Performance of Latent Growth Curve Models with Binary Variables

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Goal

- Examine estimation of latent growth curve models with diagonalized weighted least squares and categorical ML estimation approaches for binary variables

- Effects of sample size, number of time points, base rate proportion

- Examine convergence failures, parameter bias, standard error bias, Type I error, and coverage

Background

Simulation of SEMs with binary/ordinal estimators suggest:

- Weighted least squares (WLS) estimation with polychoric correlations has better small sample performance than regular ADL/WLS approach (Flora & Curran, 2004; Maydeu-Olivares, 2001; Muthén, du Toit, & Spisic, 1997)
- Diagonalized WLS more computationally practical than inversion of full weight matrix (Muthén, 1993)
- Limited information method, which may be less optimal when values are NMAR (Asparouhov & Muthén, 2010)

Background

Maximum likelihood for categorical variables

- Sometimes referred to as “marginal maximum likelihood”, uses expectation maximization (EM) algorithm with numeric integration to analyze full multiway frequency tables (Bock & Atkin, 1981; Christofersson, 1975; Muthén & Christofferson, 1981)

- Commonly employed with item response models (Demars, 2012; Kamata & Bauer, 2008)

- Parameter estimates that are interpretable as logistic regression coefficients

- Full-information estimation approach. Can be computationally intensive, but may be preferable to WLSMV when values are NMAR

Background

- DWLS method with robust adjustments outperforms unadjusted counterpart (Bandalos, 2014; DeMars, 2012), henceforth **referred to as WLSMV**
- Maximum likelihood (ML) for binary variables using robust standard errors also outperforms unadjusted ML counterpart, henceforth **referred to as MLR** (Bandalos, 2014; DeMars, 2012)
- Sample sizes of 100-150 have poor Type I error or coverage rates for WLSMV (Rhemtulla, Brosseau-Liard, & Savalei, 2012)

Background

- Several sources describe details of testing LGC models with binary or ordinal variables (e.g., Lee, Wickrama, & O'Neal, 2018; Masyn, Petras, & Liu, 2014; Mehta, Neale, & Flay, 2004; Newsom, 2015)
- Not widely used presently but popularity is likely to increase

Background

- Bandalos (2014) compared robust binary ML and DWLS and found that
  - Robust ML and WLSMV performed comparably but had high Type I error rates with low N (150) and asymmetric data (up to N = 300)
  - Robust ML performed better with asymmetric variables (rarer event)
  - Concluded WLSMV was better all-around choice because of less bias in parameter estimates

Background

Comparisons of categorical estimation methods have been based on factor models and some standard predictive models

Latent growth curve models (LGC) differ in the large number of parameter constraints, parameters of interest (factor means and variances), and potential impact of the number of time points

Few existing studies of performance of LGC models with categorical indicators, and more work is needed

Background

Muthén (1996) conducted a small simulation study with binary variables in latent growth curve models

- Using LISCOMP (unadjusted DWLS)
- Low parameter bias for N=250 and N=1000
- Fairly symmetric outcome, with baseline proportion of .64
- Type I error rates were approximately nominal for N=250 (5.4%)

Background

Finch (2017) conducted a simulation study with binary variables in latent growth curve models

- Examined WLSMV 4, 6, and 8 time points
- N = 200, 500, and N=1000 (but smaller N may be common for studies)
- Examined different baseline proportions, but neither condition particularly rare (.5 an .69)

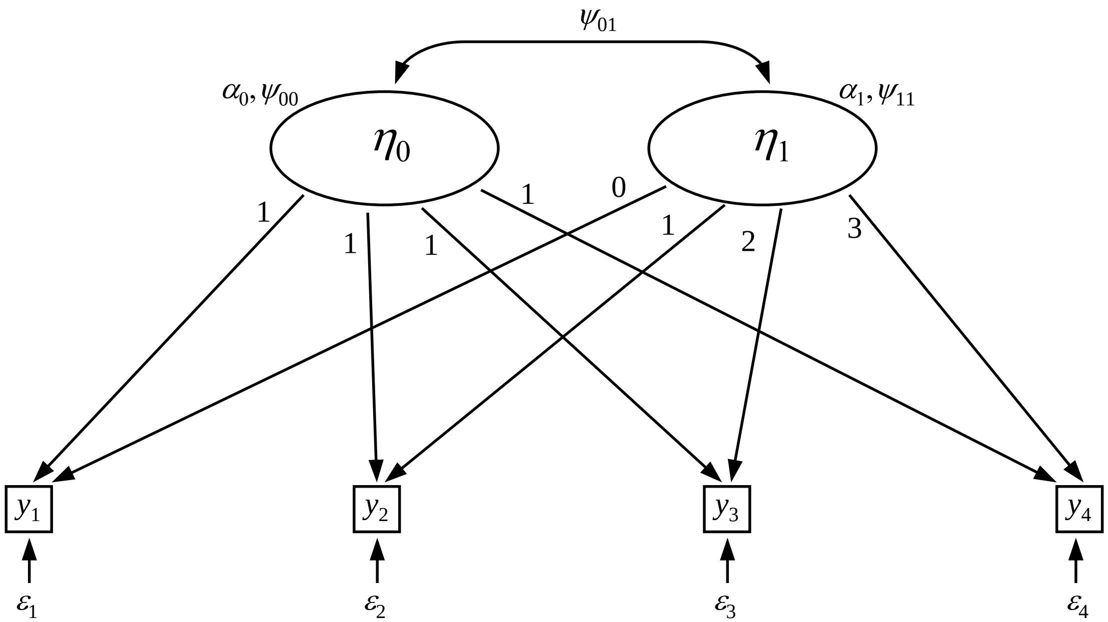
Background

Finch (2017)

- Better convergence with more time points
- Found low bias and good coverage rates in fixed effects (i.e., average intercept and slope estimates) even for N = 200
- No evaluation of random effects
- Did not examine MLR estimation

Background

The latent growth curve model



Method

Simulations

- Data were generated in SAS 9.4 using the RandomMVBinary macro (Wicklin, 2013) to produce proportions and change in proportions comparable to applied longitudinal studies (LLSE: Sorkin & Rook, 2004; HRS: Heeringa & Connor, 1995)

- Simulations were analyzed using the Mplus Version 8 (Muthén & Muthén, 1998-2017) MONTECARLO feature

Design

Dependent Measures

- % bias slope means (fixed effects)
- % bias slope variances (random effects)
- Bias greater than 5% was considered unacceptable (Hoogland & Boomsma, 1998)

Design

Bias Computation for Average Slopes

- Percent relative bias

$$\%Bias\left(\hat{\theta}\right)=\frac{\bar{\hat{\theta}}-\theta}{\theta}\bullet 100$$

- Computation when parameter value equals 0

$$\%Bias\left(\hat{\theta}\right)=\Phi^{-1}\left(z_{\bar{\hat{\theta}}}\right)-\Phi^{-1}\left(z_{\theta}\right)$$

where Φ^{-1} is the cumulative standard normal probability, $z_{\hat{\theta}}=\left(\hat{\theta}-\theta\right)/SD_{\hat{\theta}}$ estimated from standard score values for the parameter, z_{θ}, with the percentile for the population value for θ_i equal to .5

Design

Dependent Measures

- Bias Computation for Variances

$$\%Bias\left(SE\right) = \frac{E\left(SE\right) - SD}{SD} \bullet 100$$

Design

Dependent Measures

- Type I error (% samples significant) – for conditions in which the slope is zero
- 95% Coverage (% samples in which population value falls within confidence interval) – for all other conditions

Design

Independent Variables

- Sample size: 100, 200, 500, 1000
- # time points: T = 3, 5, 7
- 84,000 replications, 500 per cell (e.g., Paxton et al., 2001)
- Average intercept: small proportion (.11) vs. medium proportion (.45)
- Average slope: 0 vs. mod effect (increase of p = .025 per wave)
- Slope variance: small vs. medium effect

Design Summary

	Average Intercept, α_I	Average Slope, α_S	Slope Variance, ψ_s
SSS	Small	Small(Zero)	Small
SMS	Small	Medium	Small
SMM	Small	Medium	Medium
MSS	Medium	Small(Zero)	Small
MSM	Medium	Small(Zero)	Medium
MMS	Medium	Medium	Small
MMM	Medium	Medium	Medium

