autocorrelation and residual variance. *Behavior Research Methods. 45.* 65-74.

$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$

A design-comparable *d*-Type Effect Size

Transforming Equation (1) into Statistical Models for SCD research

MB1:

VARYING INTERCEPTS, FIXED EFFECTS, NO TRENDS

$$\delta_{AB} = \frac{E\big(Y_{iB}(A)\big) - E(Y_{iB}(N))}{\sqrt{Var(Y_{iB}(N))}}$$

$$\Downarrow$$

$$\frac{\gamma_{10}}{\sqrt{\tau_0^2 + \sigma^2}}$$

MB3:

VARYING INTERCEPTS, FIXED EFFECTS, FIXED TRENDS

$$\delta_{AB} = \frac{E\big(Y_{iB}(A)\big) - E(Y_{iB}(N))}{\sqrt{Var(Y_{iB}(N))}}$$

$$\Downarrow$$

$$\frac{\gamma_{10} + \gamma_{20}(B - A)}{\sqrt{\tau_0^2 + \sigma^2}}$$

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Simulation Study

Is an Asymptotic Defense of Estimates in Single-Case Research feasible?

Dependent Variable:

- Point Estimates (Bias, RMSE)
- Interval Estimates (i.e., Coverage, Precision)

Independent Variable:

- Estimation (REMLE with Hedges corrections for effect size estimate)

Jerzy Neyman

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

PROC MIXED

A SAS procedure for Gaussian Mixed Models

```
Proc mixed data=sample asycov cl;
     class IDLEVEL2;
     model y = phase / s cl alpha = .05 ddfm = kenward covb;
     random int / sub = IDLEVEL2;
     repeated / sub = IDLEVEL2 type=ar(1);
     run;
```

Note. We used PROC Mixed output to assemble the effect size but we then adjusted the estimate using Hedge’s formula for corrections. Please consultant the following reference for correction formulas:

Pustejovsky, J.E, Hedges, L.V., & Shadish, W.R. (2014). Design-Comparable Effect Sizes in Multiple Baseline Designs: A General Modeling Framework. *Journal of Educational and Behavioral Statistics. 39.* 368-393.

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Simulation Study

Is an Asymptotic Defense of Estimates in Single-Case Research feasible?

Experimental Conditions

- Actual Model Configuration and Parameters:

