Sample Considerations for Detecting Person, Dyad, and Contextual Effects Using the Common Fate Model for Dyadic Analysis

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Modern Modeling Methods Conference
June 28, 2023

Outline

3 Parts

- The Common-Fate Model and the Between-Within CFM
- Simulation Design
- Results and Implications

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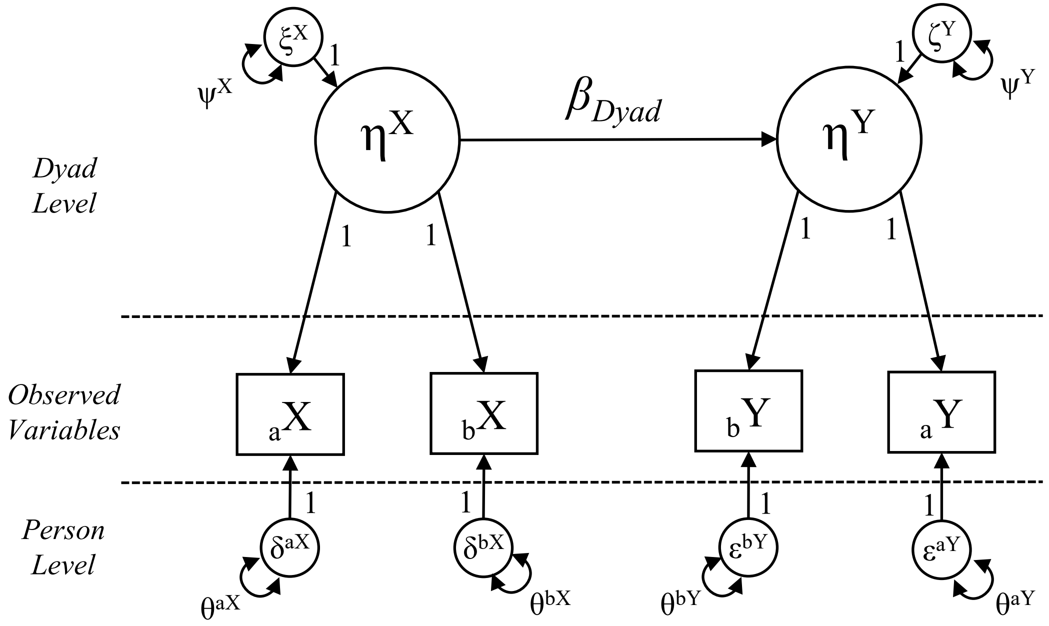
The Common-Fate Model and BW-CFM

Dyadic Analysis Models

- Several analytic approaches have been developed for dyadic data
- The Actor-Partner Interdependence Model (APIM; Kenny, 1996)
  - Most widely applied
  - Focuses on Person level relationships
  - Also accommodates Dyad-level predictors
- The Common-Fate Model (CFM; Kenny, 1996)
  - Increasing in popularity over the past decade
  - Focuses primarily on Dyad level relationships
  - Ledermann & Kenny (2011) expanded to incorporate Person-level associations
- Hybrid Actor-Partner Common-Fate Models (Wickham & Macia, 2019)

