Performance of the LMS Procedure in Estimating Latent Interaction Models with Ordered-Categorical Indicators

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INTRODUCTION

Several procedures have been developed and extensively examined to estimate latent interaction models with continuous data (see Marsh et al., 2004, for a review). There are also three specialized estimators for estimating such models with categorical indicators, which theoretically have identical asymptotic properties. Among these three, a maximum likelihood estimator called the Latent Moderated Structural Equations (LMS; Klein & Moosbrugger, 2000) is readily available in commercial software, easier to implement, and can be used with both dichotomous and ordered-categorical data. The purpose of this simulation study was to investigate for the first time performance of LMS method in comparison with the Unconstrained Product Indicator method (Marsh, Wen, & Hau, 2004) in estimating latent interaction models with ordered-categorical data.

METHOD

We studied the structural and measurement models below:

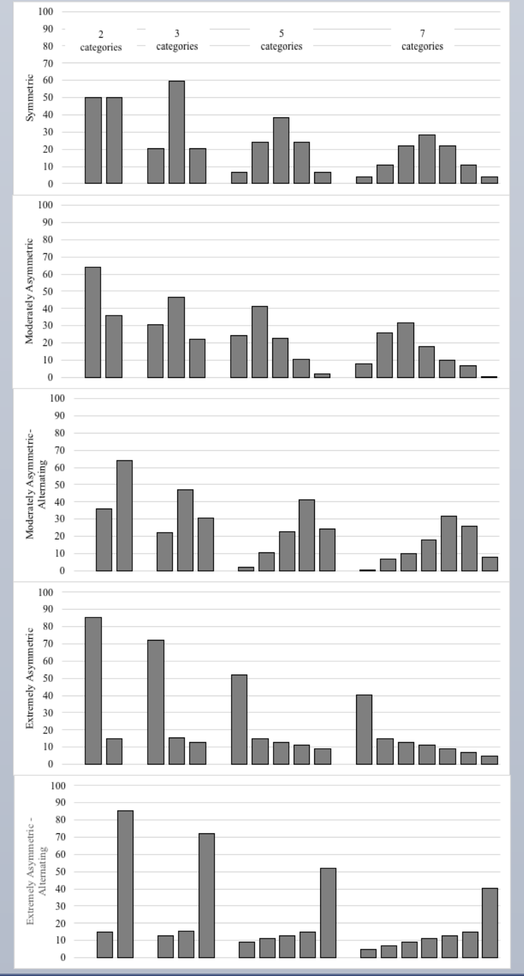
$$\eta = \gamma_1\xi_1 + \gamma_2\xi_2 + \gamma_3(\xi_1\xi_2) + \zeta$$

$$\begin{pmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \\ X_5 \\ X_6 \end{pmatrix} = \begin{pmatrix} \lambda_{X1} & 0 \\ \lambda_{X2} & 0 \\ \lambda_{X3} & 0 \\ 0 & \lambda_{X4} \\ 0 & \lambda_{X5} \\ 0 & \lambda_{X6} \end{pmatrix} \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} + \begin{pmatrix} \delta_{X1} \\ \delta_{X2} \\ \delta_{X3} \\ \delta_{X4} \\ \delta_{X5} \\ \delta_{X6} \end{pmatrix}$$

- The two latent factors (ξ_1 and ξ_2) are standardized, and the correlation between them is set to 0.3.
- The intercept γ_0, and the main effects γ_1 and γ_2 were set to 0.1, 0.3 and 0.1 respectively.
- The interaction effect γ_3 was set to 0, 0.1040, and 0.1589, to represent three conditions of the interaction effect.
- $\gamma_3 = 0$ corresponds to ρ^2 increase of 0 from linear to interaction model with disturbance variance of 0.2753, whereas $\gamma_3 = 0.1040$ correspond to ρ^2 increase of 0.03 (disturbance variance = 0.2635), and $\gamma_3 = 0.1589$ corresponds to ρ^2 increase of 0.07 (disturbance variance = 0.2478).

