Structure after Measurement Estimation of Latent Interactions in Partially Nested Structural Equation Models

Kyle Cox (Charlotte)

Ben Kelcey (University of Cincinnati)

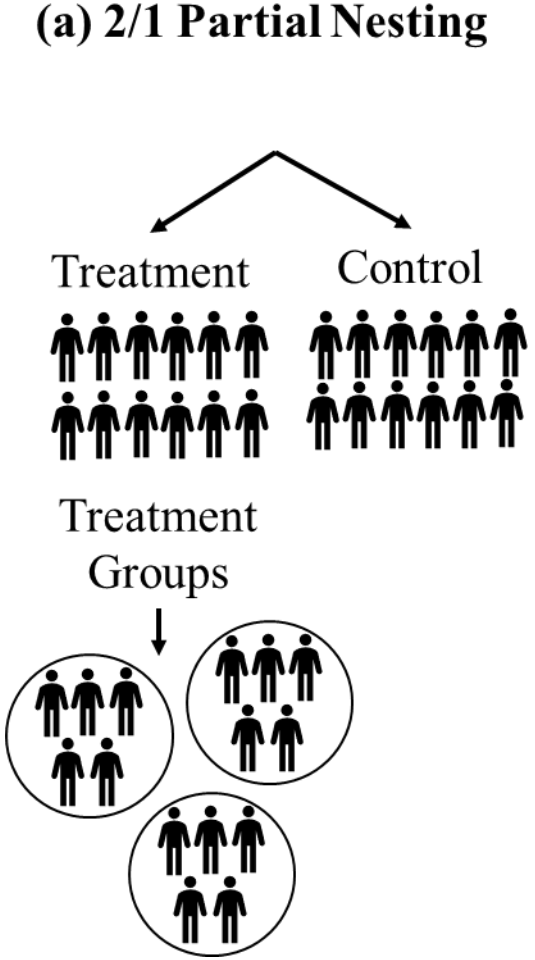
UNIVERSITY OF NORTH CAROLINA CHARLOTTE

Partial Nesting

WHAT: Study arms (e.g., treatment and control groups) that have disparate nesting structures (e.g., multilevel vs. single level).

WHEN: Interventions, programs, and policies of interest often allow mutual interactions among participants in a group (i.e., small group instruction, tutoring group) or a shared facilitator (i.e., teacher, counselor) leading to dependencies among participant outcomes.

Dependencies among participant outcomes are not present in, for example, a waitlisted comparison group.



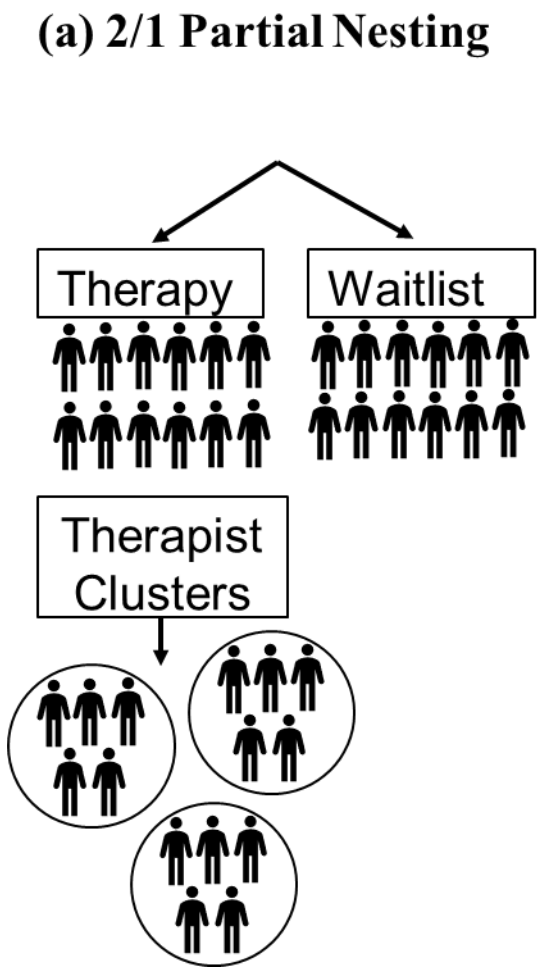
Partial Nesting: Working Example

Hypothetical study based on Heider et al. (2018)

Examine the relationships between symptom severity (Y), baseline-symptom intensity (X) and a patient's readiness to change (Z) during therapeutic treatment for somatoform disorders.