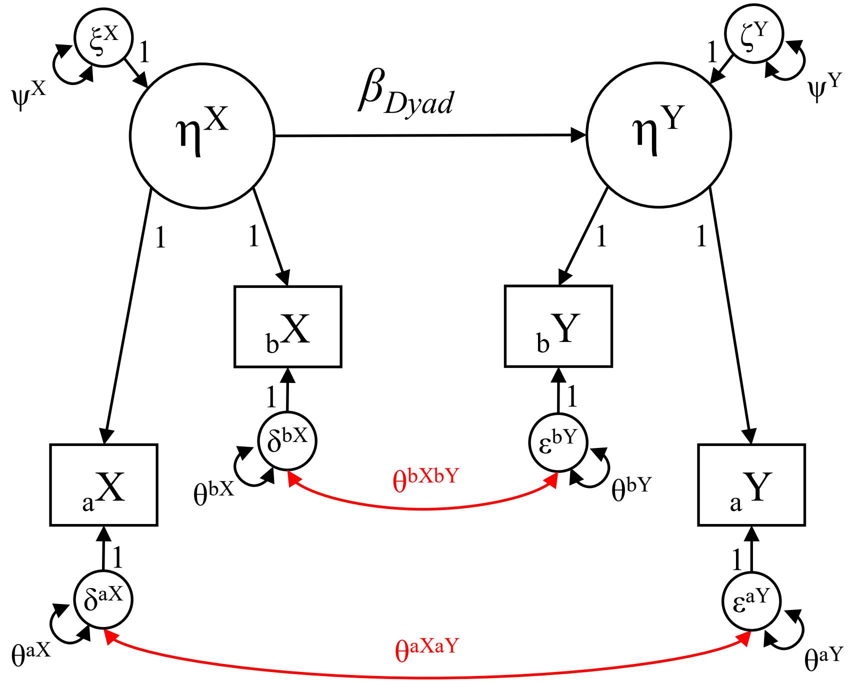
The Common-Fate Model

- 2 Variables (X, Y) × 2 Persons (a, b):
  - Observed Variables $_{a}X$, $_{b}X$, $_{a}Y$, $_{b}Y$
- Latent Variables (η^X, η^Y) measured by Observed
  - Partitions variance into Dyad and Person level components
- β_{Dyad} describes regression of Dyad level outcome (η^Y) on predictor (η^X)
- df = 3 → Over-identifying constraints:
  - cov($_{a}X$,$_{a}Y$) = cov($_{a}X$,$_{b}Y$) = cov($_{b}X$,$_{b}Y$) = cov($_{b}X$,$_{a}Y$)



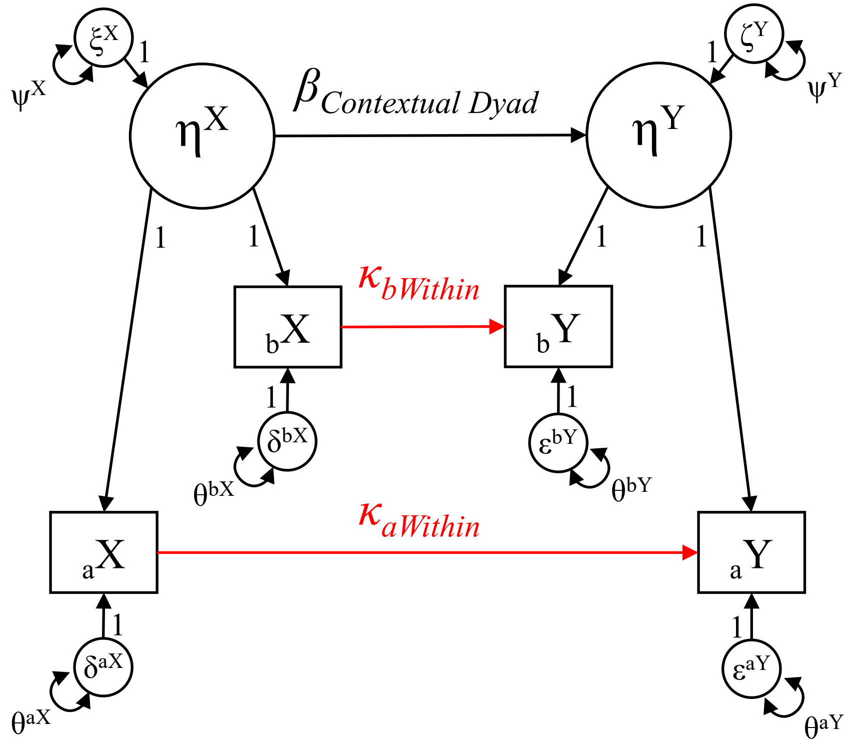
The Common-Fate Model

- Residual Person level relationships must be modeled when over-identifying constraints are untenable
  - Otherwise β_{Dyad} is biased
- Can incorporate covariance parameters to obtain unbiased estimate of β_{Dyad}
  - e.g., θ^{axay}, θ^{bxby}
- Unable to compare of magnitude of Dyad and Person level associations



