Performance of Latent Growth Curve Models with Binary Variables

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Goal

• Examine estimation of latent growth curve models with diagonalized weighted least squares and categorical ML estimation approaches for binary variables

• Effects of sample size, number of time points, base rate proportion

• Examine convergence failures, parameter bias, standard error bias, Type I error, and coverage
Background

Simulation of SEMs with binary/ordinal estimators suggest:

- Weighted least squares (WLS) estimation with polychoric correlations has better small sample performance than regular ADL/WLS approach (Flora & Curran, 2004; Maydeu-Olivares, 2001; Muthén, du Toit, & Spisic, 1997)

- Diagonalized WLS more computationally practical than inversion of full weight matrix (Muthén, 1993)

- Limited information method, which may be less optimal when values are NMAR (Asparouhov & Muthén, 2010)
Background

Maximum likelihood for categorical variables

• Sometimes referred to as “marginal maximum likelihood”, uses expectation maximization (EM) algorithm with numeric integration to analyze full multiway frequency tables (Bock & Atkin, 1981; Christofersson, 1975; Muthén & Christofferson, 1981)

• Commonly employed with item response models (Demars, 2012; Kamata & Bauer, 2008)

• Parameter estimates that are interpretable as logistic regression coefficients

• Full-information estimation approach. Can be computationally intensive, but may be preferable to WLSMV when values are NMAR
Background

- DWLS method with robust adjustments outperforms unadjusted counterpart (Bandalos, 2014; DeMars, 2012), henceforth referred to as WLSMV

- Maximum likelihood (ML) for binary variables using robust standard errors also outperforms unadjusted ML counterpart, henceforth referred to as MLR (Bandalos, 2014; DeMars, 2012)

- Sample sizes of 100-150 have poor Type I error or coverage rates for WLSMV (Rhemtulla, Brosseau-Liard, & Savalei, 2012)
Background

• Several sources describe details of testing LGC models with binary or ordinal variables (e.g., Lee, Wickrama, & O’Neal, 2018; Masyn, Petras, & Liu, 2014; Mehta, Neale, & Flay, 2004; Newsom, 2015)

• Not widely used presently but popularity is likely to increase
Background

• Bandalos (2014) compared robust binary ML and DWLS and found that

  • Robust ML and WLSMV performed comparably but had high Type I error rates with low N (150) and asymmetric data (up to N = 300)
  
  • Robust ML performed better with asymmetric variables (rarer event)

  • Concluded WLSMV was better all-around choice because of less bias in parameter estimates
Background

Comparisons of categorical estimation methods have been based on factor models and some standard predictive models

Latent growth curve models (LGC) differ in the large number of parameter constraints, parameters of interest (factor means and variances), and potential impact of the number of time points

Few existing studies of performance of LGC models with categorical indicators, and more work is needed
Background

Muthén (1996) conducted a small simulation study with binary variables in latent growth curve models

- Using LISCOMP (unadjusted DWLS)
- Low parameter bias for N=250 and N=1000
- Fairly symmetric outcome, with baseline proportion of .64
- Type I error rates were approximately nominal for N=250 (5.4%)
Background

Finch (2017) conducted a simulation study with binary variables in latent growth curve models

- Examined WLSMV 4, 6, and 8 time points
- $N = 200, 500, \text{ and } N=1000$ (but smaller $N$ may be common for studies)
- Examined different baseline proportions, but neither condition particularly rare ($0.5 \text{ an } 0.69$)
Background

Finch (2017)

• Better convergence with more time points

• Found low bias and good coverage rates in fixed effects (i.e., average intercept and slope estimates) even for $N = 200$

• No evaluation of random effects

• Did not examine MLR estimation
Background

The latent growth curve model

![Diagram of the latent growth curve model with paths and variables labeled]

\[ y_1, \varepsilon_1 \]
\[ y_2, \varepsilon_2 \]
\[ y_3, \varepsilon_3 \]
\[ y_4, \varepsilon_4 \]

\[ \eta_0, \alpha_0, \psi_{00} \]
\[ \eta_1, \alpha_1, \psi_{11} \]

\[ \psi_{01} \]
Method

Simulations

• Data were generated in SAS 9.4 using the RandomMVBBinary macro (Wicklin, 2013) to produce proportions and change in proportions comparable to applied longitudinal studies (LLSE: Sorkin & Rook, 2004; HRS: Heeringa & Connor, 1995)

• Simulations were analyzed using the Mplus Version 8 (Muthén & Muthén, 1998-2017) MONTECARLO feature
Design

Dependent Measures

• % bias slope means (fixed effects)

• % bias slope variances (random effects)

• Bias greater than 5% was considered unacceptable (Hoogland & Boomsma, 1998)
Design

Bias Computation for Average Slopes

• Percent relative bias

\[
\%Bias(\hat{\theta}) = \frac{\hat{\theta} - \theta}{\theta} \times 100
\]

• Computation when parameter value equals 0

\[
\%Bias(\hat{\theta}) = \Phi^{-1}(z_{\hat{\theta}}) - \Phi^{-1}(z_{\theta})
\]

where \( \Phi^{-1} \) is the cumulative standard normal probability, \( z_{\hat{\theta}} = (\hat{\theta} - \theta) / SD_{\hat{\theta}} \) estimated from standard score values for the parameter, \( z_{\theta} \) with the percentile for the population value for \( \theta \) equal to .5
Design