MBI: Parameters					
δ_{ab}	γ_{00}	γ_{10}	ϕ	σ^2	τ_0^2
7	0	10	0	1	1

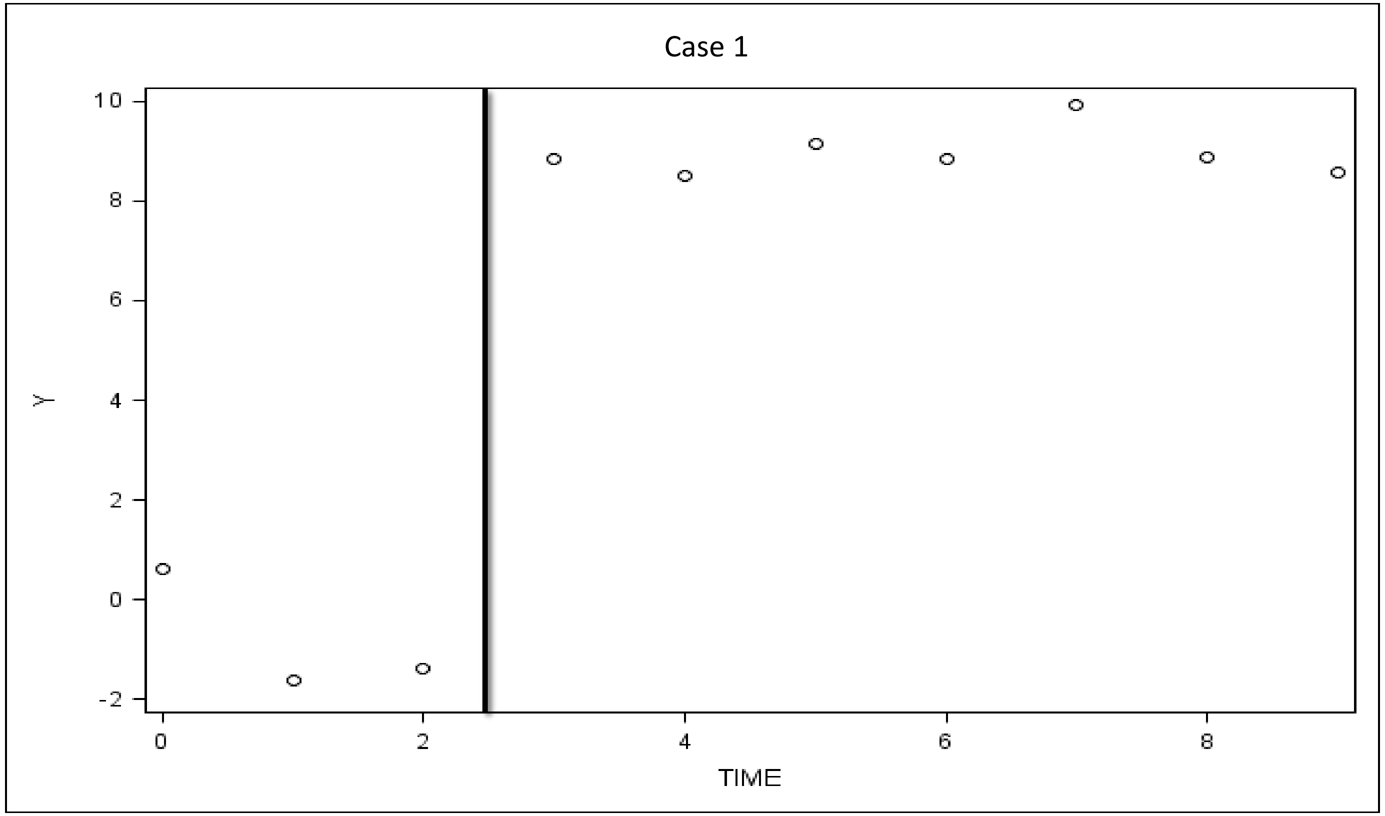
- Multiple Baseline Design
  - 4 cases; 10 repeated measures; different phase shifts
- Simulated Repetitions of Experiment
  - N=500

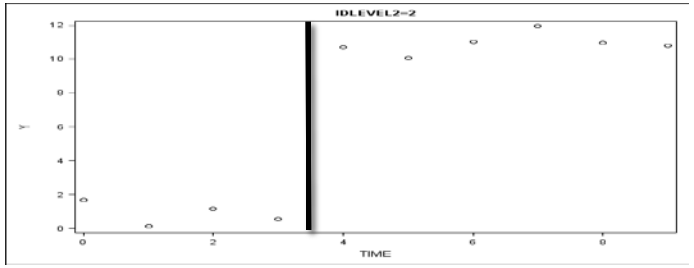
B.F. Skinner

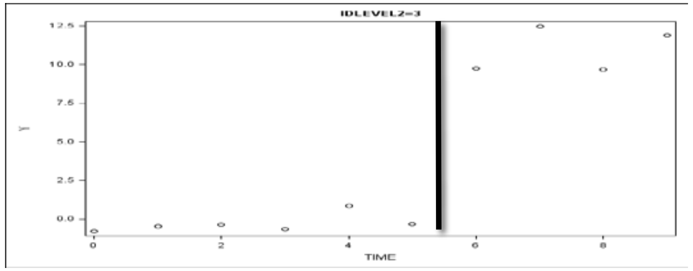
$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

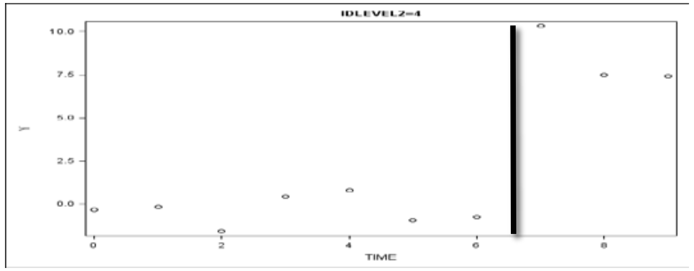
Illustrative Example

One iteration of SCD research from our simulation study









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Larry Hedges
Statistician

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Results

Is an Asymptotic Defense of Estimates in Single-Case Research feasible?