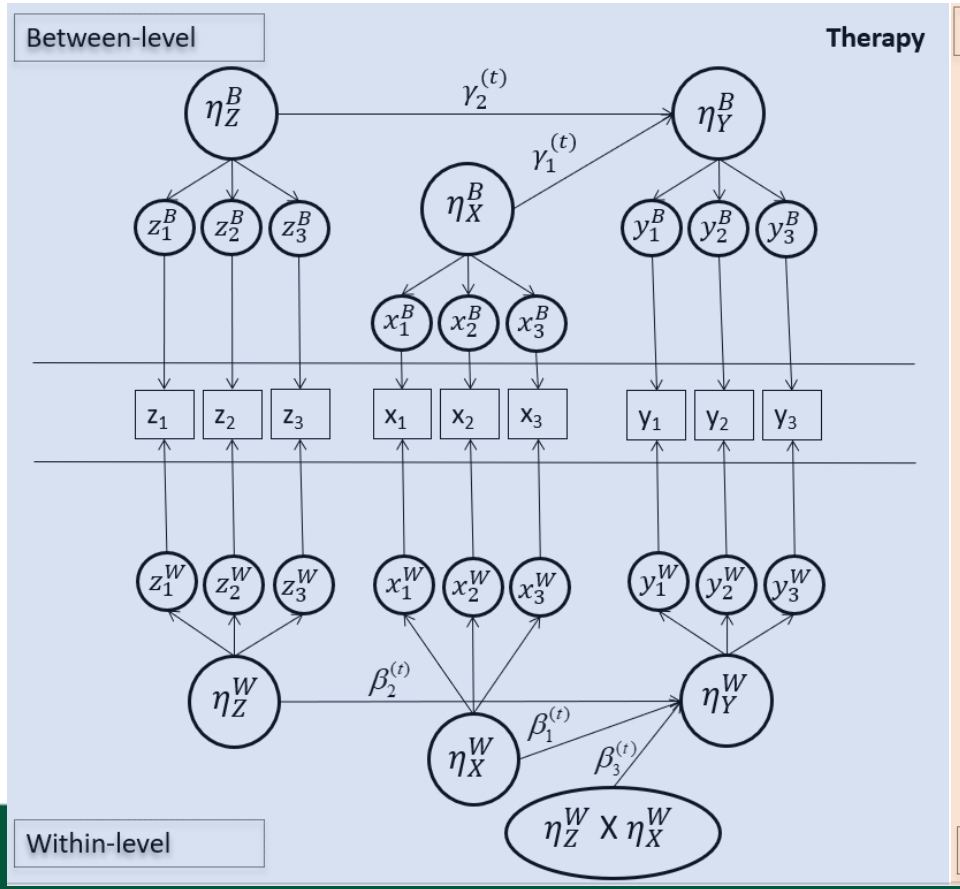
Individuals who participate in the therapy are clustered within therapists

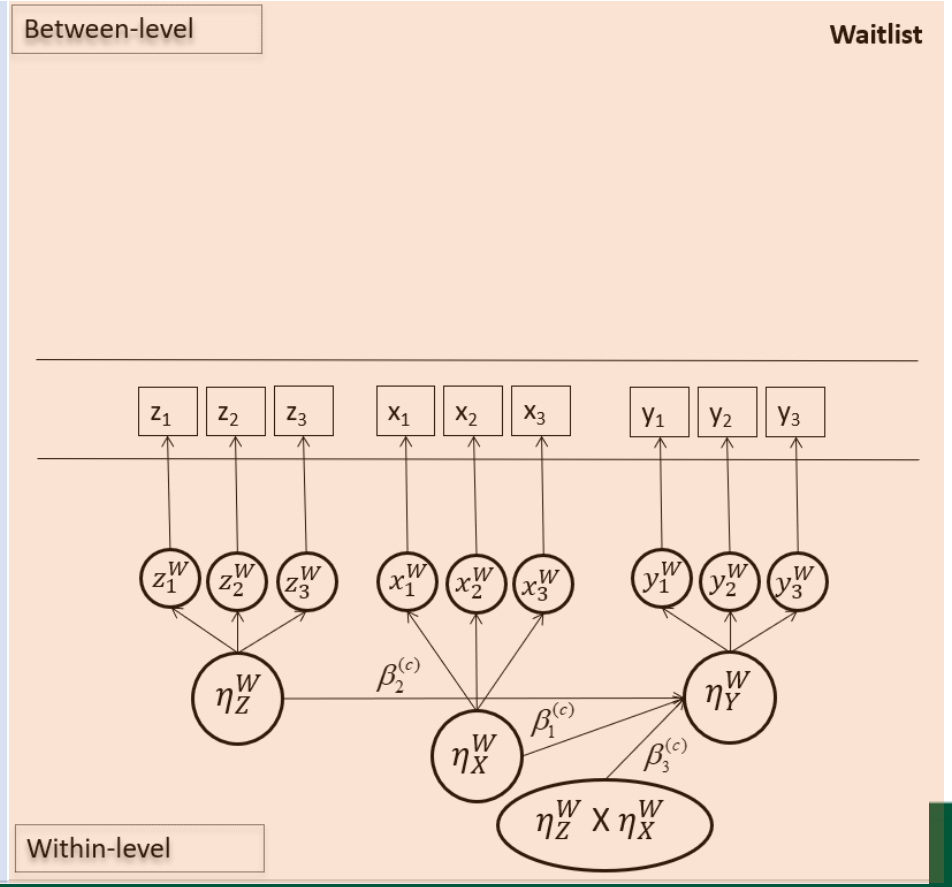
Comparison group individuals remain unclustered or ungrouped on a waitlist.

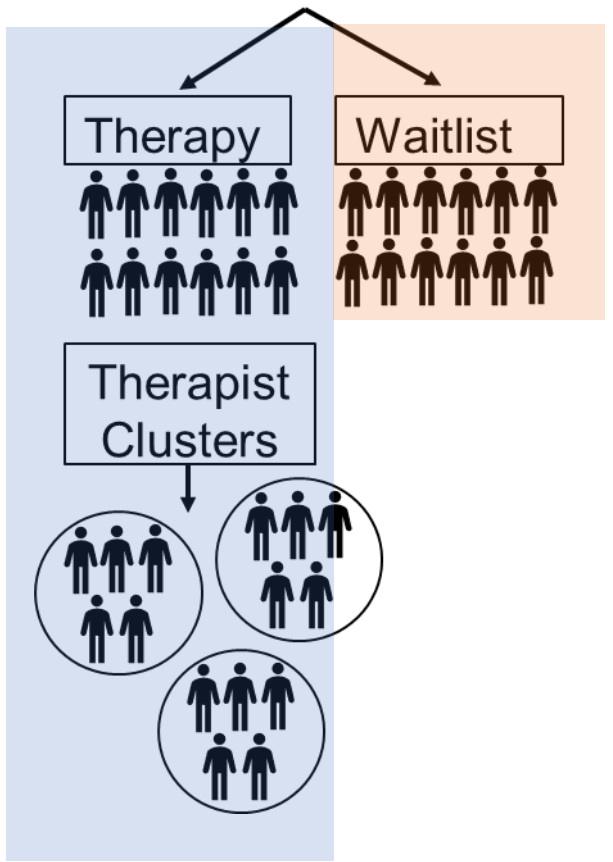


Two-Level Partial Nesting

Specification 1: Under a multiple-arm multilevel SEM for partially nested data (MSEM-PN) framework (Lachowicz, et al. 2015;Sterba et al., 2014)