Dependent Measures
• Bias Computation for Variances

\[ \%Bias(SE) = \frac{E(SE) - SD}{SD} \times 100 \]
Design

Dependent Measures

• Type I error (% samples significant) – for conditions in which the slope is zero

• 95% Coverage (% samples in which population value falls within confidence interval) – for all other conditions
Design

Independent Variables

- Sample size: 100, 200, 500, 1000
- # time points: T = 3, 5, 7
- 84,000 replications, 500 per cell (e.g., Paxton et al., 2001)
- Average intercept: small proportion (.11) vs. medium proportion (.45)
- Average slope: 0 vs. mod effect (increase of p = .025 per wave)
- Slope variance: small vs. medium effect
## Design Summary

<table>
<thead>
<tr>
<th></th>
<th>Average Intercept, $\alpha_I$</th>
<th>Average Slope, $\alpha_S$</th>
<th>Slope Variance, $\psi_s$</th>
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<tbody>
<tr>
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Results

Convergence
• Models converged successfully for models in all conditions for both WLSMV and MLR

Improper solutions
• Improper solutions (e.g., negative error variances) were common, sometimes as high as 80% of samples with small N and T
• Especially high for the low baseline proportion conditions (SSS, SMS, and SMM)
Improper Solutions

SSS

MSS

MSM

WLSMV

MLR
Improper Solutions

SMS

MMS

WLSMV

MLR

SMM

MMM

Paper Presented at the Modern Modeling Methods Conference, May 2018, Storrs, CT
Average Slope
% Bias
(zero slope)

Average Slope
% Bias
(medium slope)

Paper Presented at the Modern Modeling Methods Conference, May 2018, Storrs, CT
Average Slope SE
% Bias
(zero slope)

SSS

MSS

MSM

N = 100  N = 200  N = 500  N = 1000

WLSMV  T = 3

T = 5

T = 7

MLR  T = 3

T = 5

T = 7

Paper Presented at the Modern Modeling Methods Conference, May 2018, Storrs, CT
Average Slope

% Type I Error

(zero slope)
Average Slope
95% Coverage (medium slope)

SMS

WLSMV

- T = 3
- T = 5
- T = 7

SMM

MLR

- T = 3
- T = 5
- T = 7
Slope Variance

% Bias

(small variance)
Slope Variance
% Bias
(medium variance)

SMM

MSM

MMM

N = 100  N = 200  N = 500  N = 1000

T = 3
T = 5
T = 7

WLSMV
MLR

Paper Presented at the Modern Modeling Methods Conference, May 2018, Storrs, CT
Slope Variance SE

% Bias

(small variance)

Paper Presented at the Modern Modeling Methods Conference, May 2018, Storrs, CT
Slope Variance
95% Coverage
(small variance)

Paper Presented at the Modern Modeling Methods Conference, May 2018, Storrs, CT
Slope Variance
95% Coverage
(medium variance)

SMM

MSM

MMM

N = 100  N = 200  N = 500  N = 1000

WLSMV  T = 3  MLR  T = 5

T = 7
Summary

Convergence and Improper Solutions

• All models converged for all conditions with WLSMV and MLR

• Improper solutions decreased with larger N and more time points

• Improper solutions were generally uncommon for MLR for all conditions and far more common for WLSMV, particularly for N < 500

• Even with N = 1000, WLSMV 30% of samples had improper solutions when T = 3
Summary

Average Slope

• % bias was not problematic for either estimation method when $N \geq 200$ for both WLSMV and MLR, but was unacceptable for $N = 100$ when $T = 3$

• % bias for SEs was generally acceptable except for $N = 100$ and $T = 3$ with MLR

• Type I errors were near nominal levels in all conditions, particularly when $T > 3$

• 95% coverage rates were appropriate for $N \geq 200$ for all $T$ and both estimators
Summary

Slope Variance

• % bias was erratic when T=3 for both MLR and WLSMV

• With T > 3, % bias was acceptable for WLSMV if N > 200 and MLR if N > 500

• % bias for SEs was poor for T=3 for both MLR and WLSMV, with SEs better estimated by MLR than WLSMV when N = 200

• 95% coverage was poor for T = 3 for both WLSMV and MLR, and better for MLR when T = 5
Discussion

Strengths

- Parameter estimates and standard errors in context of LGC models, where means and variances are of principal interest

- Variance estimates and robust ML for LGC models with binary variables has not be investigated

- Asymmetric distributions, comparing a low baseline proportion of .1 to larger baseline proportion .45.

- Included smaller sample size condition, N = 100. Smallest sample size studied by Finch (2017) was N = 200
Discussion

Recommendations

- Convergence and improper solutions provide information of importance for practicing researchers:
  - More than three time points needed
  - MLR has fewer improper solutions

- \( N = 100 \) generally unacceptable when \( T = 3 \) for accurate slope SEs, Type I error, and coverage

- \( T = 3 \) insufficient for slope variance estimates
Discussion

Limitations

• Limited to binary variables, more ordinal categories may improve estimation problems (Finch, 2007)

• Correctly specified models only, misspecifications should be examined (e.g., nonlinear models)

• Quadratic effects may require \( N = 1,000 \) or more (Finch, 2007)

• Estimation improved for fixed effects with ordinal variables and more time points \( (T = 6) \)
Thank you!

Please contact Jason Newsom, newsomj@pdx.edu, with comments or questions.

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