Results

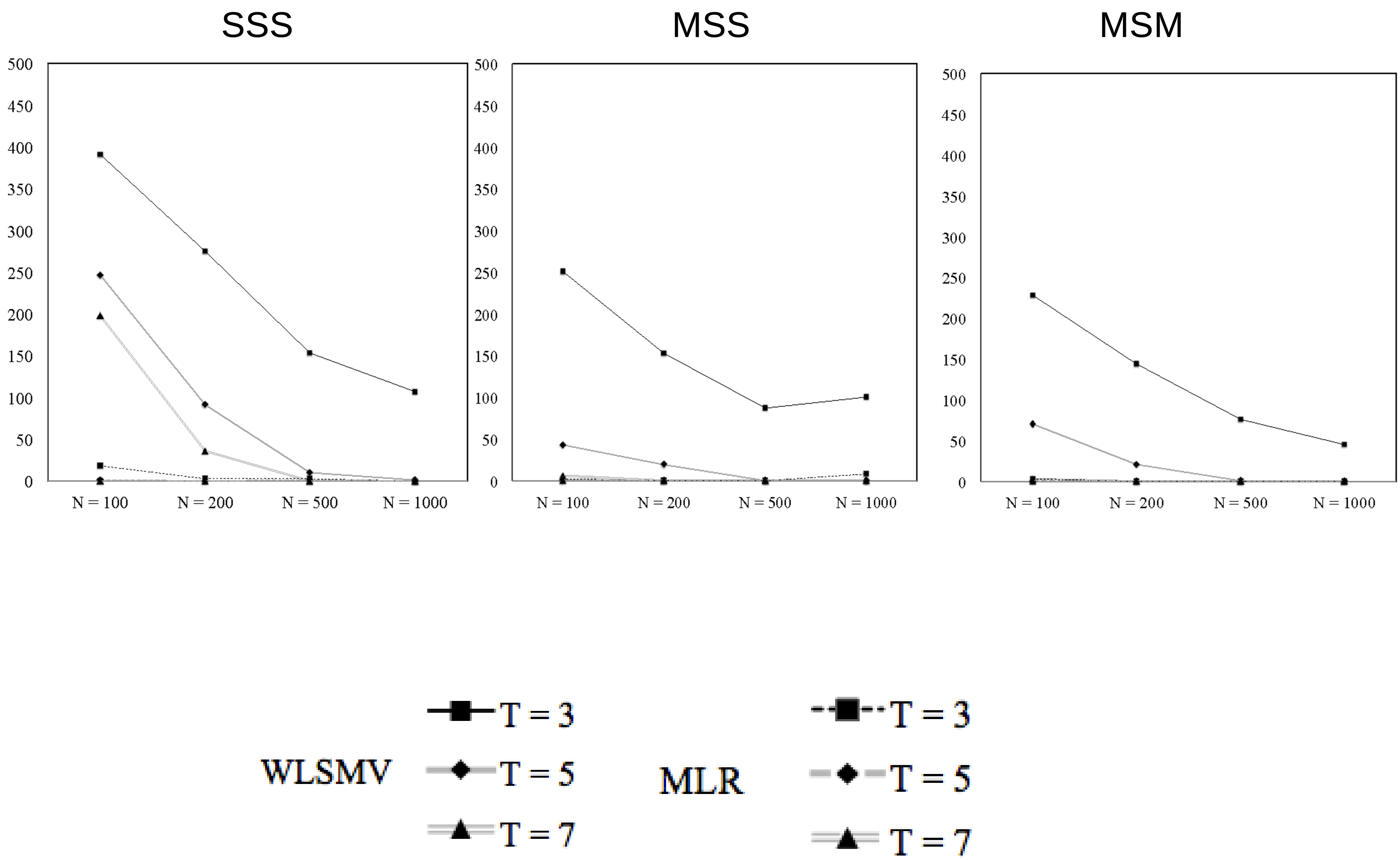
Convergence

- Models converged successfully for models in all conditions for both WLSMV and MLR

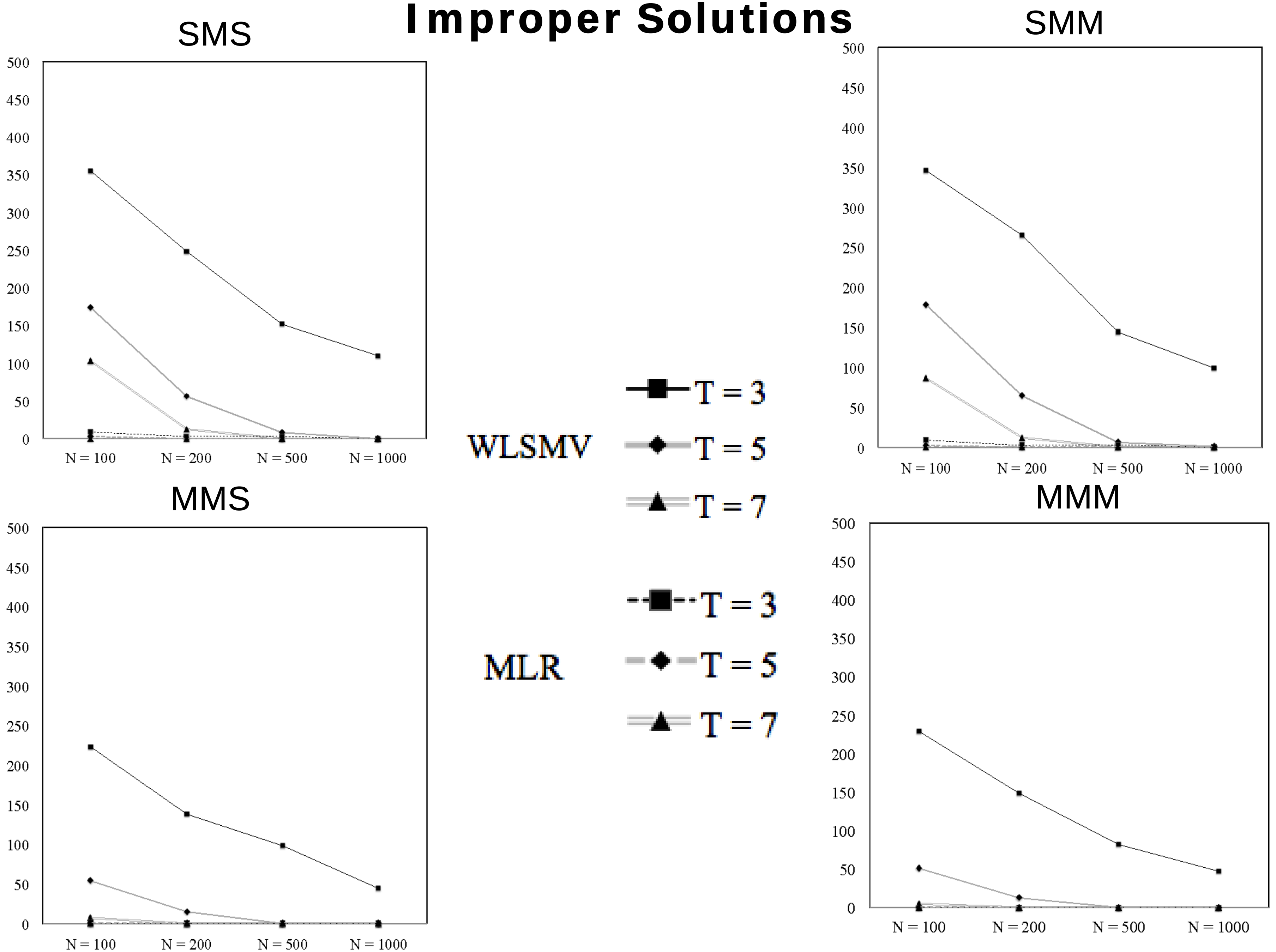
Improper solutions

- Improper solutions (e.g., negative error variances) were common, sometimes as high as 80% of samples with small N and T
- Especially high for the low baseline proportion conditions (SSS, SMS, and SMM)