Experimental Conditions:

- Number of categories of indicators (4)
- Symmetry of category thresholds (5)
- Missing data scenario (2) and rate (3: complete, 25% and 40% missing)
- Interaction effect size (3: 0, 0.03, 0.07)
- Sample size (3: N = 200, 500, 1000)

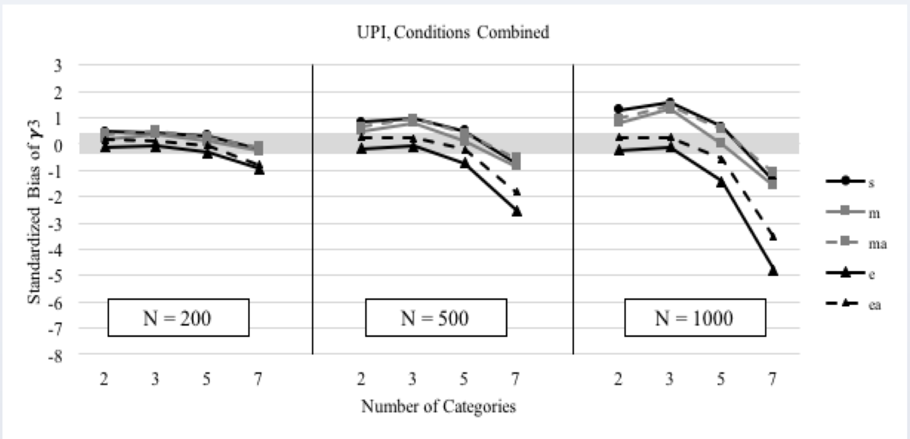


We randomly generated 500 replications for each of the 900 conditions.

RESULTS: THE INTERACTION EFFECT (UPI)

Standardized Bias $\dfrac{\bar{\hat{\theta}} - \theta}{sd(\hat{\theta})}$

- In LMS, SBs were acceptable (<.4) across all conditions.
- In UPI (right), large biases were observed in many conditions.



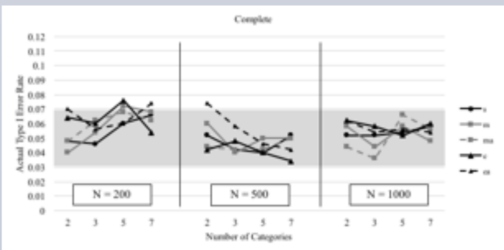
Relative Bias of the SE $\dfrac{\overline{SE(\hat{\theta})} - SD(\hat{\theta})}{SD(\hat{\theta})} \times 100\%$

- LMS made unbiased SE estimates in all conditions of the symmetric and moderately asymmetric indicators. A few large RBSEs were observed only when N= 200, and indicators were moderately-alternating, extremely, or extremely-alternating asymmetric with 2 categories.
- In UPI, inflated RBSEs were observed in many conditions. Smaller N, and lower number of categories caused extreme biases in both complete and MAR data sets, especially when coupled with extreme non-symmetry of the indicators.

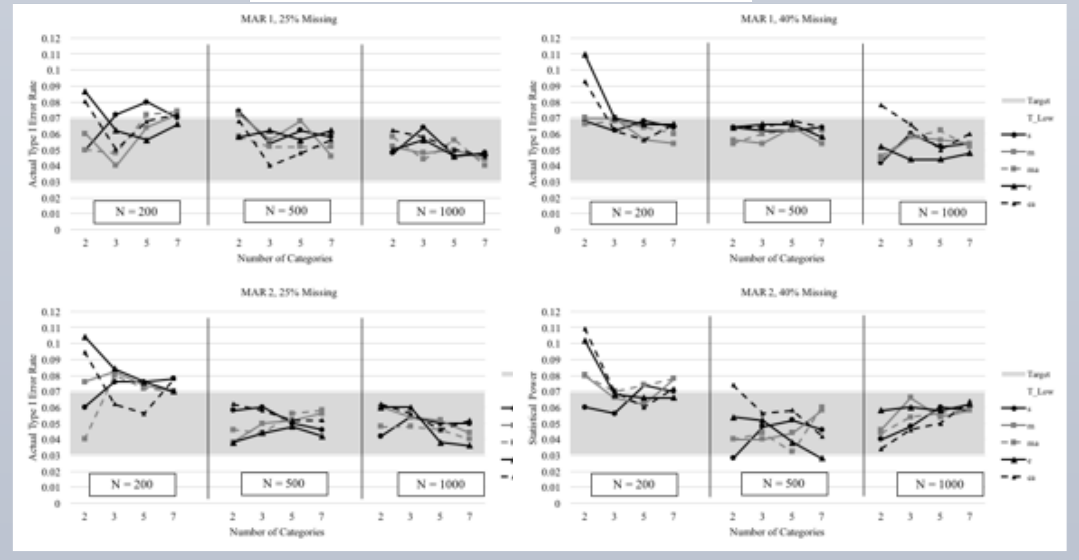
Mean Squared Error $\left(\bar{\hat{\theta}} - \theta\right)^2 + \left(SD(\hat{\theta})\right)^2$