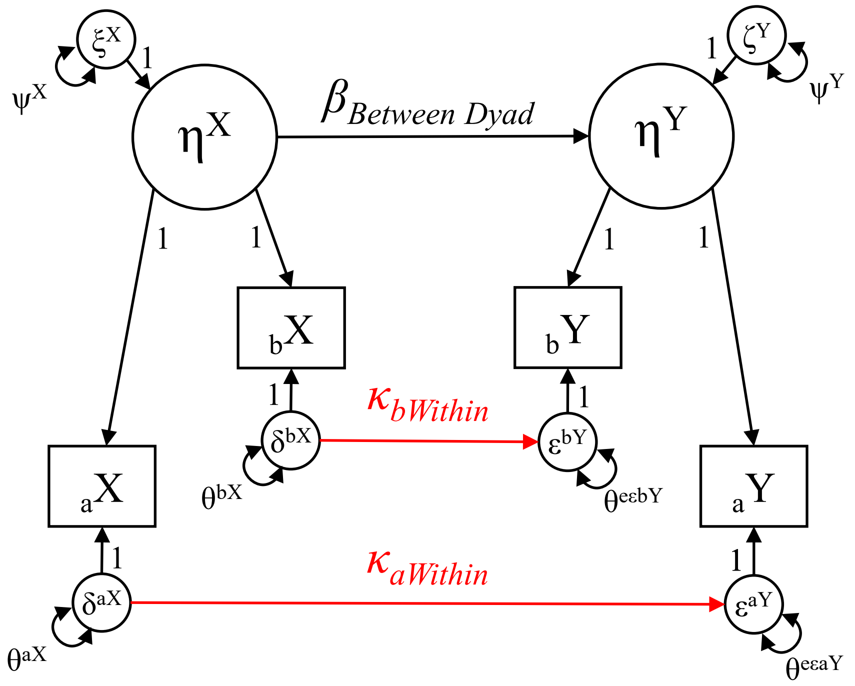
Contextual Effects Model

- Ledermann & Kenny (2011) described a variation on the CFM incorporating Person level relationships
  - Features regressions among observed Person level variables: aY on aX, bY on bX
- Under this parameterization:
  - $\beta_{Contextual\ Dyad}$ is the <u>Contextual</u> effect
    - $\beta_{Contextual\ Dyad} = \beta_{Between\ Dyad} - (.5*\kappa_{aWithin} + .5*\kappa_{bWithin})$
  - $\kappa_{aWithin}$ and $\kappa_{bWithin}$ are <u>pure</u> <u>Within-Dyad</u> effects



The Between-Within Common-Fate Model (BW-CFM)

- Wickham (2023) described the specification of a Between-Within Common-Fate Model (BW-CFM)
  - Features regressions among ‘explicit’ Person level residuals: ε^{aY} on δ^{aX}, ε^{bY} on δ^{bX}
- Under the BW-CFM parameterization:
  - $\beta_{Between\ Dyad}$ is the pure Between-Dyad effect
  - $\kappa_{aWithin}$ and $\kappa_{bWithin}$ are pure Within-Dyad effects
- Contextual effects can be obtained by specifying auxiliary parameters
  - e.g., $\gamma_{aContextual} = \beta_{Between\ Dyad} - \kappa_{aWithin}$



The Present Study

- Dyadic data structure represents a special case of the standard multilevel design where $N_{level\ 2} = 2*N_{level\ 1}$
- BW-CFM is specified as a ‘wide’ (single-level) SEM featuring regressions among latent variables
  - Documented sample size requirements for standard multilevel designs likely inadequate
- Present study reports results of Monte Carlo simulation to aid researchers in selection of sample size for studies utilizing the BW-CFM
  - Naturally, sample size and magnitude of X-Y relationships will be positively associated with power
  - Proportion of variance at Dyad (vs. Person) levels also related to power at each level

Simulation Design

BW-CFM Parameterization

- Deriving meaningful population parameter values requires elaboration regression and variance parameters at each level
- Assuming unit-variance for observed variables $_aX$ and $_bX$:
  - $Var(\eta^X) = \psi^X$ = the Intra-Class Correlation = r_{aXbX}
- Furthermore: $R^2_{Dyad} = \dfrac{\beta^2_{Between\ Dyad} * \psi^X}{r_{aYbY}}$
  - Rearranging to obtain: $\beta_{Between\ Dyad} = \sqrt{\dfrac{\psi^X * R^2_{Dyad}}{r_{aXbX}}}$
- And, $ResVar(\eta^Y) = \psi^Y = r_{aYbY} - \beta^2_{Between\ Dyad} * \psi^X$

BW-CFM Parameterization

- Because observed variables are set to z-scale
  - Variances of explicit residuals for Person level X variables, δ^{aX} and δ^{bX} are equal to $1 - r_{aXbX}$
- Furthermore: $R^2_{Person\ a} = \frac{\kappa^2_{aWithin} * \delta^{aX}}{1 - r_{aYbY}}$
- Rearranging to obtain: $\kappa_{aWithin} = \sqrt{\frac{R^2_{Person\ a} * (1 - r_{aYbY})}{\delta^{aX}}}$
- And $ResVar(\varepsilon^{aY}) = \theta^{\varepsilon aY} = (1 - r_{aXbX}) - \kappa^2_{aWithin} * \delta^{aX}$