Problem

Estimation of model parameters with latent interactions

Maximum likelihood (ML) and Bayes estimation have demonstrated

(1) broad range of computational challenges (e.g., high dimensional integration, non-convergence, implausible parameter values)

(2) Instability and/or bias in parameter estimates

Issues exacerbated with limited sample sizes

(Asparouhov & Muthen, 2020; Bogaert et al., 2022; Cox & Kelcey, 2021; Cox et al., 2023; Devlieger & Rosseel, 2017, 2020; Kelcey, 2019)
(e.g., Bogaert et al., 2022; Cox & Kelcey, 2021; Cox et al., 2023; Devlieger & Rosseel, 2017, 2020; Kelava et al., 2011; Klein & Muthén, 2007; Rosseel, 2020; Smid & Rosseel, 2020)

Possible Solution

Structure-After-Measurement approaches (SAM) have shown promise overcoming these issues…Croon's bias corrected factor score path analysis (SAM-Croon's)

SAM-Croon's can accommodate latent variables, latent interactions, and partially nested data under the multiple-arm MLSEM framework for partially nested data (MLSEM-PN)

(Lachowicz, et al. 2015; Sterba et al., 2014)
(Cox & Kelcey, 2021, Cox et al., 2023; Rosseel & Loh, 2022)

Purpose

Extend and investigate a local SAM approach utilizing factor score regression with Croon's correction (SAM-Croon's) to estimate latent interactions in partially nested structural equation models

(Lachowicz et al., 2015; Sterba et al., 2014)

Analytic Models

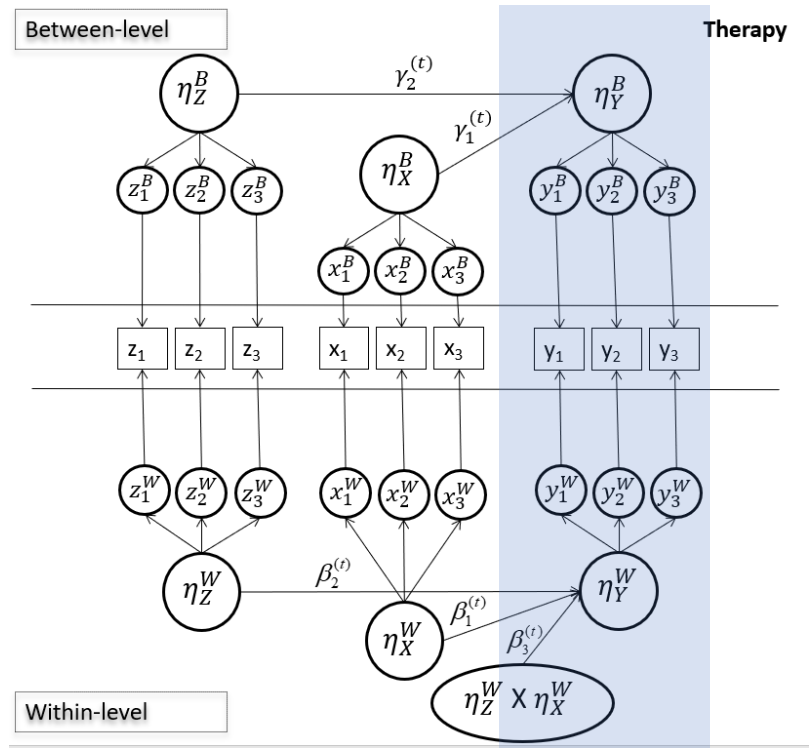
Latent interactions with 2/1 partially nested data

A1 latent interaction: individual level interaction involving the within part of a level-one moderator and within part of a level-one predictor

Analytic Models: Measure

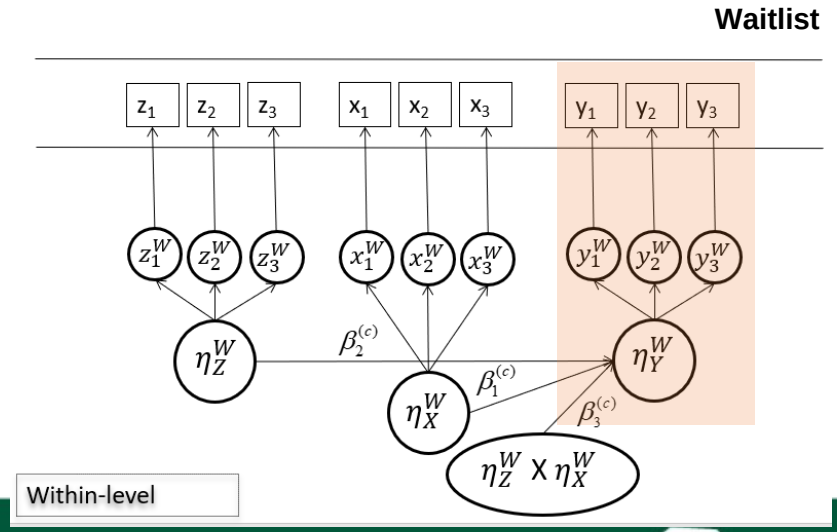
Multilevel Factor Model:

$$\mathbf{y}_{ij} = \boldsymbol{\mu}_{Y_j} + \boldsymbol{\Lambda}^{(t)}_{Y^B} \eta^{(t)}_{Y^B_j} + \boldsymbol{\Lambda}^{(t)}_{Y^W} \eta^{(t)}_{Y^W_{ij}} + \boldsymbol{\varepsilon}^{(t)}_{Y^W_{ij}} + \boldsymbol{\varepsilon}^{(t)}_{Y^B_j}$$