- Across all conditions, LMS produced low MSEs although extreme non-symmetry of 2-category indicators caused slight inflation when N=200 with missing data.
- In UPI, large MSEs were observed overall. Extremely large values (>1) were observed when number of indicator categories were <5.

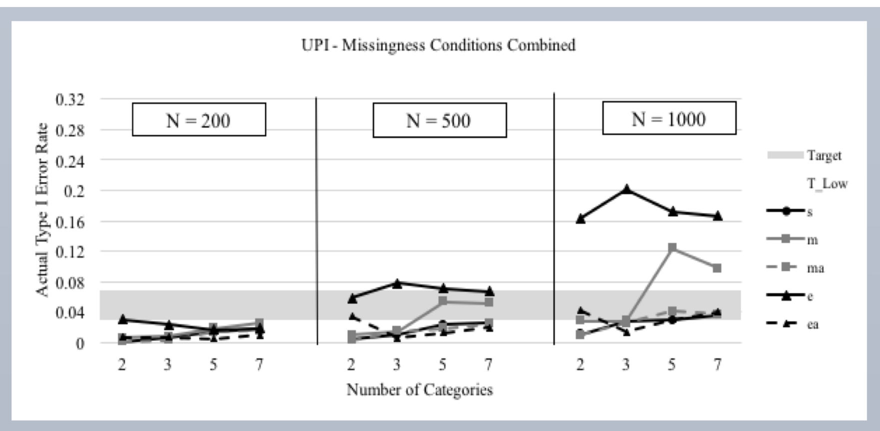
Type I Error



Overall, Type I error rates were within the acceptable limit for LMS models, except when 2-category indicators were extremely asymmetric in MAR data sets with N = 200.