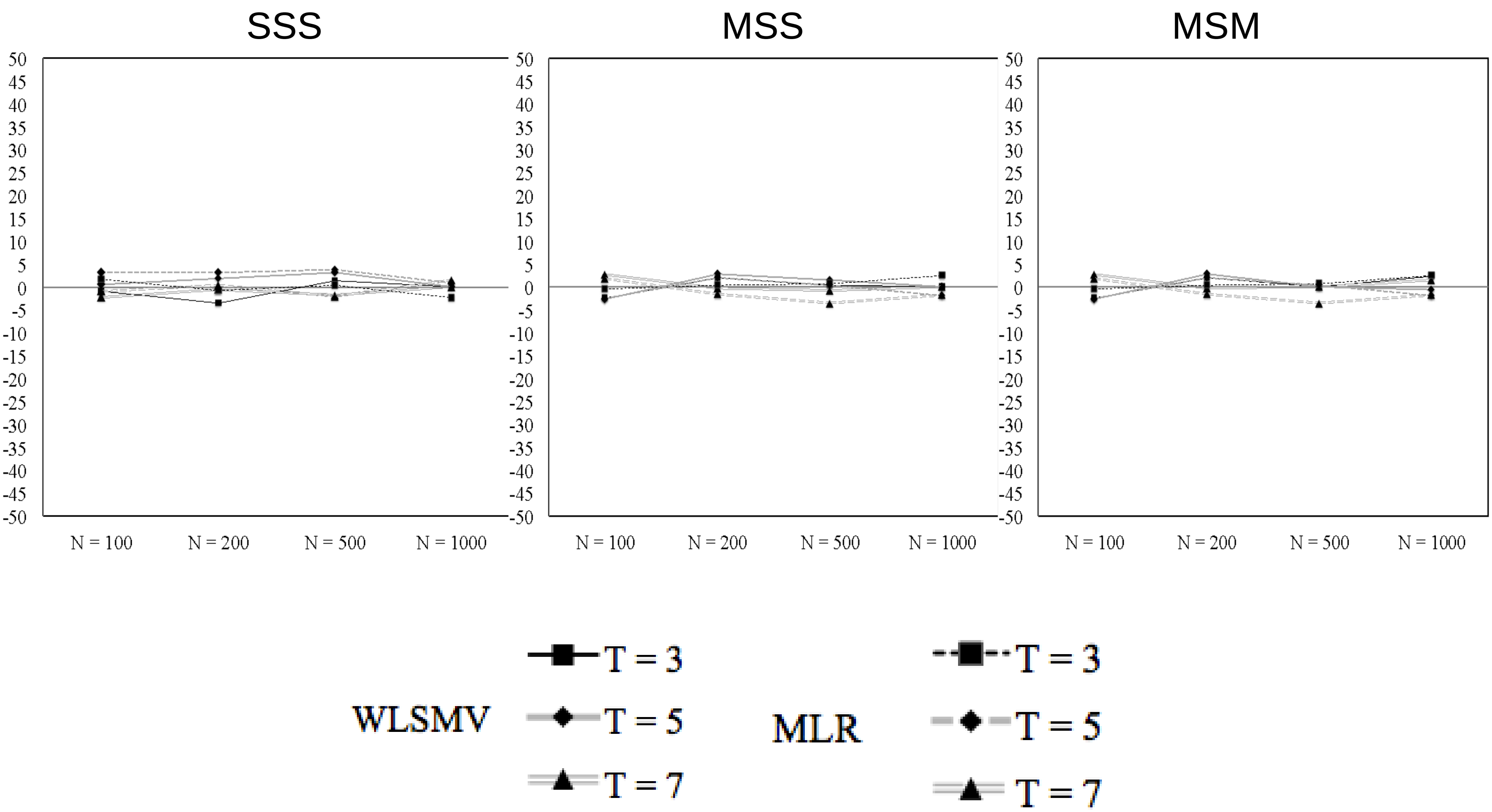
Improper Solutions



Improper Solutions

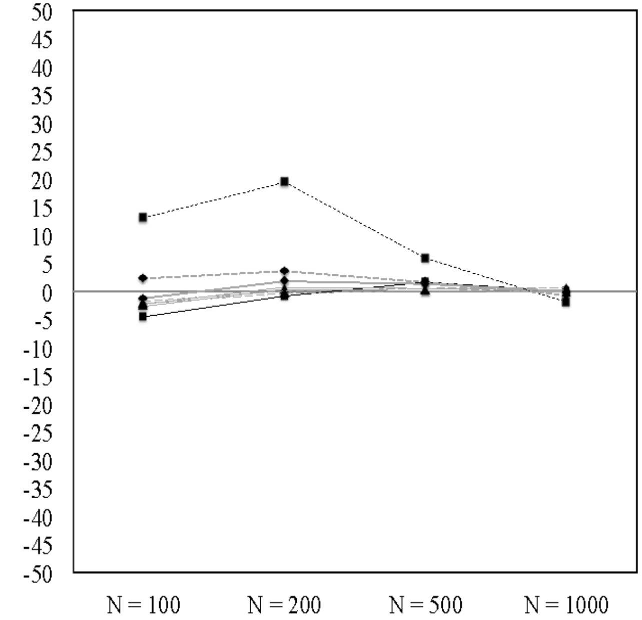


Average Slope
% Bias
(zero slope)

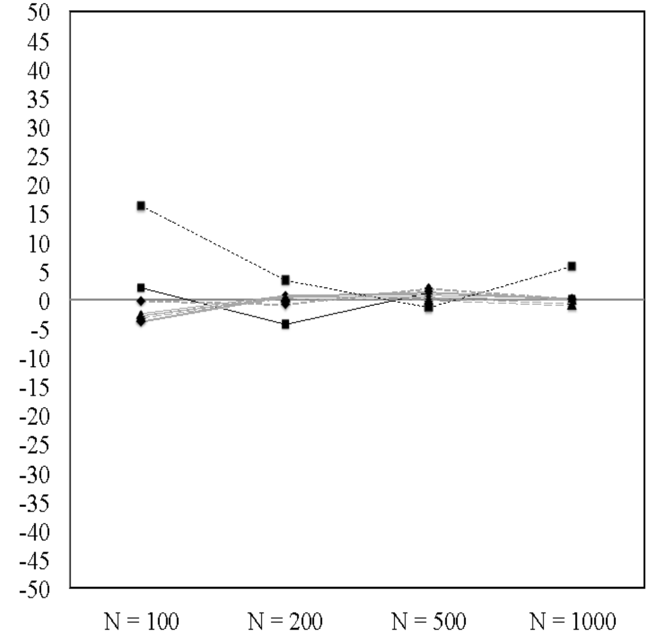


Average Slope
% Bias
(medium slope)