Common Factor model:

$$\mathbf{y}_i = \boldsymbol{\mu}_Y + \boldsymbol{\Lambda}_Y \eta^{(c)}_Y + \boldsymbol{\varepsilon}^Y_i$$



Analytic Models: Structural

Therapy-Outcome:

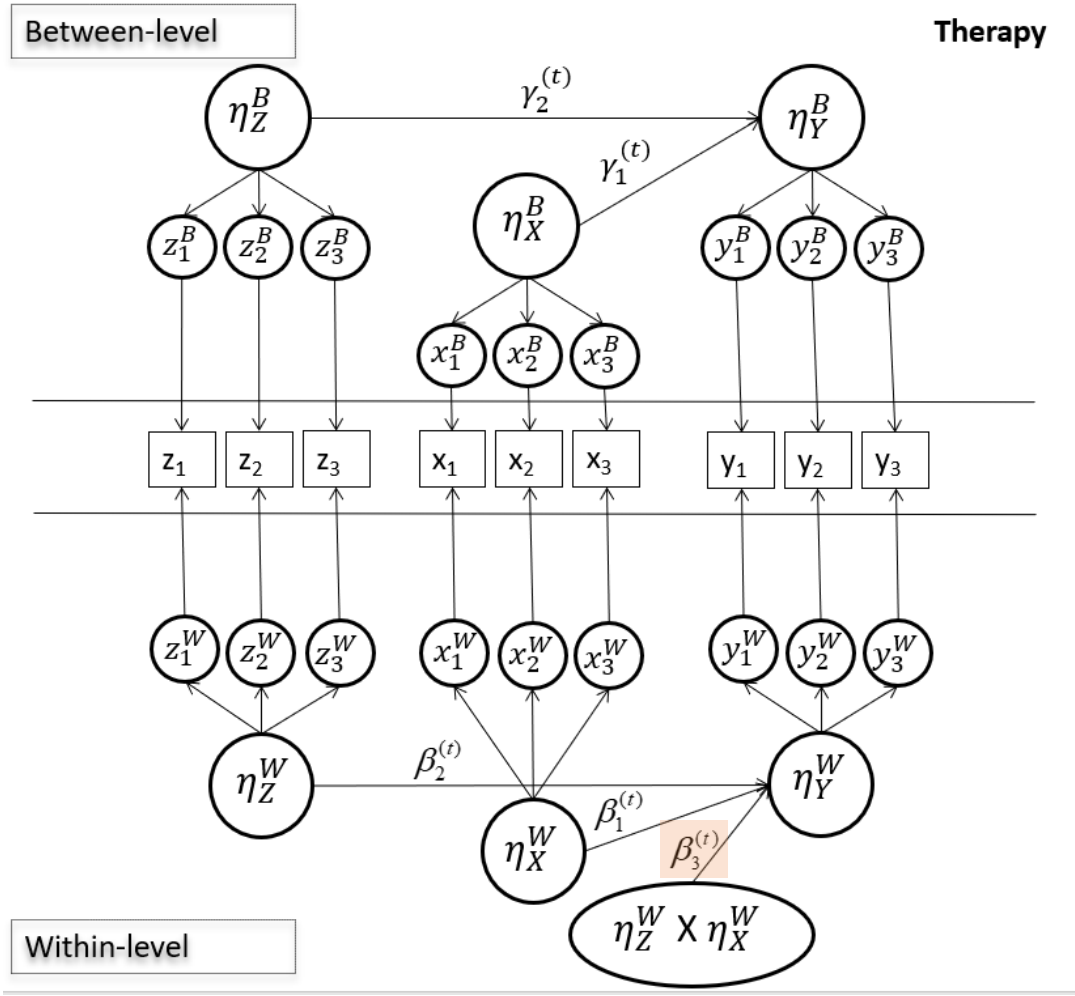
$$\eta_{Y^W}^{(t)} = \beta_0^{(t)} + \beta_1^{(t)}\eta_X^W + \beta_1^{(t)}\eta_Z^W + \beta_3^{(t)}\eta_X^W\,\eta_Z^W + \varepsilon_{ij}^{(t)}$$

$$\eta_{Y^B}^{(t)} = \gamma_0^{(t)} + \gamma_1^{(t)}\eta_X^B + \gamma_1^{(t)}\eta_Z^B + u_j^{(t)}$$

Residuals:

$$\varepsilon_{ij}^{(t)} \sim N(0, \sigma^2_{Y_|^{(t)}})$$

$$u_j^{(t)} \sim N(0, \tau^2_{Y_|^{(t)}})$$



Analytic Models: Structural

Between-level

Waitlist

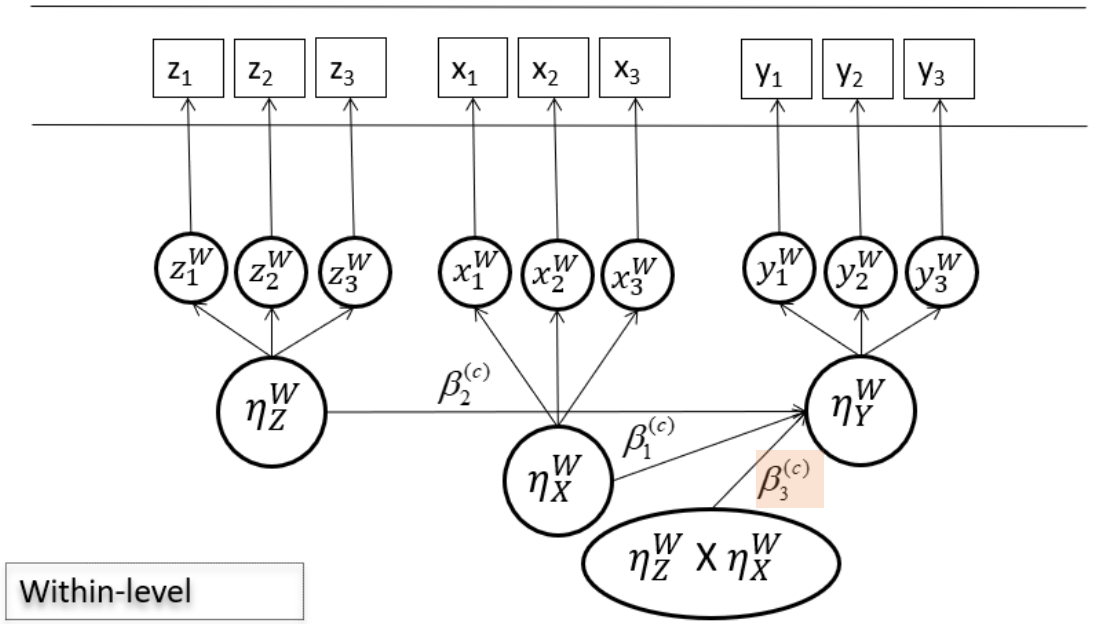
Waitlist-Outcome:

$$\eta_{Y^W}^{(c)} = \beta_0^{(c)} + \beta_1^{(c)}\eta_X^W + \beta_1^{(c)}\eta_Z^W + \beta_3^{(c)}\eta_X^W\,\eta_Z^W + \varepsilon_{ij}^{(c)}$$

$$\eta_{Y^B}^{(c)} = 0$$

Residuals:

$$\varepsilon_i^{(c)} \sim N(0, \sigma^2_{Y_|^{(c)}})$$



Within-level

Latent Interactions in Partially Nested SEM

Under the MSEM-PN framework any two-level SEM latent moderation effect specification from Preacher et al. (2016) and utilized in Asparouhov & Muthen (2020) can be considered in the nested/clustered study arm

Latent interactions in the waitlist (non-nested study arm) are limited to single-level interactions

We considered an individual level interaction involving the within part of a level-one moderator and within part of a level-one predictor possible in both study arms (A1)

SAM Croon’s for Latent Interactions in 2/1 Partially Nested SEMs

SAM Croon’s

SAM Croon's Estimation

(1) Maximum likelihood estimation of measurement models

(2) Estimate covariance matrix of factor scores and observed variables

(3) Measurement model results from step one are leveraged to create a method of moments correction for the biased covariance matrix produced in Step (2)

(4) Estimate structural parameters with maximum likelihood using corrected covariance

SAM Croon's Estimation: Latent Interactions

SAM Croon's Corrections adjust the covariance between latent variables and latent interactions

They are dependent on the location of the variables and interaction (i.e., between- or within-level).

For two-level SEMs with latent interactions, six corrections are needed for the seven possible moderation effects. One of these corrections is also applicable to single-level latent interaction corrections

SAM Croon's Estimation: Latent Interactions

Example A1: Correct the covariance of the interaction involving the Within Part of Level-One Moderator and the Within Part of Level-One Predictor (A1) with the outcome.

More formally…

We must correct $\text{cov}(\eta_Z^W \eta_X^W, \eta_Y^W)$ where $\text{cov}(\hat{\eta}_Z^W \hat{\eta}_X^W, \hat{\eta}_Y^W) = cov(\tilde{Z}^W \cdot \tilde{X}^W, \tilde{Y}^W)$

with $\tilde{Z}^W, \tilde{X}^W \text{ and, } \tilde{Y}^W$ representing the within-level factor scores

SAM Croon's Estimation: Latent Interactions

Correction:

$$\operatorname{cov}(\eta_Z^W \eta_X^W, \eta_Y^W) = \frac{\operatorname{cov}(\hat{\eta}_{Z_i}^W \hat{\eta}_{X_i}^W, \hat{\eta}_{Y_{ij}}^W)}{\mathbf{A}_Z^W \mathbf{A}_X^W \boldsymbol{\Lambda}_Z^W \boldsymbol{\Lambda}_X^W \boldsymbol{\Lambda}_Y^W \mathbf{A}_Y^W}$$

with latent variable interaction variance found with

$$\operatorname{var}(\eta_Z^W \eta_X^W) = \operatorname{var}(\eta_Z^W) \operatorname{var}(\eta_X^W) + \operatorname{cov}(\eta_Z^W, \eta_X^W)^2$$

All components of the covariance are located at the within-level eliminating the need for any consideration of unreliability of indicator cluster means.

This correction operates as a correction for a single-level latent interaction and is applicable to the waitlist condition

(see Cox et al., 2023 and Cox & Kelcey, 2021)

Simulation Study

SAM Croon's for Latent Interactions in 2/1 Partially Nested SEMs

Simulation Study: Conditions

Path coefficient values:

$$\text{Within: } \beta_1^{(t)} = \beta_1^{(c)} = \beta_2^{(t)} = \beta_2^{(c)} = 0.5$$

$$\beta_3^{(t)} = \beta_3^{(c)} = 0.3$$

$$\text{Between: } \gamma_1^{(t)} = \gamma_2^{(t)} = 0.0$$

Factor loadings of 1.0, 1.5, and 0.66

Indicator and outcome error variance decomposition: 0.8 **W**ithin, 0.2 **B**etween

Various Sample Sizes (as low as n2=30 and n1=10)

Simulation Study: Results

Outcomes:

- Convergence rate
- Bias
- Efficiency of estimates (SD across simulation runs and RMSE)

Approaches:

- SAM-Croon's
- Uncorrected factor scores

Method:

- Data were generated in R and analyzed in Mplus 8.3(MplusAutomation)

Results

SAM Croon's for Latent Interactions in 2/1 Partially Nested SEMs

Results: Software and Estimator Limitations

Data structure recommendation: MSEM-PN (Long Format) from Sterba et al. (2014) and Lachowicz et al. (2015) using multiple group model…