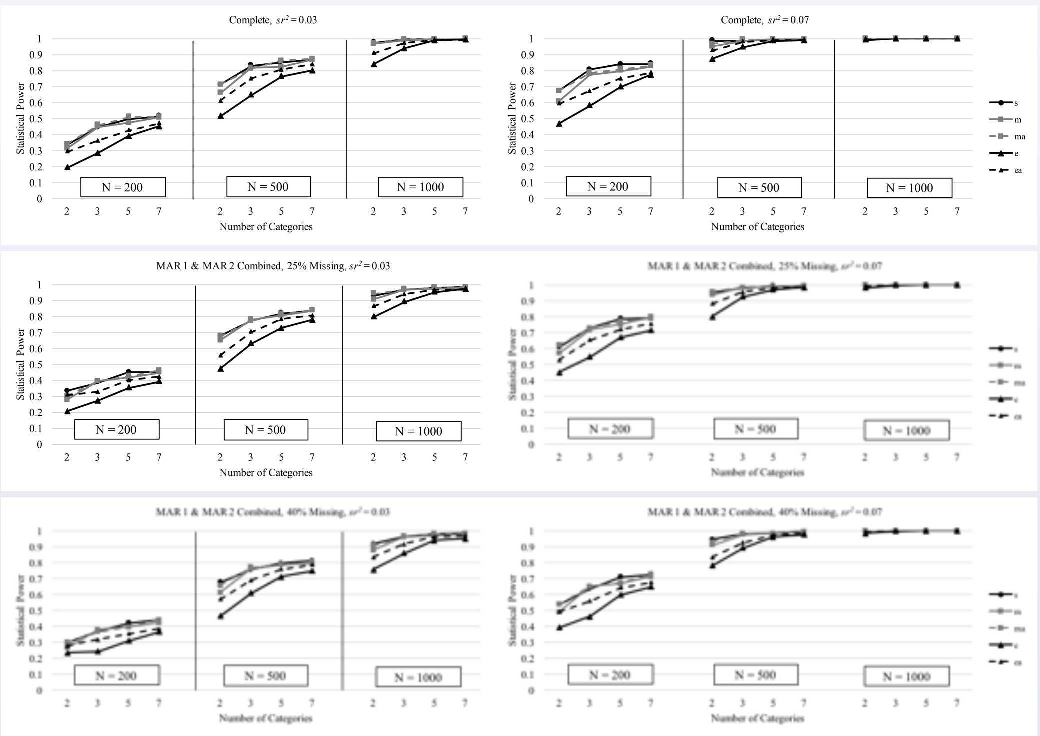


In UPI, N < 1000 caused unacceptably low Type I error for almost all types of indicators.

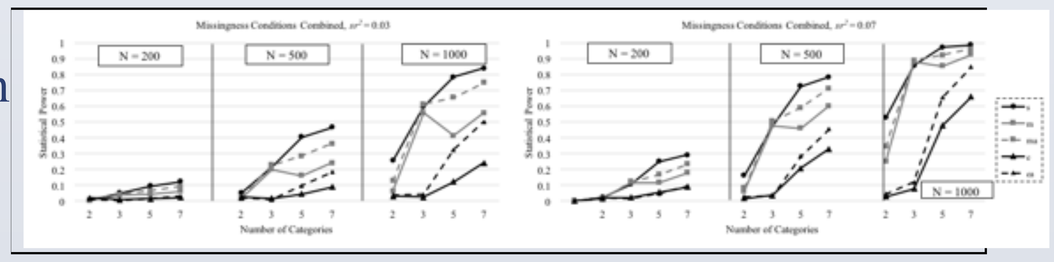


Power

LMS: N = 1000 was required for $sr^2 = 0.03$, and N >= 500 was required for $sr^2 = 0.07$ for a power level >.90. Extreme, and extreme-alternating non-symmetry caused lower power especially with smaller number of categories of indicators.



UPI models resulted in much lower statistical power.



RESULTS: LOWER ORDER EFFECTS

- LMS produced acceptable standardized biases under all conditions for γ_0, γ_1, and γ_2, whereas UPI produced very large standardized biases for γ_1 and γ_2 in most conditions, especially with larger N.
- LMS showed inflated RBSE for γ_0, γ_1 and γ_2 with MAR data when the indicators were extremely or extremely-alternating asymmetric with 2- and 3-categories.
- With LMS, N >=500 frequently resulted in acceptable power levels with indicators with >=3 categories.

RESULTS: FACTOR LOADINGS AND CATEGORY THRESHOLDS

- LMS produced very large (frequently > 1) standardized biases for factor loadings in all conditions, while UPI produced acceptable standardized biases in almost all conditions.
- LMS made biased category threshold estimates of indicators.

CONCLUSIONS

- We recommend using the LMS procedure when estimating latent variable interactions and lower order effects with ordered-categorical indicators. However, factor loading estimates should not be interpreted in these models.
- Larger N (>=500), smaller skewness and kurtosis, and larger number of indicator categories should be preferred if possible.
- The effect of data sparseness and scaling method on the efficiency of structural and measurement parameter estimates in LMS procedure should be examined in future research.

REFERENCES

Klein, A. G., & Moosbrugger, H. (2000). Maximum likelihood estimation of latent interaction effects with the LMS method. *Psychometrika*, *65*, 457-474.

Marsh, H. W., Wen, Z., & Hau, K. -T. (2004). Structural equation models of latent interactions: Evaluation of alternative estimation strategies and indicator construction. *Psychological Methods*, *9*, 275-300.

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