Simulation Design

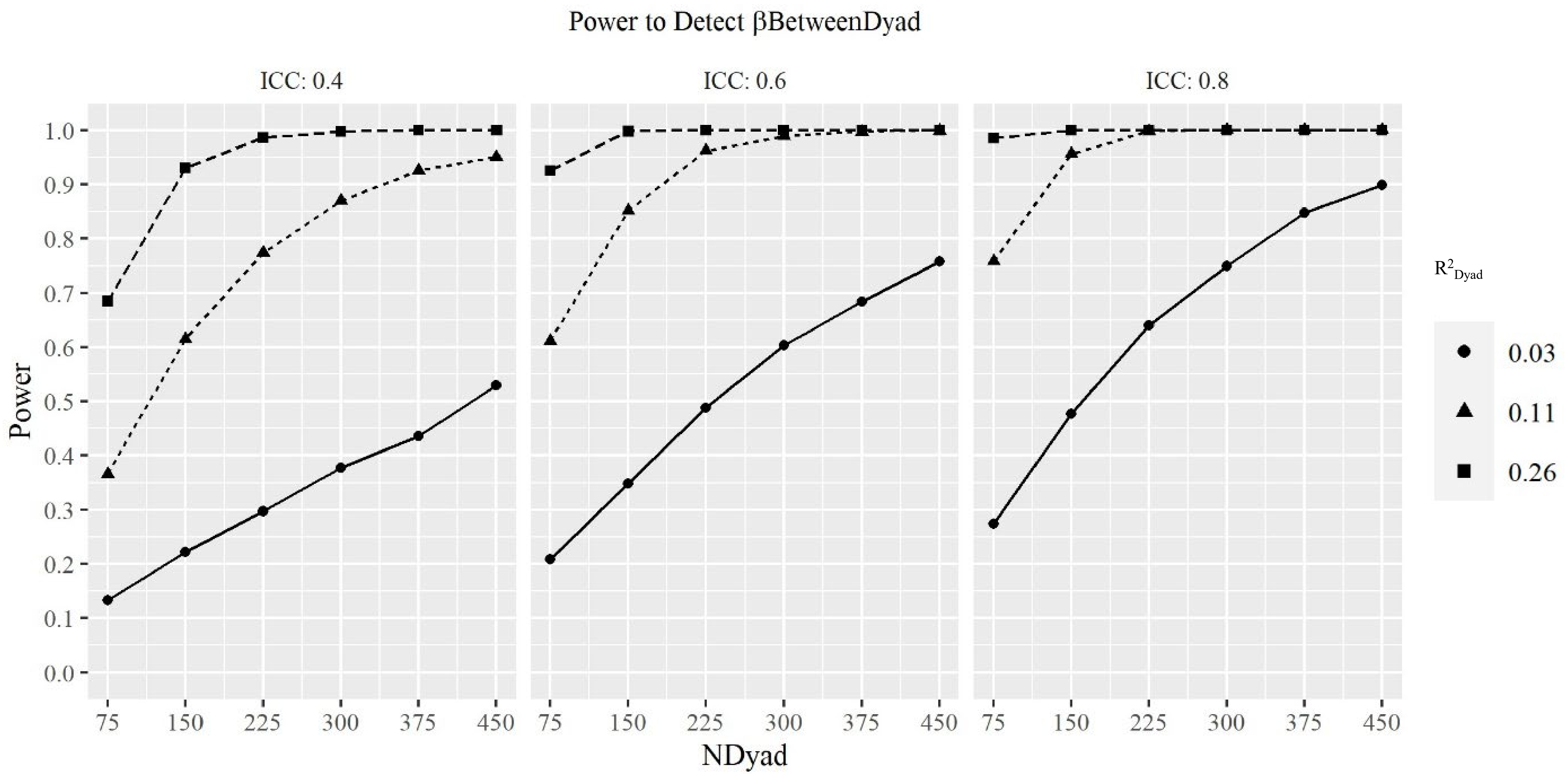
- An internal Monte Carlo conducted using *Mplus 8.9*
- Formulae described in previous slides used to derive population values
- Design:
  - 6 N_{Dyad} [75|150|225|300|375|450] ×
  - 3 ICC [.40|.60|.80] × → $r_{aXbX} = r_{aYbY}$
  - 3 R^2_{Dyad} [.03|.11|.26] × → Small, Medium, Large
  - 2 $R^2_{Person\ a}$ [.03|.11] × 2 $R^2_{Person\ b}$ [.11|.26]
- 216 cells @1000 reps per cell → All reps converged
- .out files extracted and complied using R/*MplusAutomation* (Halquist & Wiley, 2018)
- Visualizations using R/*ggplot2* (Wickham, 2016)

Results

Results Summary: Dyad Level Power

- When ICC = 0.4:
  - N_{Dyad} > 450 to detect small effects
  - N_{Dyad} >= 150 to detect medium effects
  - N_{Dyad} >= 150 to detect large effects
- When ICC = 0.6:
  - N_{Dyad} ≈ 450 to detect small effects
  - N_{Dyad} >= 150 to detect medium effects
  - N_{Dyad} >= 75 to detect large effects
- When ICC = 0.8:
  - N_{Dyad} >= 300 to detect small effects
  - N_{Dyad} >= 75 to detect medium and large effects