SMS



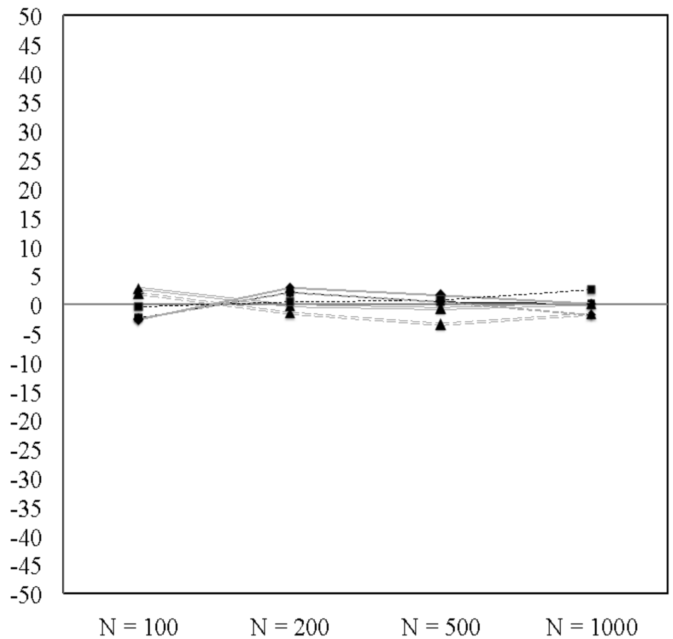
SMM



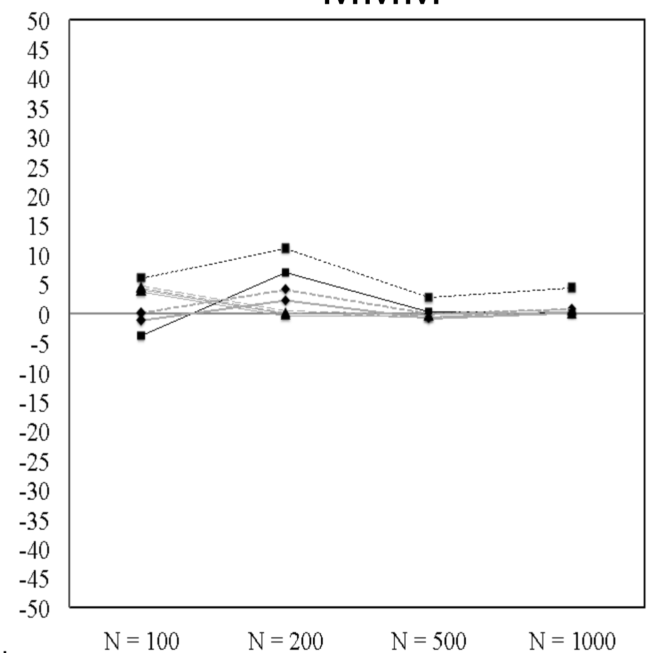
WLSMV

T = 3
T = 5
T = 7

MMS



MMM

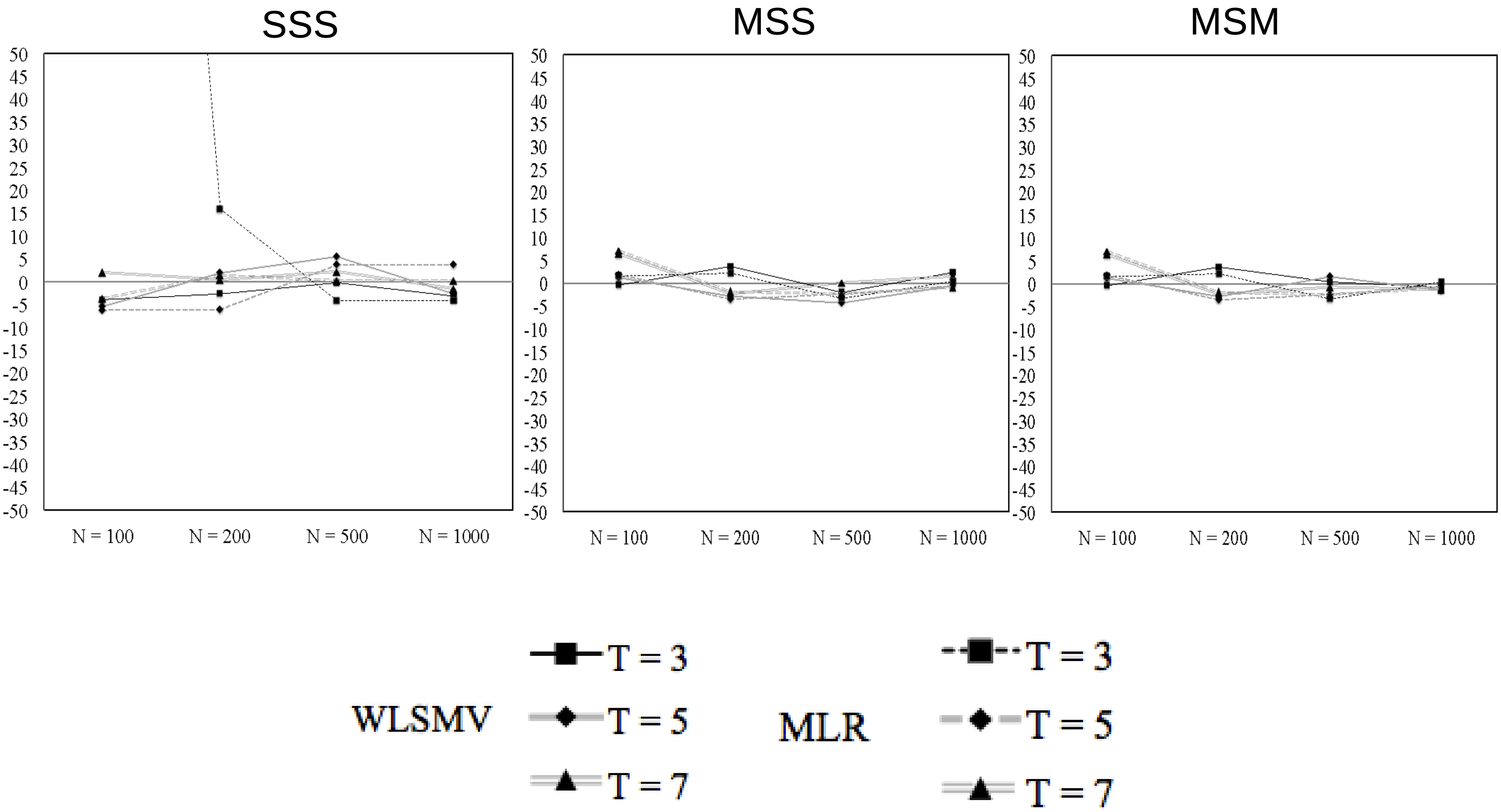


MLR

T = 3
T = 5
T = 7

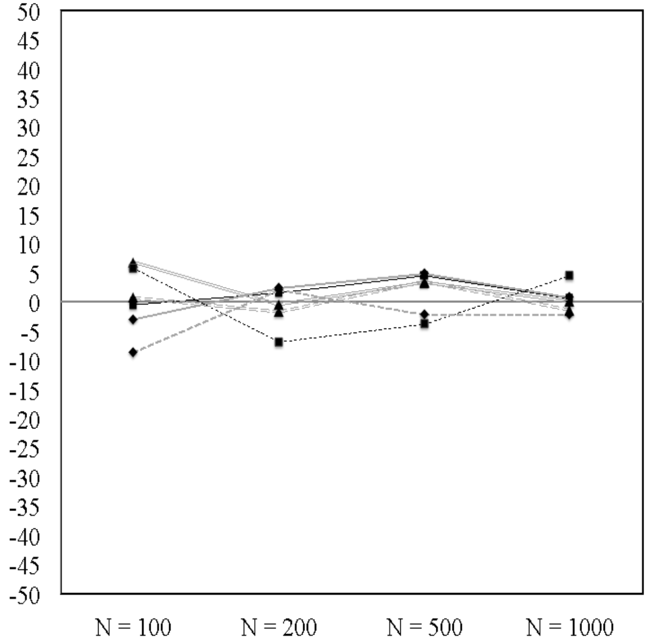
Average Slope SE

% Bias
(zero slope)

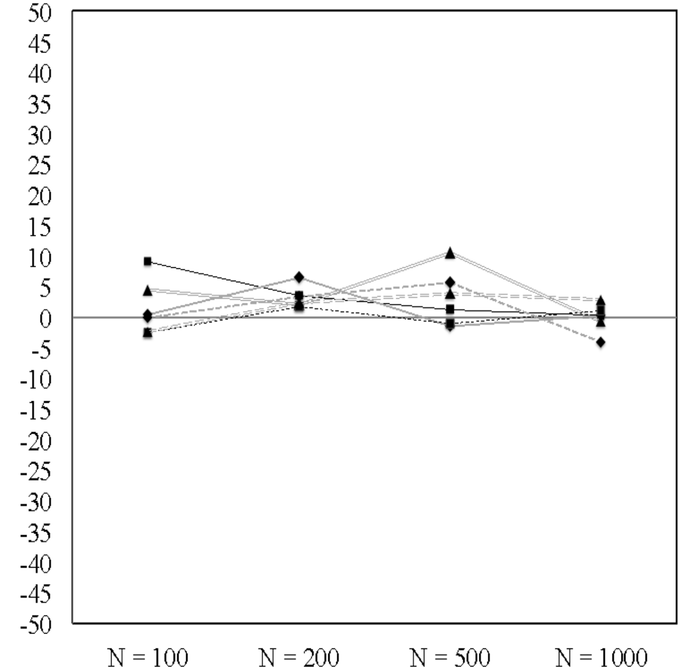


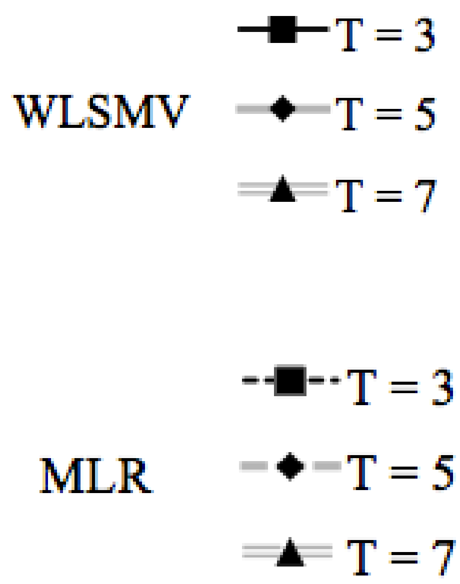
Average Slope SE
% Bias
(medium slope)

