SYNTHESIZING SINGLE-CASE STUDIES VIA MULTILEVEL MODELS: LIMITATION OF MODEL COMPLEXITY

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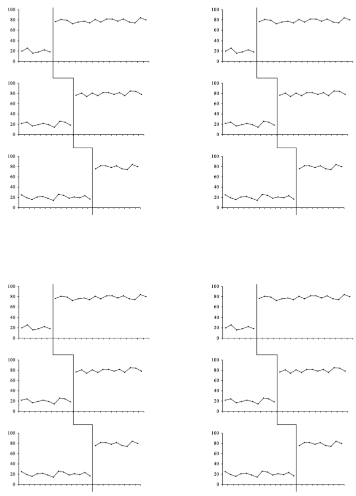
We gratefully acknowledge support from the Institute of Educational Sciences, U.S. Department of Education (through Grant R305D150007)

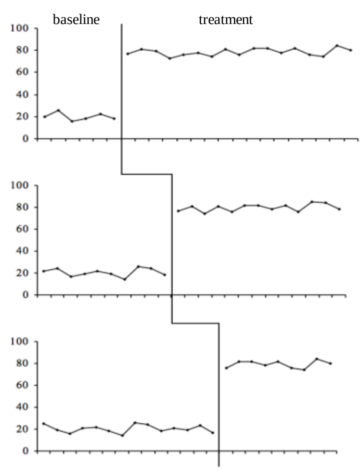
SINGLE-CASE EXPERIMENTAL DESIGNS (SCED)

- Researchers evaluate the effect of interventions by comparing repeated measurements on the same individual over time across one or more treatment conditions.
  - AB design (interrupted time-series design);
  - Withdrawal designs (e.g., ABA, ABAB, ABAC);
  - Changing criterion design (e.g., ABB', ABB'B'');
  - Multiple probe design;
  - Alternating treatments design (multi-element design);
  - Multiple baseline design.

SYNTHESIZING MULTIPLE SCED STUDIES



- Level 1: observations nested in participants
- Level 2: participants nested in studies
- Level 3: studies

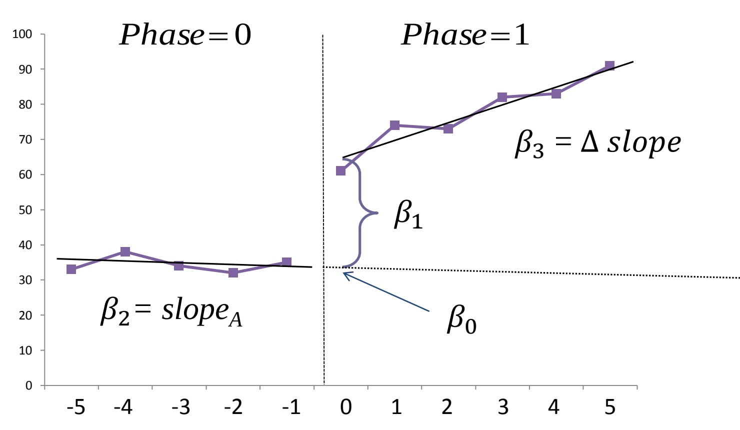


MULTILEVEL MODELS

- Multilevel models allow researchers to …
  - Estimate the average treatment effect across cases and studies
  - Quantify the changes in the treatment effect over time in treatment
  - Estimate the variation in the treatment effect across cases within studies and across studies
  - Examine potential moderators of the treatment effect

A TYPICAL MULTILEVEL MODEL

$$Y_{ijk} = \beta_{0jk} + \beta_{1jk}\ Phase + \beta_{2jk}\ Time + \beta_{3jk}\ Phase{*}Time + e_{ijk}$$