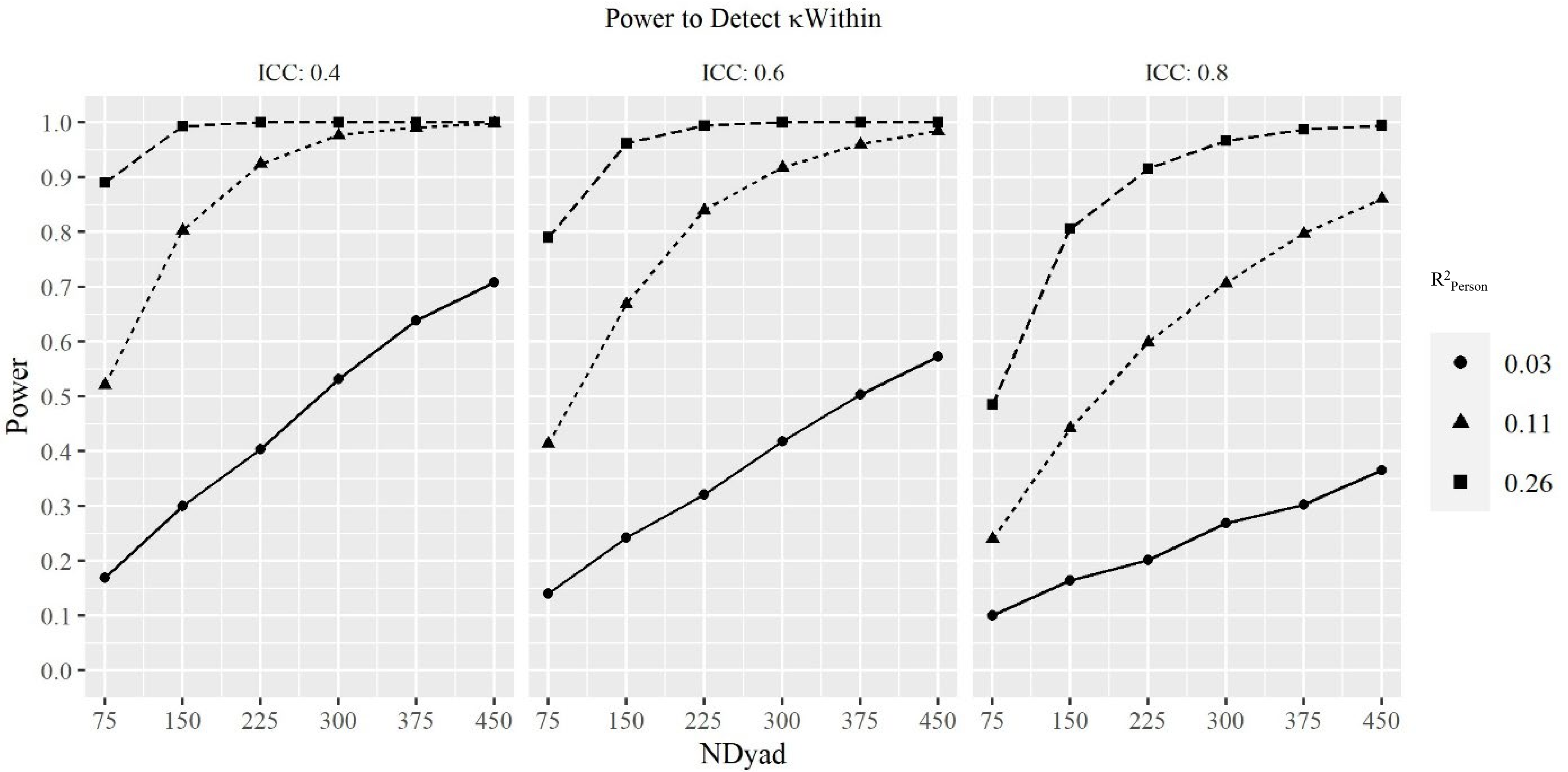
Results: Dyad Level Power



Results Summary: Person Level Power

- When ICC = 0.4:
  - N_{Dyad} > 450 to detect small effects
  - N_{Dyad} >= 225 to detect medium effects
  - N_{Dyad} >= 75 to detect large effects
- When ICC = 0.6:
  - N_{Dyad} > 450 to detect small effects
  - N_{Dyad} >= 225 to detect medium effects
  - N_{Dyad} >= 75 to detect large effects
- When ICC = 0.8:
  - N_{Dyad} > 450 to detect small effects
  - N_{Dyad} >= 375 to detect medium effects
  - N_{Dyad} >= 150 to detect large effects