SMS

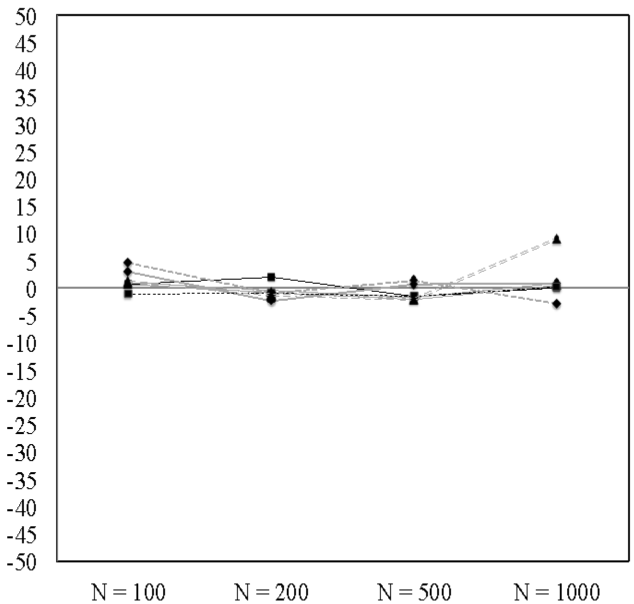


SMM

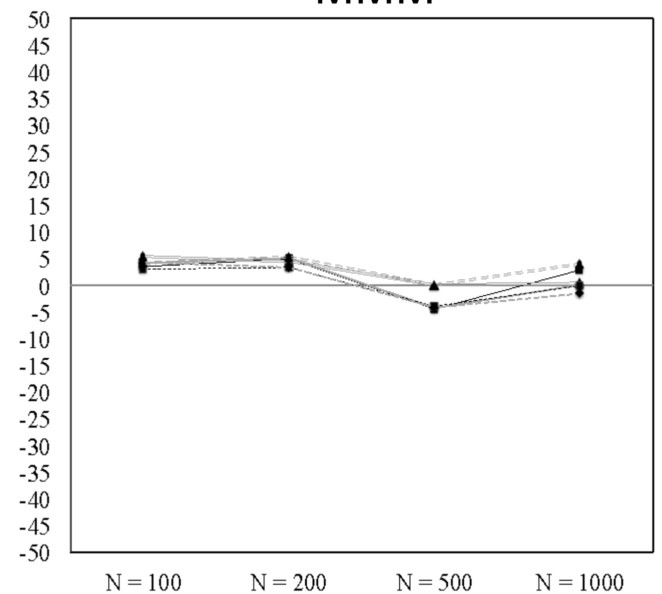




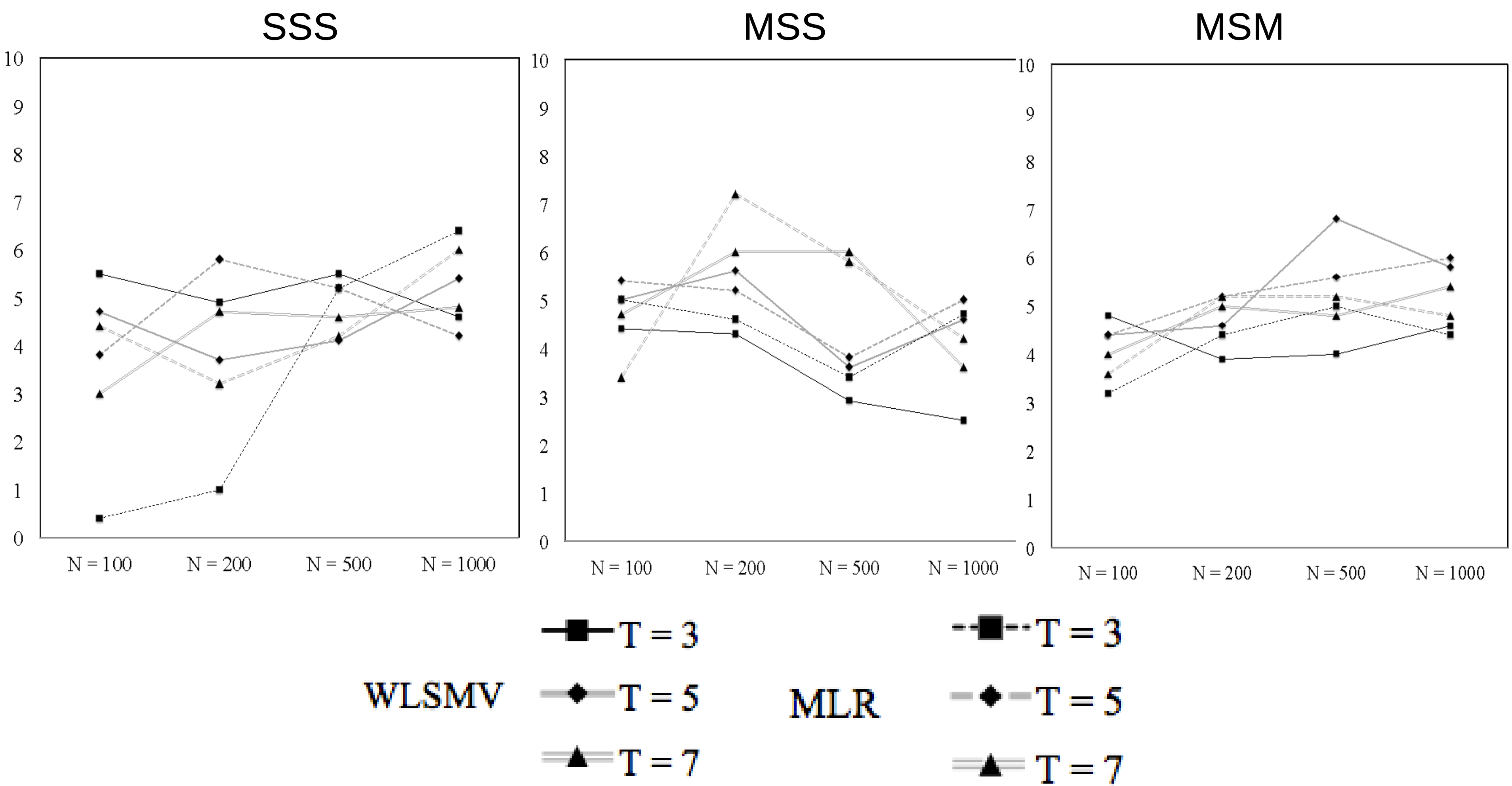
MMS



MMM

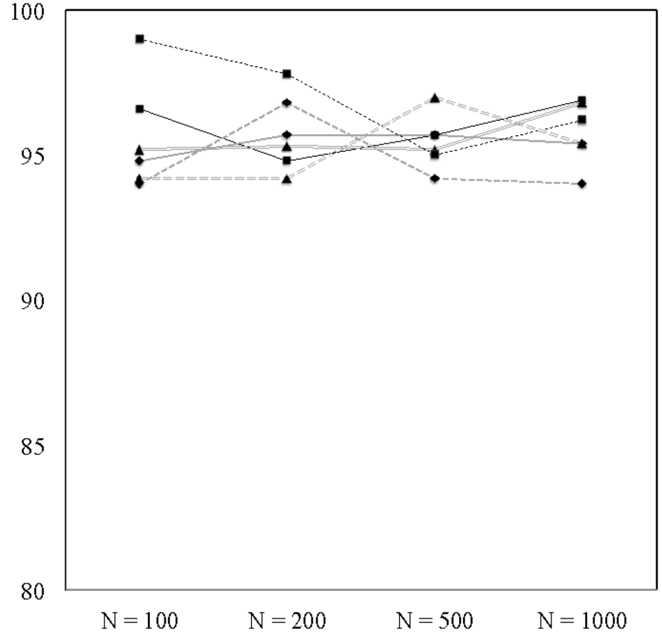


Average Slope
% Type I Error
(zero slope)

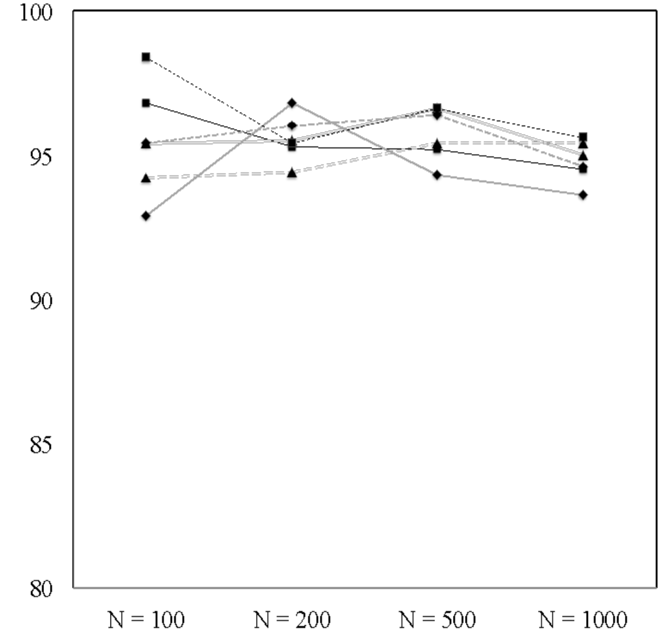


Average Slope
95% Coverage (medium slope)