-Integration for ML unavailable

-Bayes unavailable (knownclass option for multilevel mixture model problematic with partially nested structure)

-Getting factor scores at the within and between level for each group difficult for SAM-Croons

Avoid issue by separating comparison group data and analysis (e.g., two data sets, two sets of analytic models, two sets of results)…multiple arm approach

Results: Software and Estimator Limitations

Multiple arm approach:

Maximum Likelihood: Convergence time +1 hour and still results in Mplus warnings to increase integration points

Bayesian Estimation: Convergence time ~30 min

SAM-Croon's: Good to go…couple minutes to run six measurement models and two structural models that represent the therapy and waitlist groups

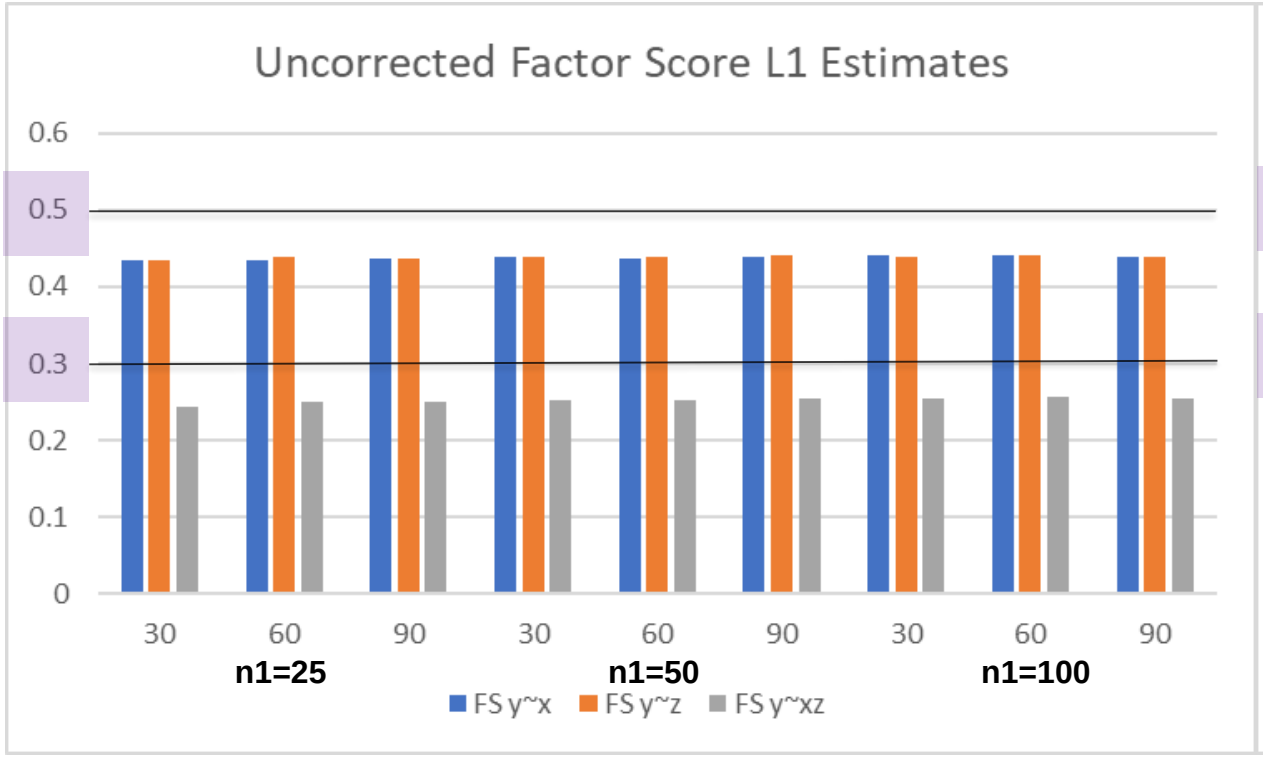
Results: Convergence

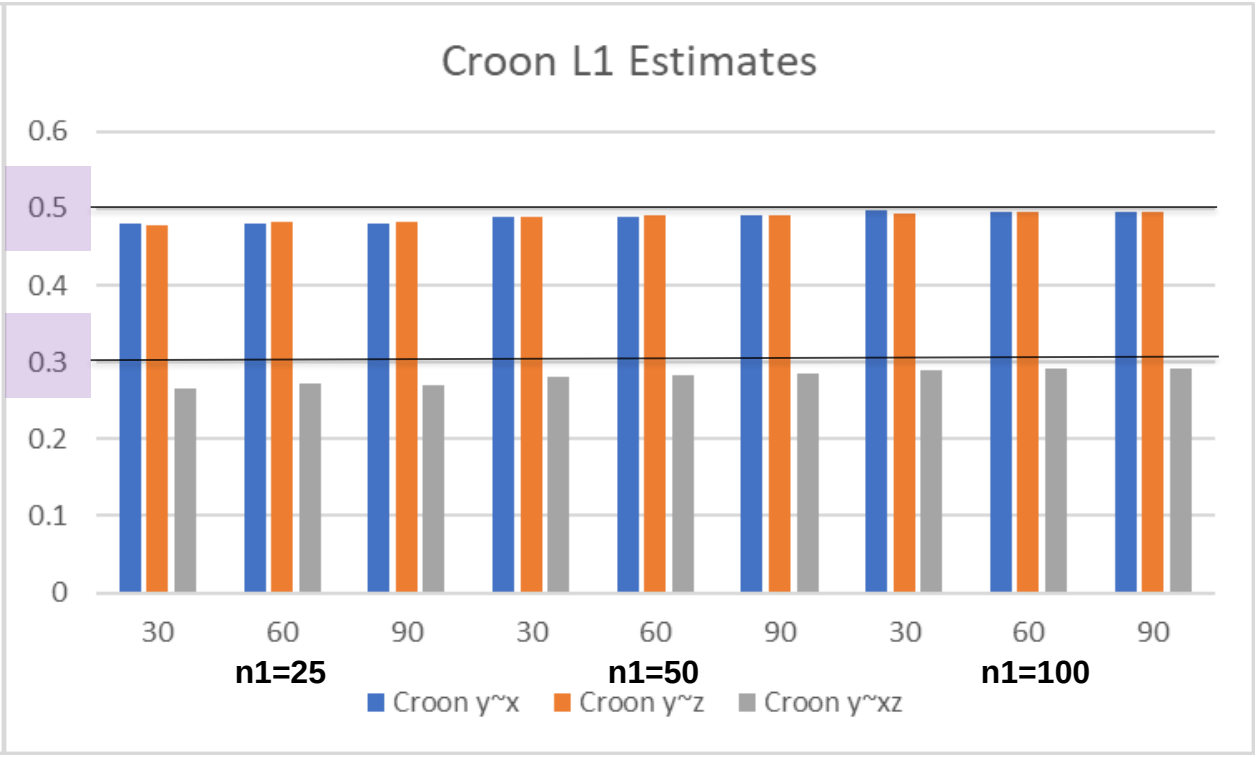
Convergence Failure Rate for Estimation of Partially Nested Structural Equation Models with a Latent Interaction (A1)