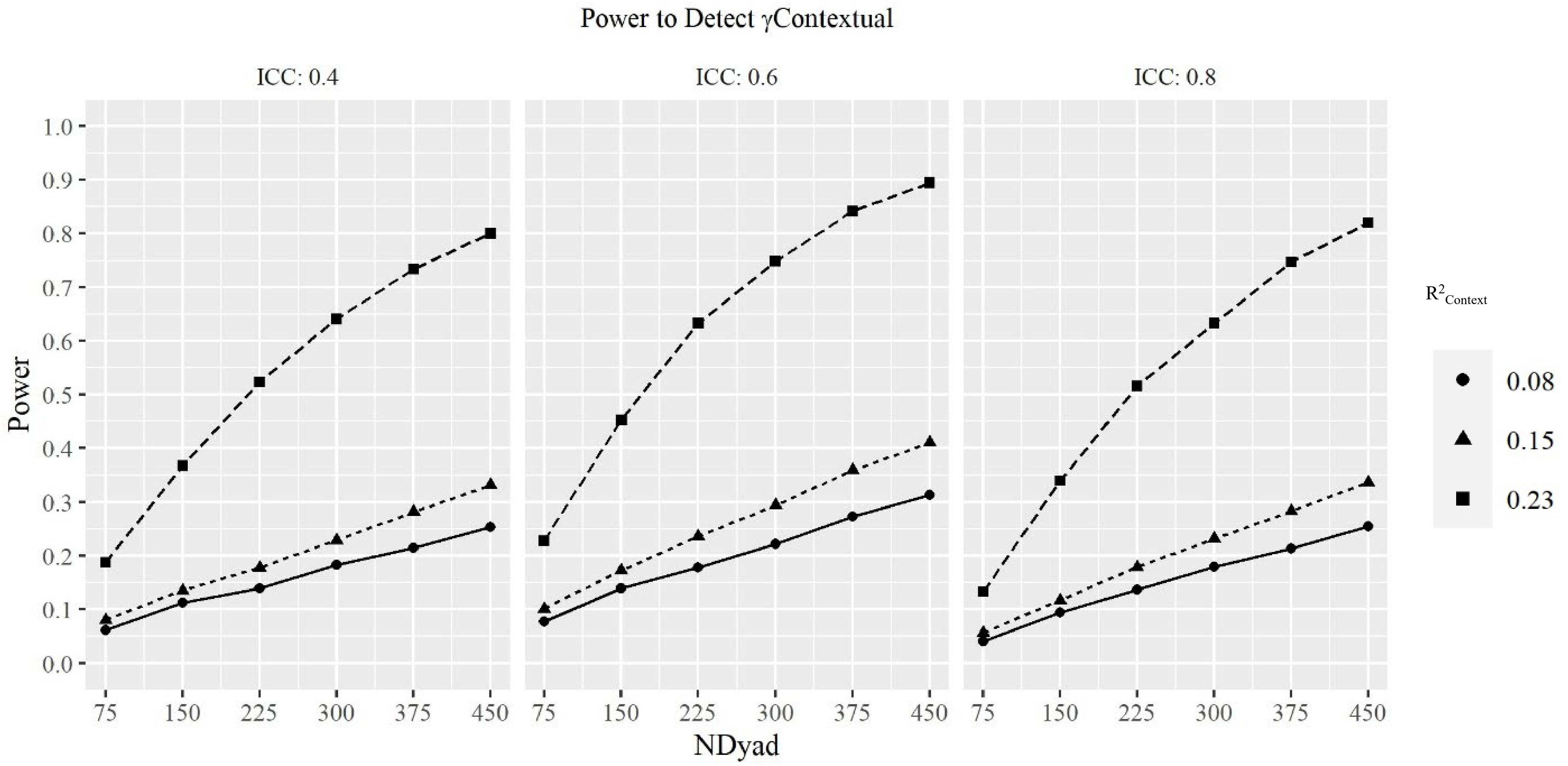
Results: Person Level Power



Results Summary: Contextual Power

- Contextual effect estimated as aux parameter using MODEL CONSTRAINT
  - i.e., $\gamma_{Contextual} = \beta_{Between\ Dyad} - \kappa_{Within}$
- Power to detect Contextual effect was generally lower than power to detect $\beta_{Between\ Dyad}$ or κ_{Within}
- Appears less sensitive to ICC
  - Some evidence that ICC = 0.6 was slightly better
  - ICC = 0.4 and 0.6 curves practically identical
- Observed Power reached (arbitrary) threshold of .80 only when N_{Dyad} = 450
- Detecting Contextual effects of smaller magnitude will be difficult unless sample size is very large!