SMS



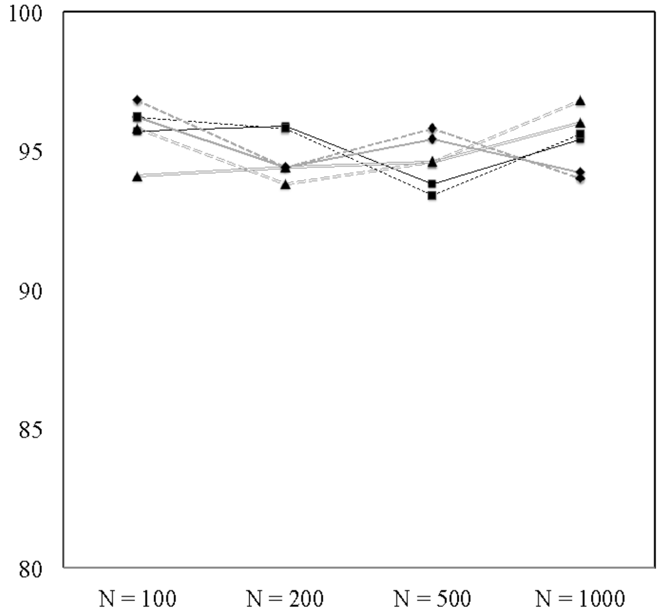
SMM



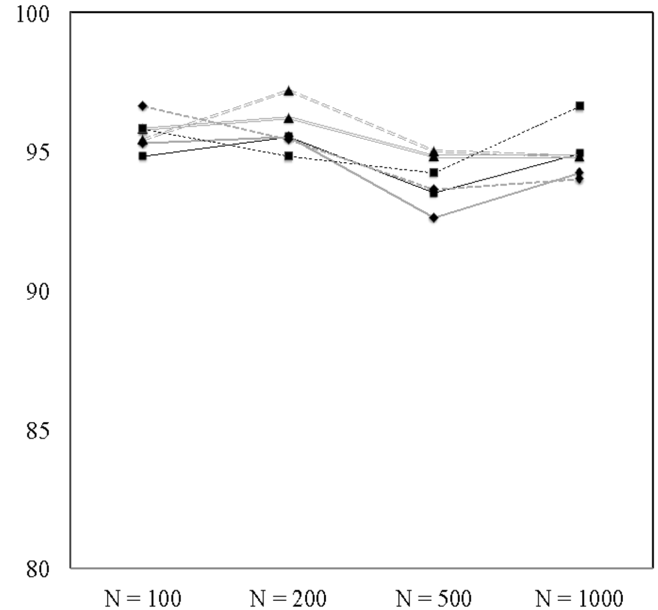
WLSMV

T = 3
T = 5
T = 7

MMS



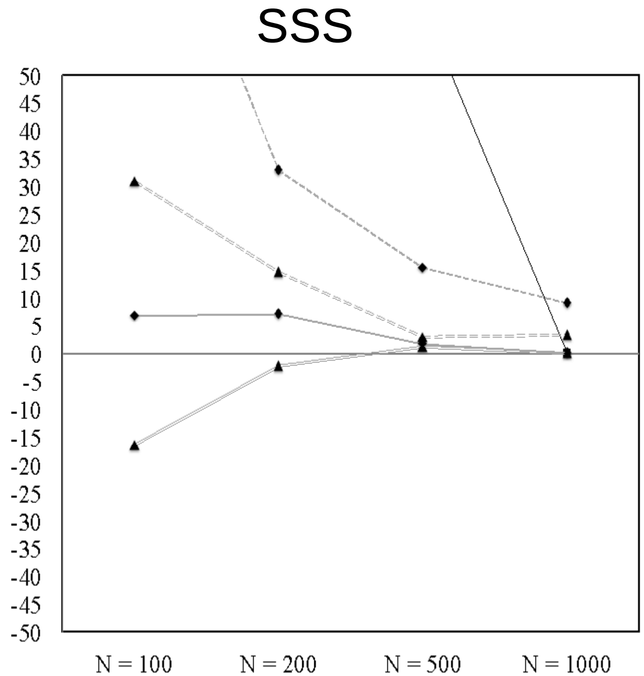
MMM

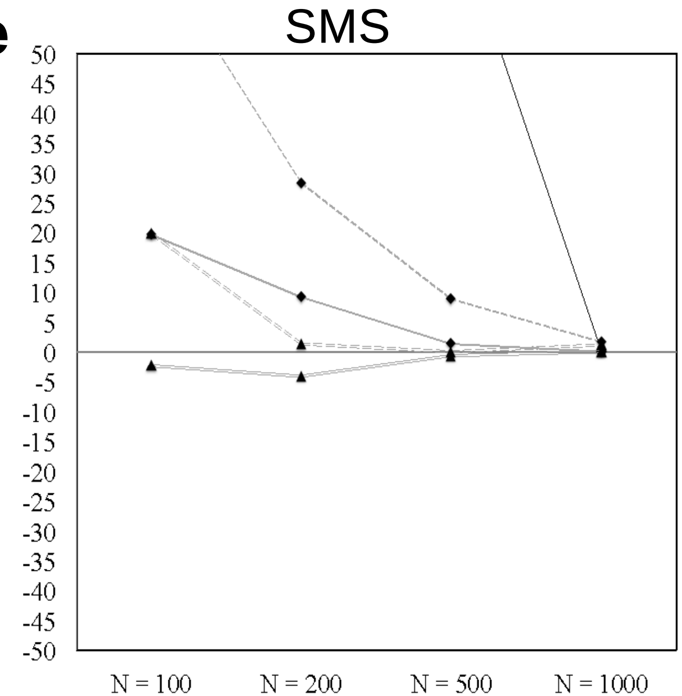


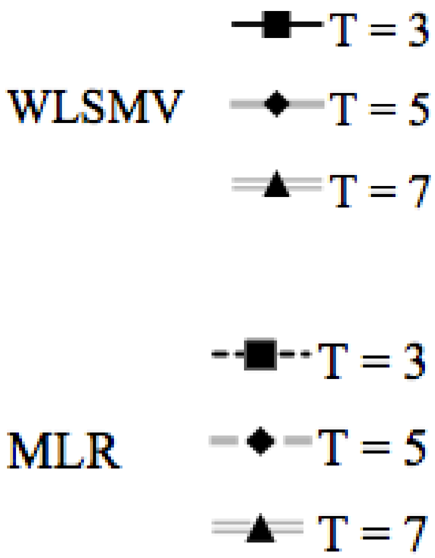
MLR

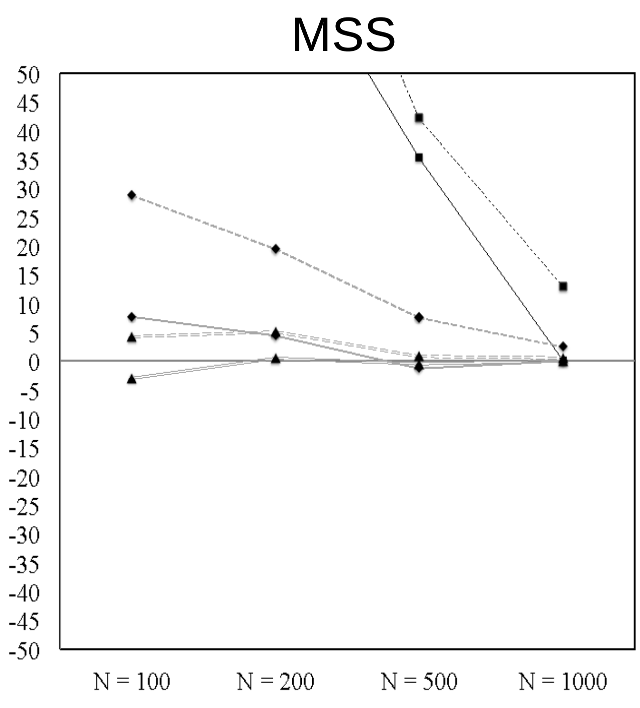
T = 3
T = 5
T = 7

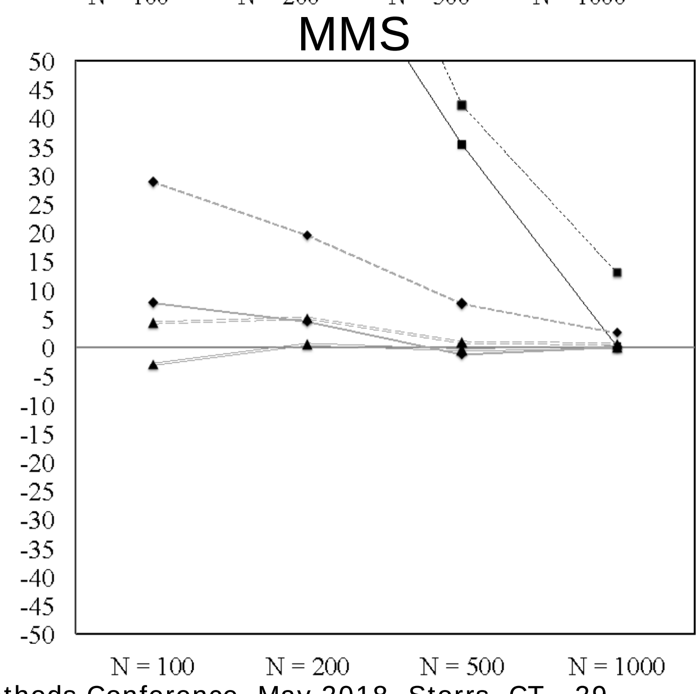
Slope Variance
% Bias
(small variance)





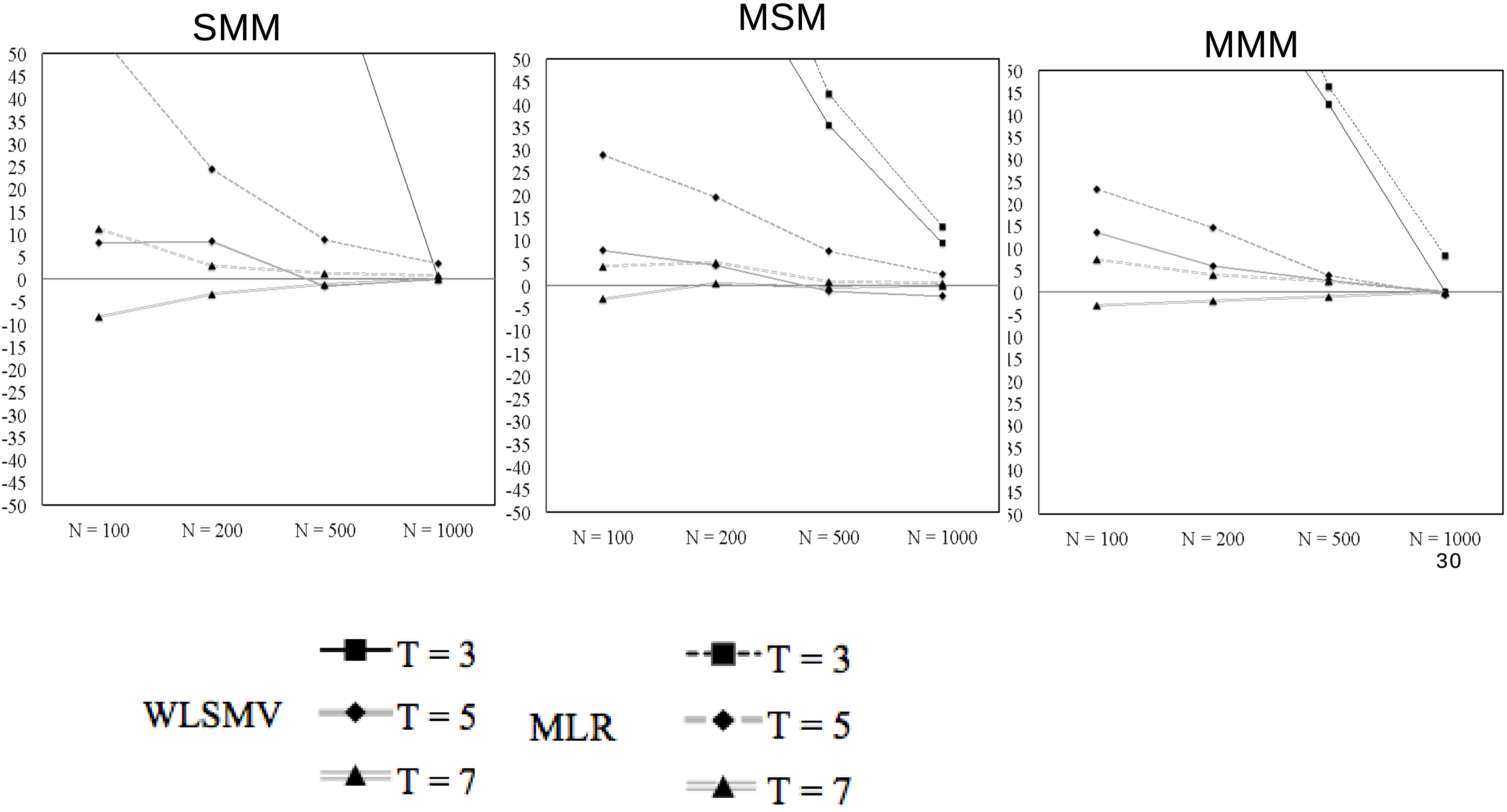






Slope Variance

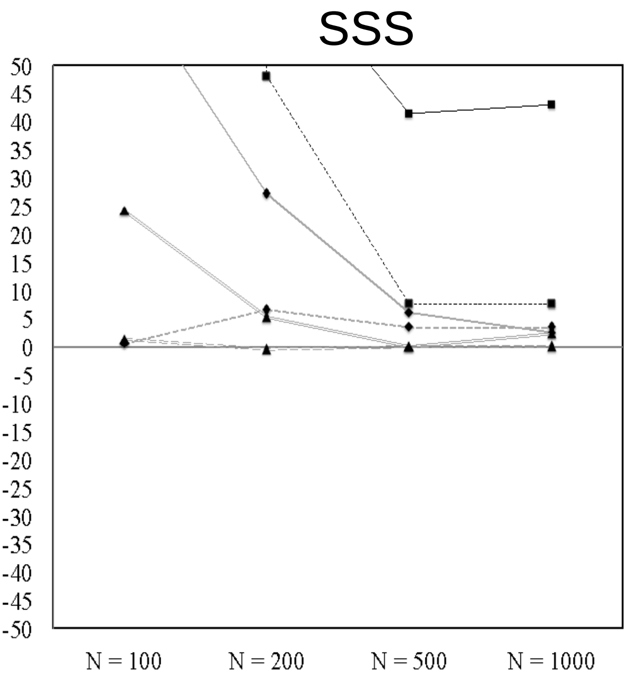
% Bias
(medium variance)

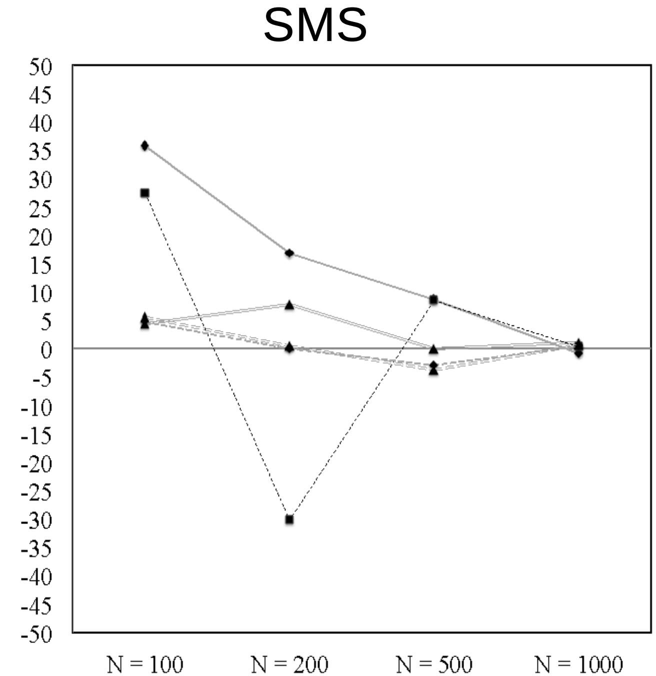


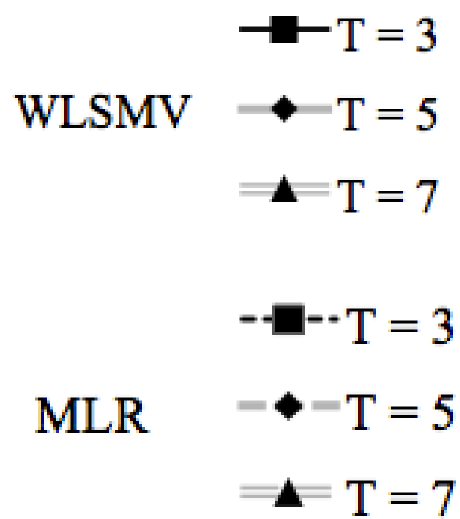
Paper Presented at the Modern Modeling Methods Conference, May 2018, Storrs, CT