N=500		Point Estimates		Interval Estimates	
		Bias	RMSE	Coverage	Precision
γ_{00}	REMLE	0.018	0.521	0.964	7.107
	MLE	0.015	0.526	0.974	5.890
γ_{10}	REMLE	-0.010	0.637	0.814	3.287
	MLE	-0.013	0.643	0.814	3.334

Inadequate Coverage

Note. We used iterative optimization technique to produce restricted maximum likelihood estimates (REMLE) and MCMC simulation to produce maximum likelihood estimates (MLE). As anticipated, REMLE and MLE results were approximate for fixed effects.

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Results

Is an Asymptotic Defense of Estimates in Single-Case Research feasible?

N=500		Point Estimates		Interval Estimates	
		Bias	RMSE	Coverage	Precision
σ^2	REMLE	0.254	0.391	0.824	1.632
	MLE	0.592	0.542	0.886	2.118
τ^2	REMLE	0.189	1.117	0.942	$\boxed{\text{Big \#}}$
	MLE	2.062	2.948	0.954	9.8774
ρ	REML	0.156	0.200	0.884	Big #
	MLE	0.195	0.201	0.828	9.8774

Note. As expected, REMLE was superior to MLE. However, MCMC simulation proved to be superior to iterative optimization. For example, optimization produced a negative variance 8% of the time and exceedingly large widths for τ^2. Thus, REMLE with MCMC simulation combination could be tested in a future simulation study to avoid the pitfalls of MLE and iterative optimization when estimating variance components from small samples.

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Results

Is an Asymptotic Defense of Estimates in Single-Case Research feasible?

N=500		Point Estimates		Interval Estimates	
		Bias	RMSE	Coverage	Precision
δ_{AB}	REMLE	-0.659	1.725	0.942	7.107
	MLE	-0.798	1.472	0.860	5.890

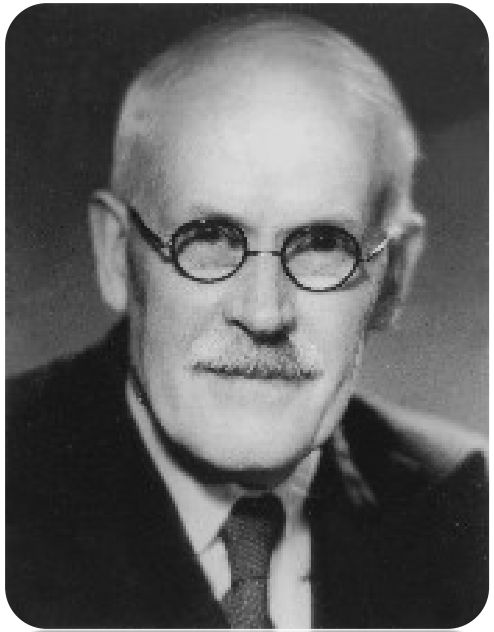
Note. MLE estimates did not make any corrections for small samples. The REMLE estimates were corrected as prescribed by Hedges. Without corrections, interval estimates will not have adequate coverage.

$$f(\theta \mid x) = \frac{f(\theta, x)}{f(x)}$$

Discussion

Central Taking Points

Harold Jeffreys



- We delimited the focus of our simulation study the *ideal* situation (i.e., very simple model). Real life is much more complex yet even in the ideal situation classical estimators were observed to fumble...

- Frequentist defenses of estimates are hard to articulate if we cannot reconstruct what happens in the abstract limits.

- ReMLE with Hedges corrections using MCMC simulation may be the best option in SCD research. This option, of course, invites Bayesian estimation with adapted likelihood function. This switch to a Bayesian estimation keep the rewards of REMLE and Hedges corrections while avoiding asymptotic considerations. We will explore this option and report findings next year…

Thanks

Tyler Hicks

tahicks@ku.edu