Results: Contextual Power



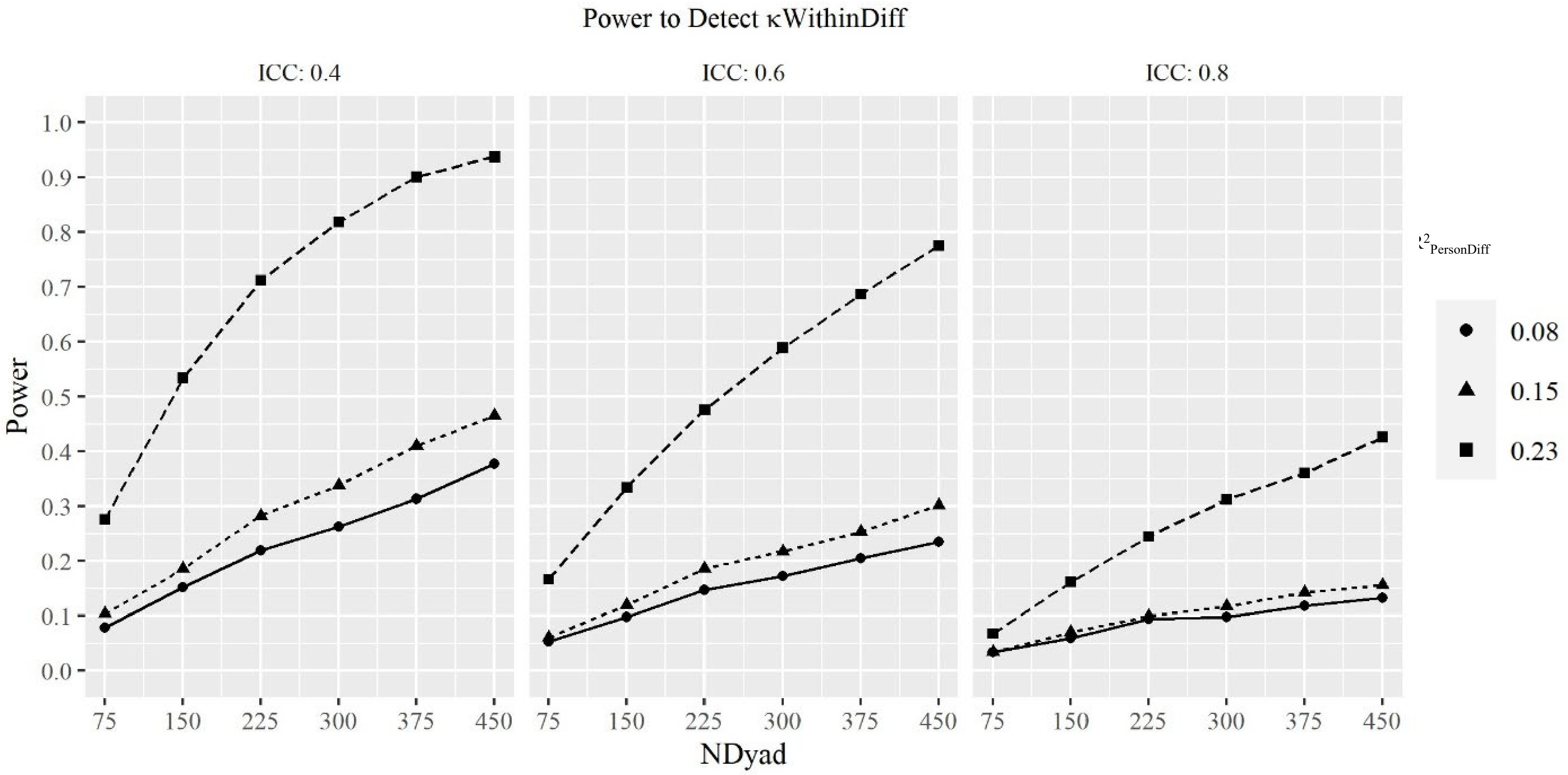
BW-CFM: Pooled Within and Contextual Effects

- $\kappa_{aWithin}$ and $\kappa_{bWithin}$ can be compared using model constraints (χ^2_{Diff} test) or auxiliary parameters:
  - $\kappa_{WithinDiff} = \kappa_{aWithin} - \kappa_{bWithin}$
- When appropriate, a pooled Person level coefficient can be estimated as an aux parameter:
  - $\kappa_{PooledWithin} = .5 * \kappa_{aWithin} + .5 * \kappa_{bWithin}$
- And the corresponding Pooled Contextual effect can also be estimated:
  - $\gamma_{PooledContextual} = \beta_{Between\ Dyad} - \kappa_{PooledWithin}$

Results Summary: Within Difference Power

- $\kappa_{WithinDiff} = \kappa_{aWithin} - \kappa_{bWithin}$ estimated using MODEL CONSTRAINT for cells where $R^2_{Person\ a} \neq R^2_{Person\ b}$
- Power to detect $\kappa_{WithinDiff}$ highest when ICC = 0.4
- Large differences can still be detected when ICC = 0.6 and N_{Dyad} >= 450
- In most cases it will be difficult to detect differences in magnitude of Person level coefficients

