Bayesian Structural Equation Models with Small Samples: A Systematic Review

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Quotes

• Rupp et al., 2004: “Bayesian parameter estimation is more appropriate than ML estimation for smaller sample sizes (…).”

• Kruschke et al., 2012: “Bayesian methods can be used regardless of the overall sample size or relative sample sizes across conditions or groups.”
Goal

Is it valid to use Bayesian instead of Maximum Likelihood estimation for SEM when the sample size is small?

• Systematic literature review
Methods – Inclusion Criteria

• Simulation study
• Bayesian parameter estimation vs Maximum Likelihood
• Small sample sizes
• Structural Equation Models

• Peer-reviewed articles
• Field: social sciences
Methods – Searches

Approach 1
Systematic review Van de Schoot et al. (2017)

Approach 2
SEMnet and multilevel mailing lists, research gate

Included in systematic review

Screening
References that meet inclusion criteria are included in systematic review and included in next search

Next search to identify new possibly relevant references
- Reference list
- Citations

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Methods – Searches

**Approach 1**
Systematic review Van de Schoot et al. (2017)

**Approach 2**
SEMnet and multilevel mailing lists, research gate

**Screening (n = 3548)**
References that meet inclusion criteria are included in systematic review and included in next search

**Included in systematic review (n = 24)**

**Next search to identify new possibly relevant references**
- Reference list
  - Citations
Results

Number of Included Simulation Studies per Model

- Autoregressive Time Series: 1
- CFA: 3
- Latent Growth: 6
- Mediation: 4
- Multilevel: 10
- Mediation Multilevel: 1
- Mixture CFA: 2
- Mixture Latent Growth: 2
## Results – Sample Size

<table>
<thead>
<tr>
<th>Studies</th>
<th>Number of clusters</th>
<th>Cluster size</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baldwin &amp; Fellingham, 2013</td>
<td>8, 16</td>
<td>5, 15</td>
</tr>
<tr>
<td>Browne &amp; Draper, 2002</td>
<td>12, 48</td>
<td>(un)balanced, mean = 18</td>
</tr>
<tr>
<td>Browne &amp; Draper, 2006</td>
<td>6, 12, 24, 48</td>
<td>(un)balanced, mean = 18</td>
</tr>
<tr>
<td>Depaoli &amp; Clifton, 2015</td>
<td>40, 50, 100, 200</td>
<td>5, 10, 20</td>
</tr>
<tr>
<td>Farrell &amp; Ludwig, 2008</td>
<td>(i) 20; (ii) 5; (iii) 80</td>
<td>(i) 20, 80, 500; (ii) 500; (iii) 20</td>
</tr>
<tr>
<td>Hox, van de Schoot &amp; Matthijsse, 2012</td>
<td>10, 15, 20</td>
<td>1755</td>
</tr>
<tr>
<td>McNeish, 2016</td>
<td>8, 10, 14</td>
<td>7-14</td>
</tr>
<tr>
<td>McNeish &amp; Stapleton, 2016</td>
<td>4, 8, 10, 14</td>
<td>7-14, 17-34</td>
</tr>
<tr>
<td>Stegmueller, 2013</td>
<td>5, 10, 15, 20, 25, 30</td>
<td>500</td>
</tr>
<tr>
<td>Tsai &amp; Hsiao, 2008</td>
<td>15</td>
<td>6</td>
</tr>
</tbody>
</table>

**Bold** = defined as a small sample size by the authors of the original paper.  
**Underlined** = not defined by the authors of original paper, defined by authors of current study.
Results – Priors

• Default prior = general prior, ‘naive’ use of Bayes
• Adapted prior = specific prior information included
• Data-dependent prior = partly based on Maximum Likelihood estimate
Results – Priors

Baldwin & Fellingham, 2013
Browne & Draper, 2002
Browne & Draper, 2006
Depaoli & Clifton, 2015
Farrell & Ludwig, 2008
Hox, van de Schoot & Matthijsse, 2012
McNeish, 2016
McNeish & Stapleton, 2016
Stegmueller, 2013
Tsai & Hsiao, 2008

- Default Prior
- Adapted Prior
- Data-Dependent Prior
Results – Bayes vs ML

n = 3 studies investigated adapted priors
n = 1: no clear difference between ML and Bayes
n = 2: Bayes adapted > ML
and ML > Bayes default
Results – Bayes vs ML

n = 3 studies investigated adapted priors
n = 1: no clear difference between ML and Bayes
n = 2: Bayes adapted > ML, and ML > Bayes default

n = 1 study investigated default and data-dependent priors
n = 1: no clear difference between ML and Bayes

n = 6 studies investigated only default priors
n = 2: ML > Bayes default
Results – Bayes vs ML

n = 3 studies investigated adapted priors
n = 1: no clear difference between ML and Bayes
n = 2: Bayes adapted > ML, and ML > Bayes default

n = 1 study investigated default and data-dependent priors
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Results – Bayes vs ML

n = 3 studies investigated adapted priors
n = 1: no clear difference between ML and Bayes
n = 2: Bayes adapted > ML, and ML > Bayes default

n = 1 study investigated default and data-dependent priors
n = 1: no clear difference between ML and Bayes

n = 6 studies investigated only default priors
n = 2: ML > Bayes default
n = 4: Bayes default > ML
Results – Bayes vs ML

With a small sample size, performance of Bayes with default priors is worse than ML!

- High bias in variance components
- Default prior ≠ noninformative prior when the sample size is small!

McNeish (2016): “With small samples, the idea of noninformative priors is more myth than reality.”
Results – Bayes vs ML

- **Latent Growth Model**: Variance of latent slope is highly biased (McNeish, 2016)

- **Mixture Model**: Prior on the class proportions seems to be really important! (Depaoli, 2012; Depaoli, 2013)

- **CFA**: Large differences in performance of 3 default priors, especially with small samples (Van Erp, Mulder, Obserski, submitted)
Conclusion

Is it valid to use Bayesian instead of Maximum Likelihood estimation for SEM when the sample size is small?

- Bayesian estimation can have advantages
- Never naively use default priors when the sample size is small!

Choose your priors carefully!
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References


References Multilevel Studies


References Other Models

**Latent Growth Model**

**CFA**

**Mixture Model**