A TYPICAL MULTILEVEL MODEL

$$Y_{ijk} = \beta_{0jk} + \beta_{1jk}\,Phase + \beta_{2jk}\,Time + \beta_{3jk}\,Phase{*}Time + e_{ijk}$$

$$\begin{aligned}\beta_{0jk} &= \theta_{00k} + u_{0jk}\\ \beta_{1jk} &= \theta_{10k} + u_{1jk}\\ \beta_{2jk} &= \theta_{20k} + u_{2jk}\\ \beta_{3jk} &= \theta_{30k} + u_{3jk}\end{aligned} \qquad \begin{bmatrix}u_{0jk}\\u_{1jk}\\u_{2jk}\\u_{3jk}\end{bmatrix} \sim \mathrm{N}\begin{bmatrix}\sigma_{u0}^2 & 0 & 0 & 0\\ 0 & \sigma_{u1}^2 & 0 & 0\\ 0 & 0 & \sigma_{u2}^2 & 0\\ 0 & 0 & 0 & \sigma_{u3}^2\end{bmatrix}$$

$$\begin{aligned}\theta_{00k} &= \gamma_{000} + v_{0k}\\ \theta_{10k} &= \gamma_{100} + v_{1k}\\ \theta_{20k} &= \gamma_{200} + v_{2k}\\ \theta_{30k} &= \gamma_{300} + v_{3k}\end{aligned} \qquad \begin{bmatrix}v_{0k}\\v_{1k}\\v_{2k}\\v_{3k}\end{bmatrix} \sim \mathrm{N}\begin{bmatrix}\sigma_{v0}^2 & 0 & 0 & 0\\ 0 & \sigma_{v1}^2 & 0 & 0\\ 0 & 0 & \sigma_{v2}^2 & 0\\ 0 & 0 & 0 & \sigma_{v3}^2\end{bmatrix}$$

TYPICAL MULTILEVEL MODEL

- the trends are non-existent or linear
- the errors within a case are homogenous and either independent or first-order autoregressive
- the case-level errors are uncorrelated
- the study-level errors are uncorrelated

Moeyaert, Ugille, Ferron, Beretvas, & Van den Noortgate, 2014; Owen & Ferron, 2011; Petit-Bois, Baek, Van den Noortgate, Beretvas, & Ferron, 2016; Moeyaert, Rindskopf, Onghena, & Van den Noortgate, 2017

TYPICAL MULTILEVEL MODEL

Estimation REML with Kenward Roger adjusted SEs and DFs

Assuming the model is correctly specified:

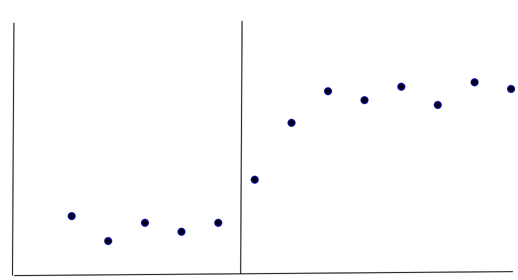
- Fixed effects are unbiased (e.g., average treatment effect)
- Fixed effect inferences are accurate (e.g., 95% CIs cover 95% of the time)
- Variance components are biased (e.g., across case variance in effect)

Bayesian Estimation

Reduces bias in variance components (if priors chosen well)

Moeyaert, Ugille, Ferron, Beretvas, & Van den Noortgate, 2014; Owen & Ferron, 2011; Petit-Bois, Baek, Van den Noortgate, Beretvas, & Ferron, 2016; Moeyaert, Rindskopf, Onghena, & Van den Noortgate, 2017

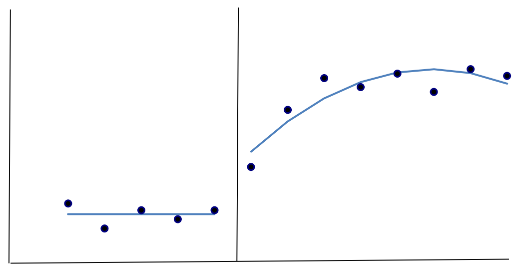
WHAT IF DATA ARE MORE COMPLEX?



Estimating logistic models creates problems given the small sample size

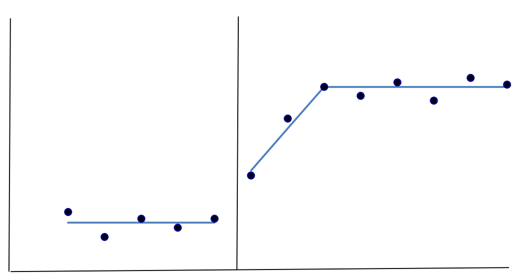
Hembry, I., Bunuan, R., Beretvas, S. N., Ferron, J. M., & Van den Noortgate, W. (2015). Estimation of a nonlinear intervention phase trajectory for multiple baseline design data. *Journal of Experimental Education*, *83*, 514-546.