Sample Size			FS			Croon's		
$n_2^{(t)}$	$n_1^{(t)}$	$n^{(c)}$	*Treatment (L2)*	*Treatment(L1)*	*Control*	*Treatment(L2)*	*Treatment(L1)*	*Control*
30	25	750	6.6%	6.6%	0.0%	7.6%	6.6%	0.0%
60	25	1500	1.8%	1.8%	0.0%	1.8%	1.8%	0.0%
90	25	2250	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
30	50	1500	5.2%	5.2%	0.0%	7.0%	5.2%	0.0%
60	50	3000	0.4%	0.4%	0.0%	0.4%	0.4%	0.0%
90	50	4500	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
30	100	3000	6.0%	6.0%	0.0%	10.2%	6.0%	0.0%
60	100	6000	2.8%	2.8%	0.0%	4.2%	2.8%	0.0%
90	100	9000	2.0%	2.0%	0.0%	2.2%	2.0%	0.0%

Results: Bias in Therapy Arm

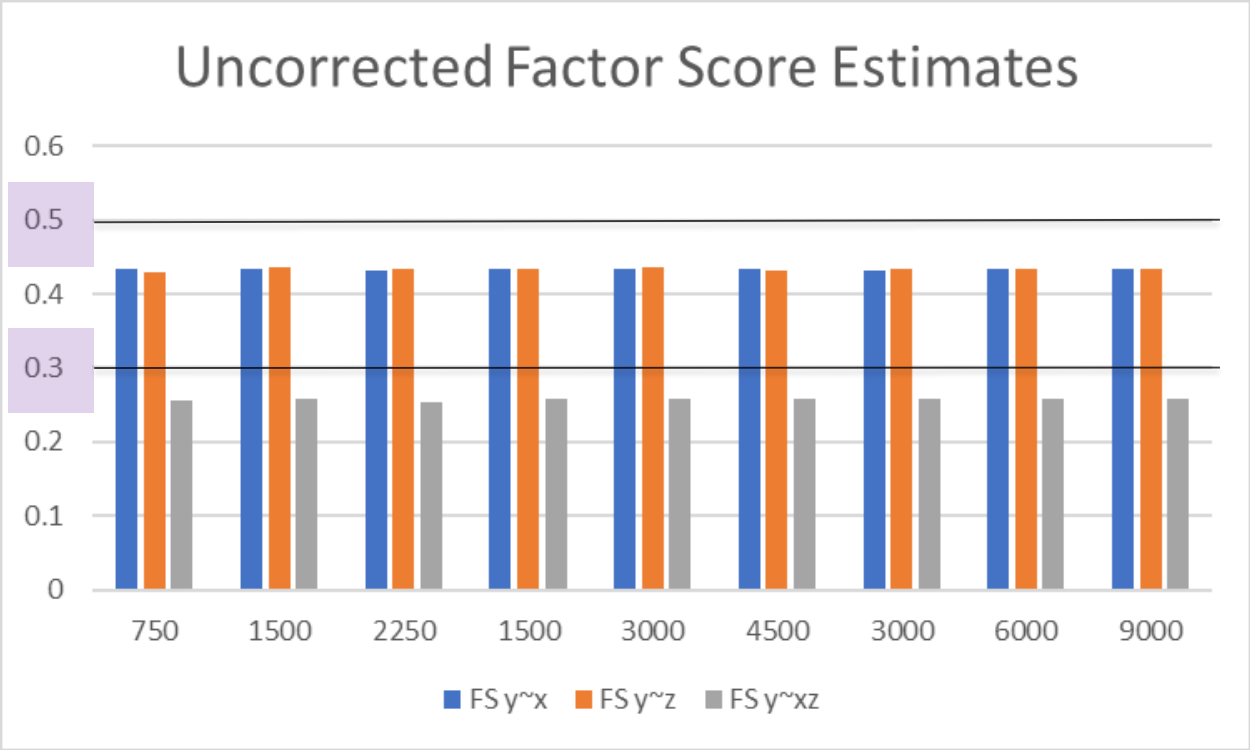
True path coefficients = 0.5
True latent interaction = 0.3