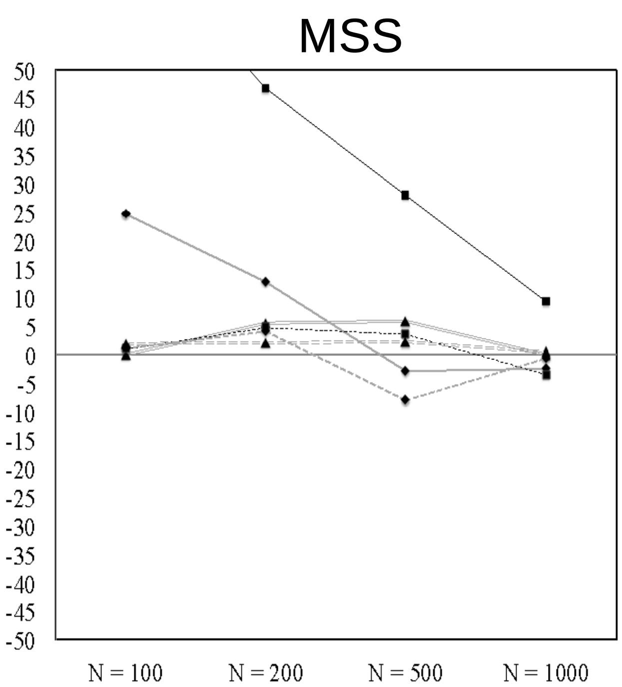
Slope Variance SE

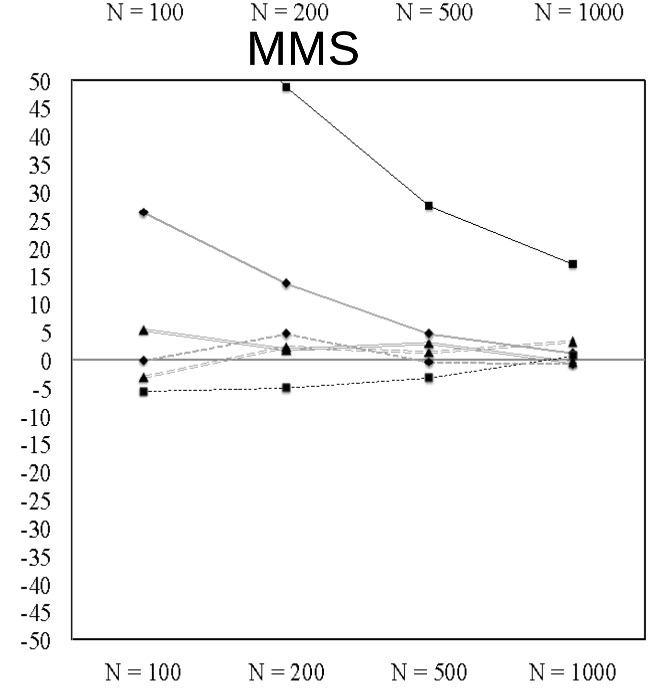
% Bias
(small variance)





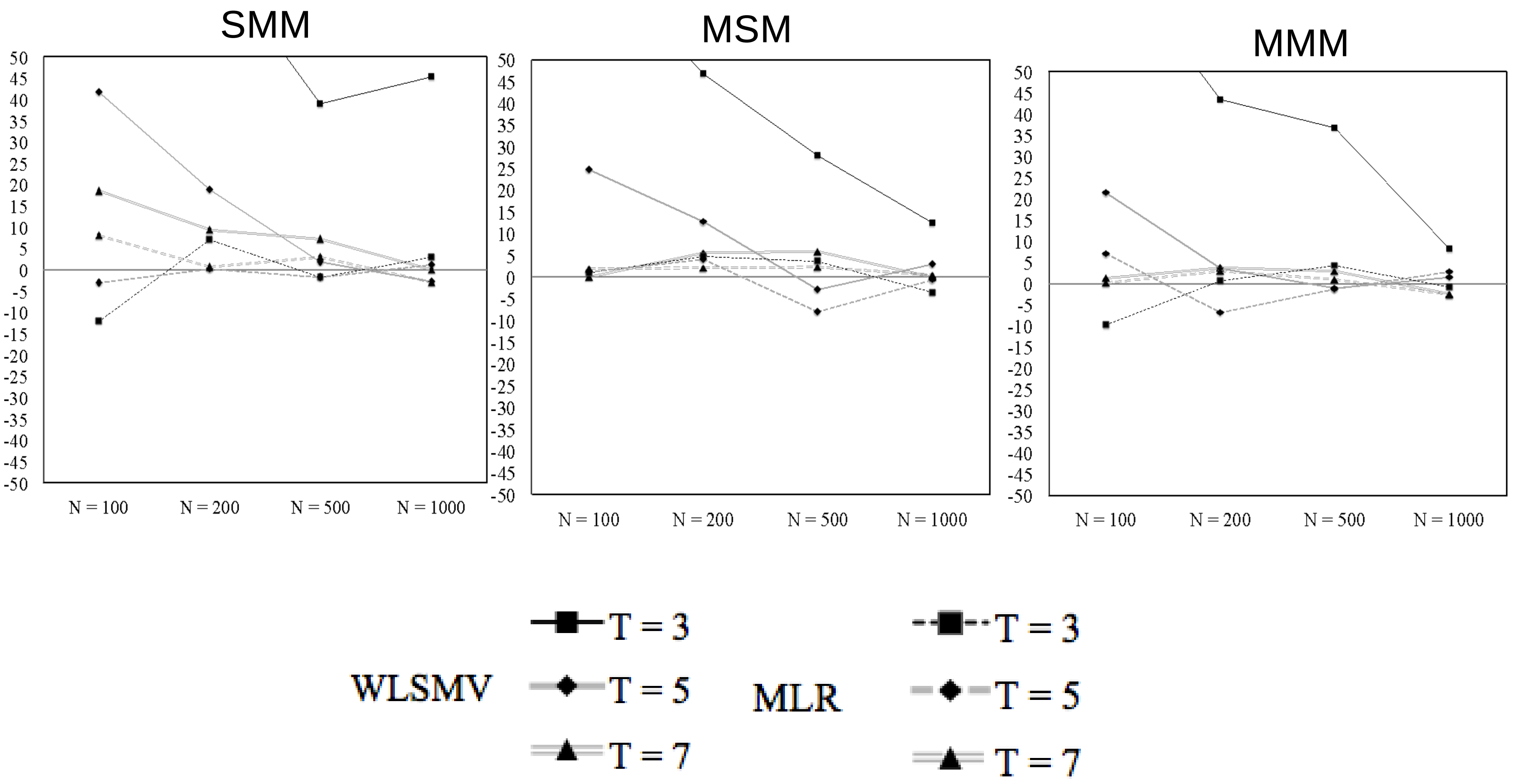






Slope Variance SE

% Bias
(medium variance)