WHAT IF DATA ARE MORE COMPLEX?



Maybe fit a quadratic trajectory to the intervention phase?

WHAT IF DATA ARE MORE COMPLEX?



Maybe fit a piecewise trajectory to the intervention phase?

WHAT IF DATA ARE MORE COMPLEX?

What if the covariance structure at level-1 is not independent and homogeneous, but rather it is:

- First-order autoregressive
- Heterogeneous across phases
- Heterogeneous across cases

What if the covariance structure at level-2 is not diagonal, but unstructured?

What if the covariance structure at level-3 is not diagonal, but unstructured?

PURPOSE OF OUR STUDY

To compare alternative multilevel models for analyzing a series of multiple baseline studies that are characterized by multiple complexities:

- treatment phase trajectories that are non-linear
- within-case (level-1) errors that are autocorrelated and heterogeneous across phases and across cases
- level-2 and level-3 errors that may have an unstructured covariance structure

OVERVIEW OF METHODS

- Monte Carlo Study

We generated data for a series of multiple-baseline studies where the underlying model had multiple complexities

We analyzed each generated data set with alternative models that differed in complexity

We compared the models on convergence rates, parameter bias, and confidence interval coverage.

REVIEWED PUBLISHED APPLIED STUDIES TO MOTIVATE DATA GENERATION MODEL

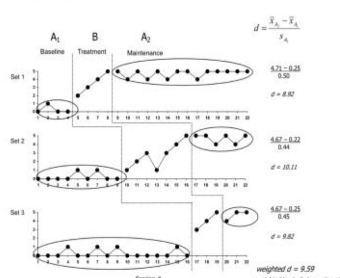
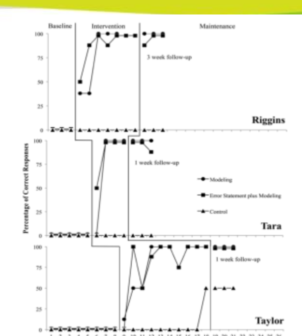


Fig. 1 Example of data from single-subject multiple baseline design. The circled data points are those that contribute to the calculation of d statistic to index changes in level of performance (see raw data in Table 1)

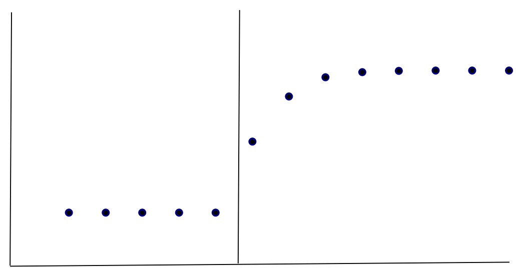


Beeson, P. M., & Robey, R. R. (2006). Evaluating single-subject treatment research: Lessons learned from the aphasia literature. *Neuropsychology review*, *16*(4), 161-169.

Townley-Cochran, D., Leaf, J. B., Leaf, R., Taubman, M., & McEachin, J. (2017). Comparing Error Correction Procedures for Children Diagnosed with Autism. *Education and Training in Autism and Developmental Disabilities*, *52*(1), 91.

DATA GENERATION

Data were generated based on a 3-level model



$$Y_{ijk} = 0.20 + u_{jk} + v_k + \frac{.5}{1+e^{-1.5(time_c)}} * Phase + e_{ijk}$$

DATA GENERATION

$$Y_{ijk} = 0.20 + u_{jk} + v_k + \frac{.5}{1+e^{-1.5(time_c)}} * Phase + e_{ijk}$$

e_{ijk}	Distributed first order autoregressive (ρ = .2)
	Baseline variance of first case: σ^2_{A1}=.0025
	Heterogeneous across phases: $\frac{\sigma^2_A}{\sigma^2_B}$ = 2
	Heterogeneous across cases: $\frac{\sigma^2_4}{\sigma^2_1}$= 4
u_{jk}	Distributed normal with variance, σ^2_u=.0025
v_k	Distributed normal with variance, σ^2_v=.0025

STUDY DESIGN

- Design Factors
  - Level 1: number of observation per participant (16 & 32)
  - Level 2: number of participant per study (4 & 8)
  - Level 3: number of study (10 & 30)

STUDY DESIGN

- Conditions

Conditions	# of Observation	# of Participant	# of Case
1	16	4	10
2	16	4	30
3	16	8	10
4	16	8	30
5	32	4	10
6	32	4	30
7	32	8	10
8	32	8	30

Number of replication in each condition: 2000

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MODELS FOR ANALYZING

Model 1

- Quadratic with simple errors structure:

Level-1 error:

- independence;
- homogeneity.