Results: Within Difference Power



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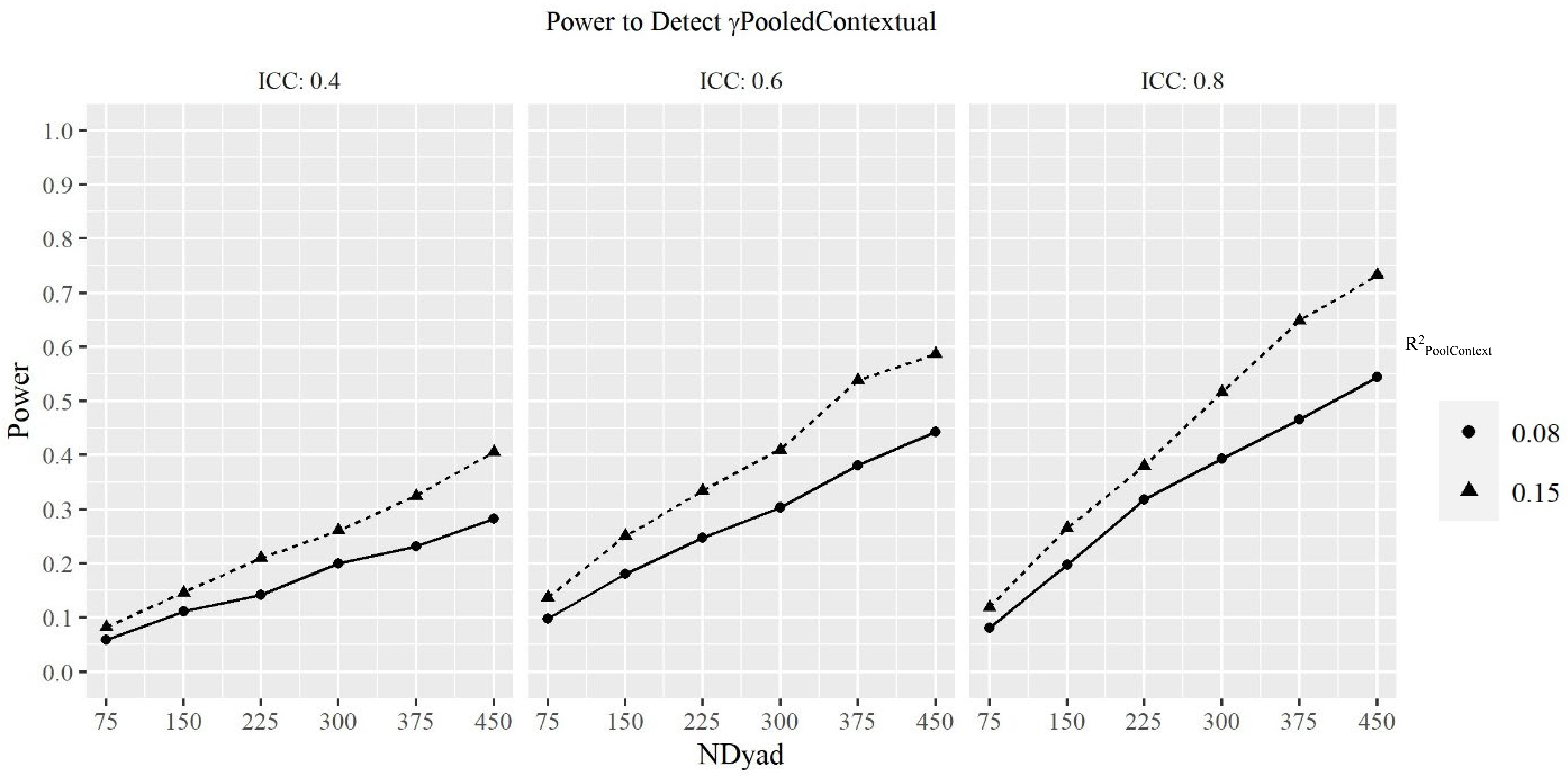
Results Summary: Pooled Contextual Power

- For cells where $R^2_{Person\ a} = R^2_{Person\ b} = 0.11$, $\kappa_{PooledWithin}$ was specified as an aux parameter using MODEL CONSTRAINT
  - i.e., $\kappa_{PooledWithin} = .5*\kappa_{aWithin} + .5*\kappa_{bWithin}$
- Because of partial offset of effect sizes across Person a and Person b, we obtain:
  - $R^2_{Dyad} - R^2_{PersonPooled}$ = .26 - .11 = **.15**
  - $R^2_{Dyad} - R^2_{PersonPooled}$ = |.03 - .11| = **.08**
- As expected, Power to detect this ‘pooled’ contextual effect was higher than the individual contextual effects
  - Positively related to ICC

Results: Pooled Contextual Power



Global Summary and Future Directions

- Whenever possible researchers should aim for N_{Dyad} >= 300 to detect meaningful effect sizes using the BW-CFM
- Like many simulations, the present study assumed ‘tidy’ data (i.e., MVN)
- Future work should explore performance under more realistic conditions
- Work in progress:
  - These findings should be submitted for peer-review in the next few weeks
  - R function and SAS MACRO allowing user-specified design features currently in dev

Thank you!

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