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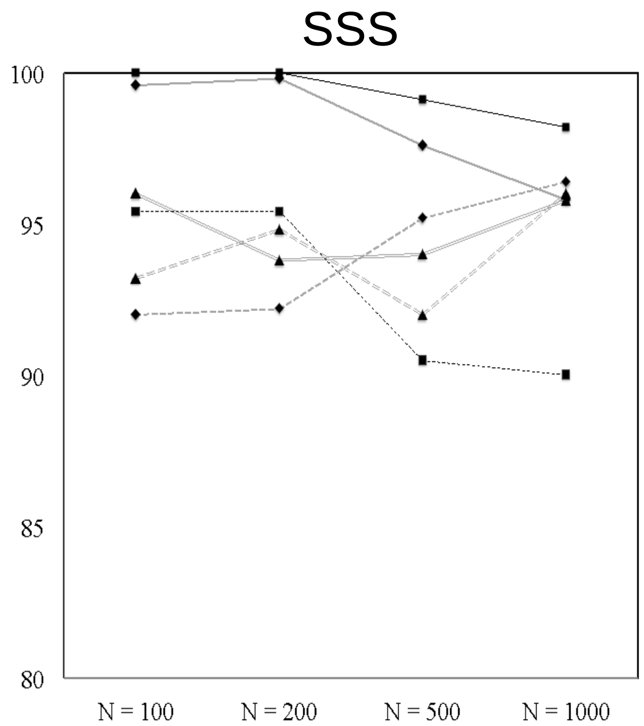
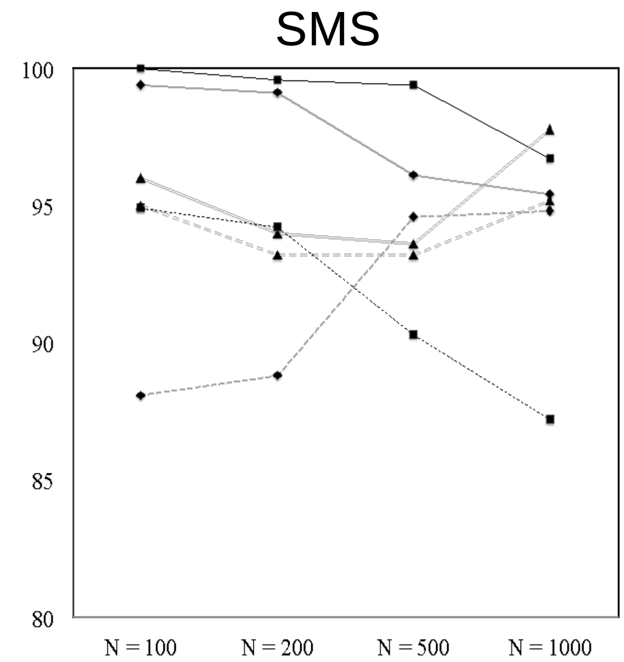
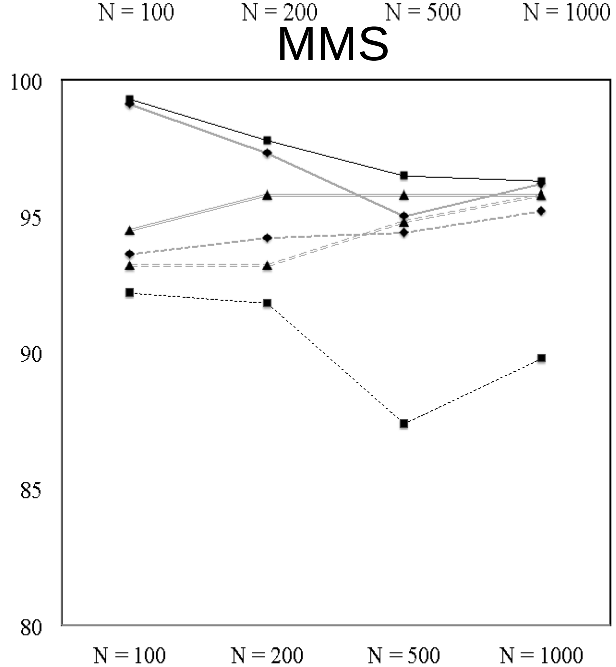
Slope Variance
95% Coverage
(small variance)

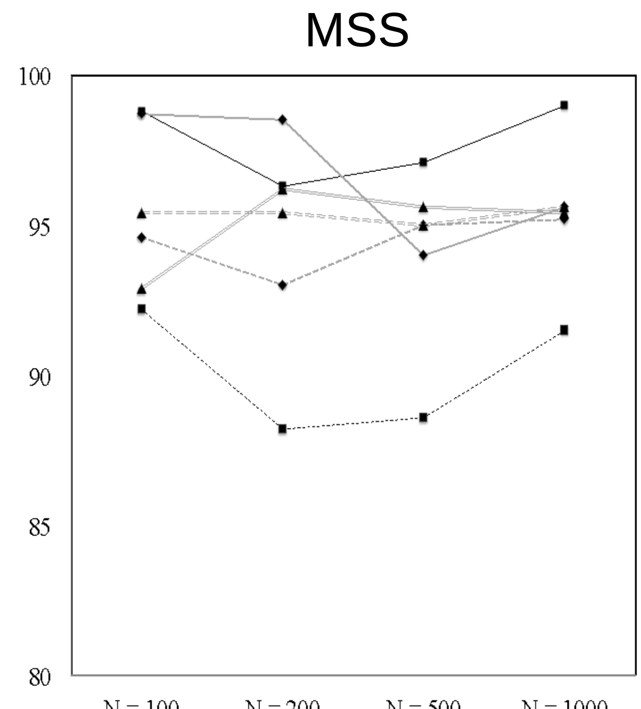




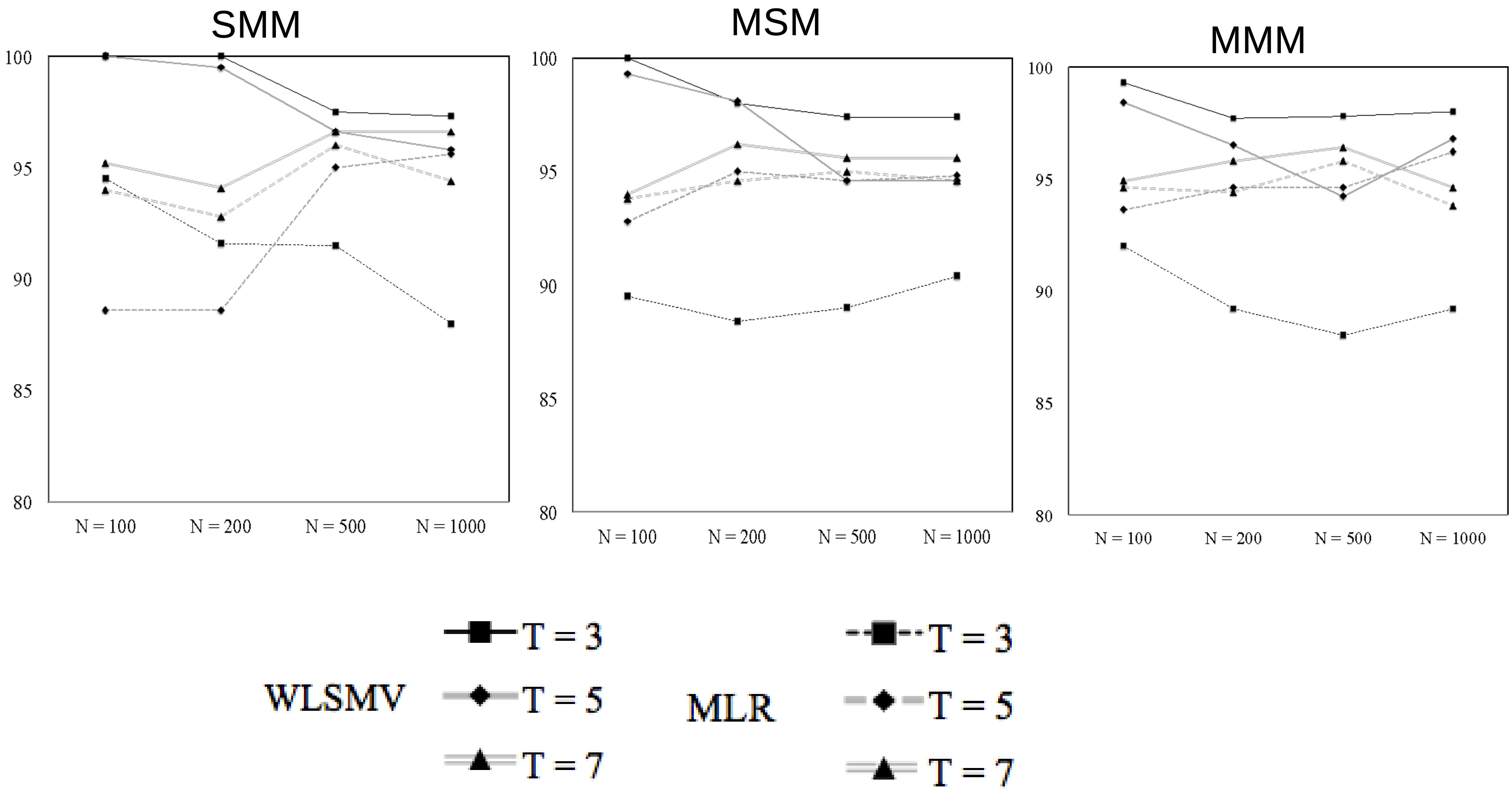
MLR
T = 3
T = 5
T = 7

WLSMV
T = 3
T = 5
T = 7





Slope Variance
95% Coverage
(medium variance)



Summary

Convergence and Improper Solutions

- All models converged for all conditions with WLSMV and MLR
- Improper solutions decreased with larger N and more time points
- Improper solutions were generally uncommon for MLR for all conditions and far more common for WLSMV, particularly for N < 500
- Even with N = 1000, WLSMV 30% of samples had improper solutions when T = 3

Summary

Average Slope

- % bias was not problematic for either estimation method when N $\geq$ 200 for both WLSMV and MLR, but was unacceptable for N = 100 when T = 3

- % bias for SEs was generally acceptable except for N = 100 and T = 3 with MLR

- Type I errors were near nominal levels in all conditions, particularly when T > 3

- 95% coverage rates were appropriate for N $\geq$ 200 for all T and both estimators

Summary

Slope Variance

- % bias was erratic when T=3 for both MLR and WLSMV
- With T > 3, % bias was acceptable for WLSMV if N > 200 and MLR if N > 500
- % bias for SEs was poor for T=3 for both MLR and WLSMV, with SEs better estimated by MLR than WLSMV when N = 200
- 95% coverage was poor for T = 3 for both WLSMV and MLR, and better for MLR when T = 5

Discussion

Strengths

- Parameter estimates and standard errors in context of LGC models, where means and variances are of principal interest

- Variance estimates and robust ML for LGC models with binary variables has not be investigated

- Asymmetric distributions, comparing a low baseline proportion of .1 to larger baseline proportion.45.

- Included smaller sample size condition, N = 100. Smallest sample size studied by Finch (2017) was N = 200

Discussion

Recommendations

- Convergence and improper solutions provide information of importance for practicing researchers:
  - More than three time points needed
  - MLR has fewer improper solutions

- N = 100 generally unacceptable when T = 3 for accurate slope SEs, Type I error, and coverage

- T= 3 insufficient for slope variance estimates

Discussion

Limitations

- Limited to binary variables, more ordinal categories may improve estimation problems (Finch, 2007)
- Correctly specified models only, misspecifications should be examined (e.g., nonlinear models)
- Quadratic effects may require N = 1,000 or more (Finch, 2007)
- Estimation improved for fixed effects with ordinal variables and more time points (T = 6)

Thank you!

Please contact Jason Newsom, newsomj@pdx.edu, with comments or questions.

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