Level-2 error:

- not correlated.

Level-3 error:

- not correlated.

Model 2

- Quadratic with complex errors structure:

Level-1 error:

- autocorrelated;
- Heterogeneous (across phases and participants).

Level-2 error:

- correlated.

Level-3 error:

- correlated.

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MODELS FOR ANALYZING

Model 3

- Piecewise with simple errors structure:

Level-1 error:

- independence;
- homogeneity.

Level-2 error:

- not correlated.

Level-3 error:

- not correlated.

Model 4

- Piecewise with complex errors structure:

Level-1 error:

- autocorrelated;
- Heterogeneous.

Level-2 error:

- correlated.

Level-3 error:

- correlated.

SIMULATION RESULTS

Model 1 Quadratic with simple errors structure

Observation	Participant	Study	Effect Est	Bias	Relative Bias	Coverage	Std. E	Convergence Rate
16	4	10	0.505	0.005	.009	.921	0.008	1
		30	0.506	0.004	.009	.841	0.004	1
	8	10	0.505	0.005	.009	.918	0.005	1
		30	0.506	0.004	.009	.735	0.003	1
32	4	10	0.456	0.044	.088	0	0.005	1
		30	0.456	0.044	.088	0	0.003	1
	8	10	0.456	0.044	.088	0	0.004	1
		30	0.456	0.044	.088	0	0.002	1

SIMULATION RESULTS

Model 2 Quadratic with complex errors structure

Sadly…IT DIDN'T CONVERGE…

SIMULATION RESULTS

Model 3 Piecewise with simple errors structure

Observation	Participant	Study	Effect Est	Bias	Relative Bias	Coverage	Std. E	Convergence Rate
16	4	10	0.496	0.005	.009	.920	0.007	1
		30	0.496	0.004	.009	.818	0.004	1
	8	10	0.496	0.004	.008	.894	0.005	1
		30	0.496	0.004	.009	.663	0.003	1
32	4	10	0.497	0.003	.005	.934	0.005	1
		30	0.497	0.003	.005	.863	0.003	1
	8	10	0.497	0.003	.006	.896	0.004	1
		30	0.497	0.003	.006	.749	0.002	1

SIMULATION RESULTS

Model 4 Piecewise with complex errors structure

Observation	Participant	Study	Effect Est	Bias	Relative Bias	Coverage	Std. E	Convergence Rate
16	4	10	0.496	.004	.007	.96	0.007	.90
		30	0.496	.004	.007	.87	0.004	.99
	8	10	0.497	.003	.007	.93	0.005	.71
		30			Simulation is still running			
32	4	10	0.498	0.002	.005	.96	.005	.95
		30	0.498	0.003	.006	.89	.003	.97
	8	10	0.496	0.004	.007	.76	.003	.84
		30			Simulation is still running			

SIMULATION RESULTS

Summary

Model 1: 32 Observations -- more bias, and coverage is 0.

Model 2: Did not converge.

Model3: 30 Studies -- the coverage was lower than the nominal .95.

Model 4: Will conclude it after get the final simulation results.

- Piecewise trajectory has less bias and better 95% confidence interval coverage across all the conditions than quadratic trajectory in the treatment phase.

- For the model with a piecewise trajectory in the treatment phase and a simpler error structure, the coverage was lower than the nominal .95.

FURTHER RESEARCH DESIGN

1. Only limited conditions were examined;
2. Future research should examine:
   - Other non-linear treatment trajectories
   - Non-normally distributed error structure (level-1, level-2, and level-3)
   - Other methods of estimating the model (such as Bayesian estimation)
   - Other dependent error structures in level-1 (moving average; 2nd order autoregressive)

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