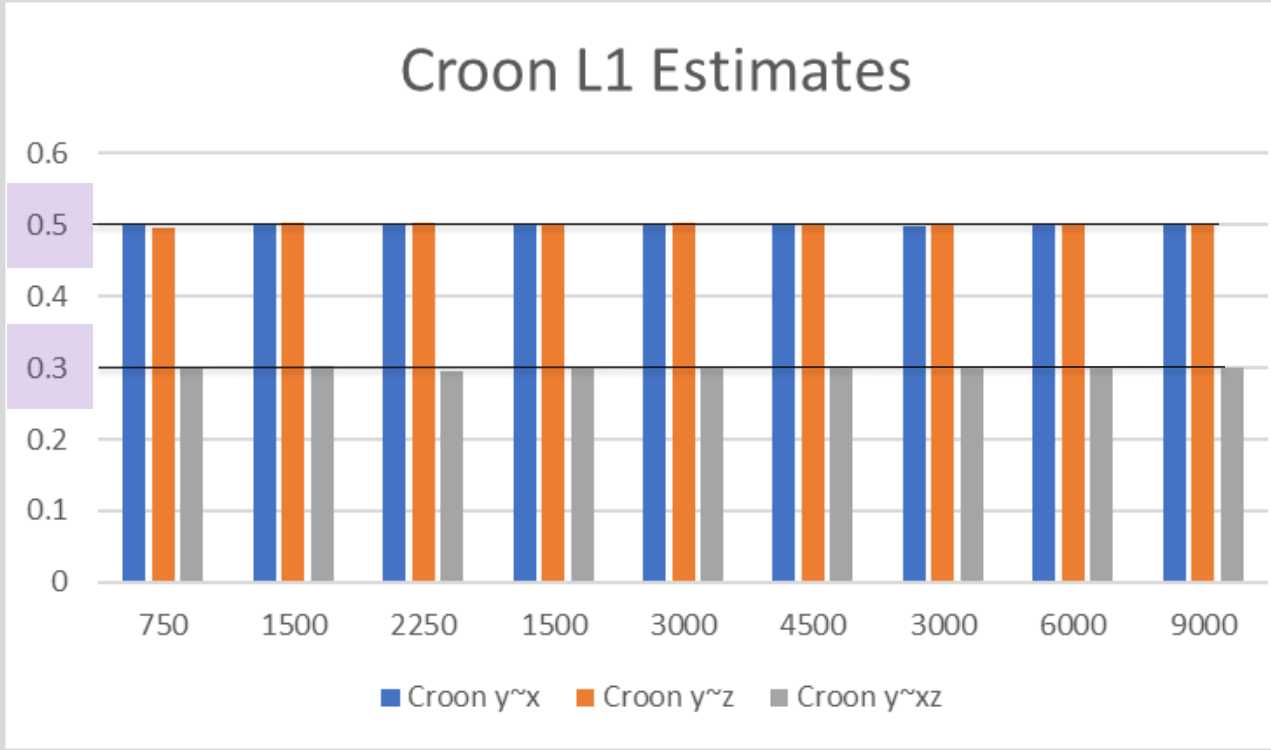




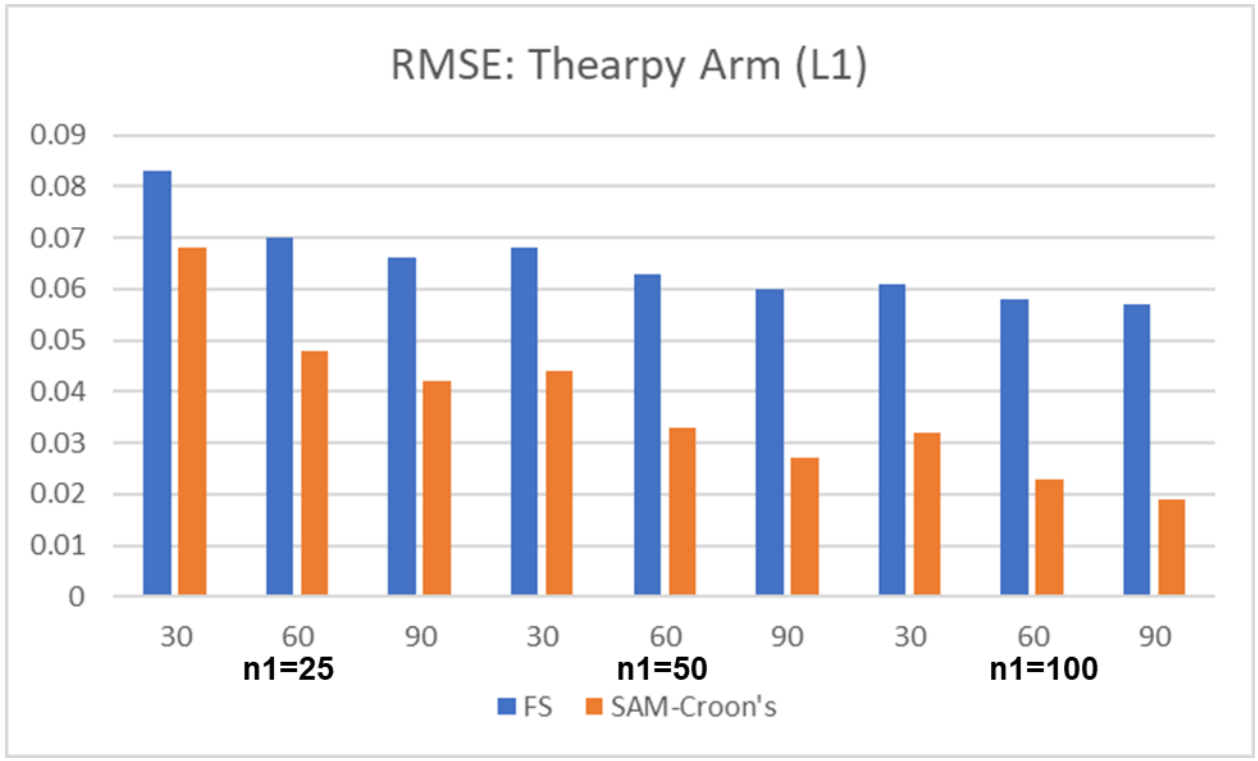
Results: Bias in Waitlist Arm

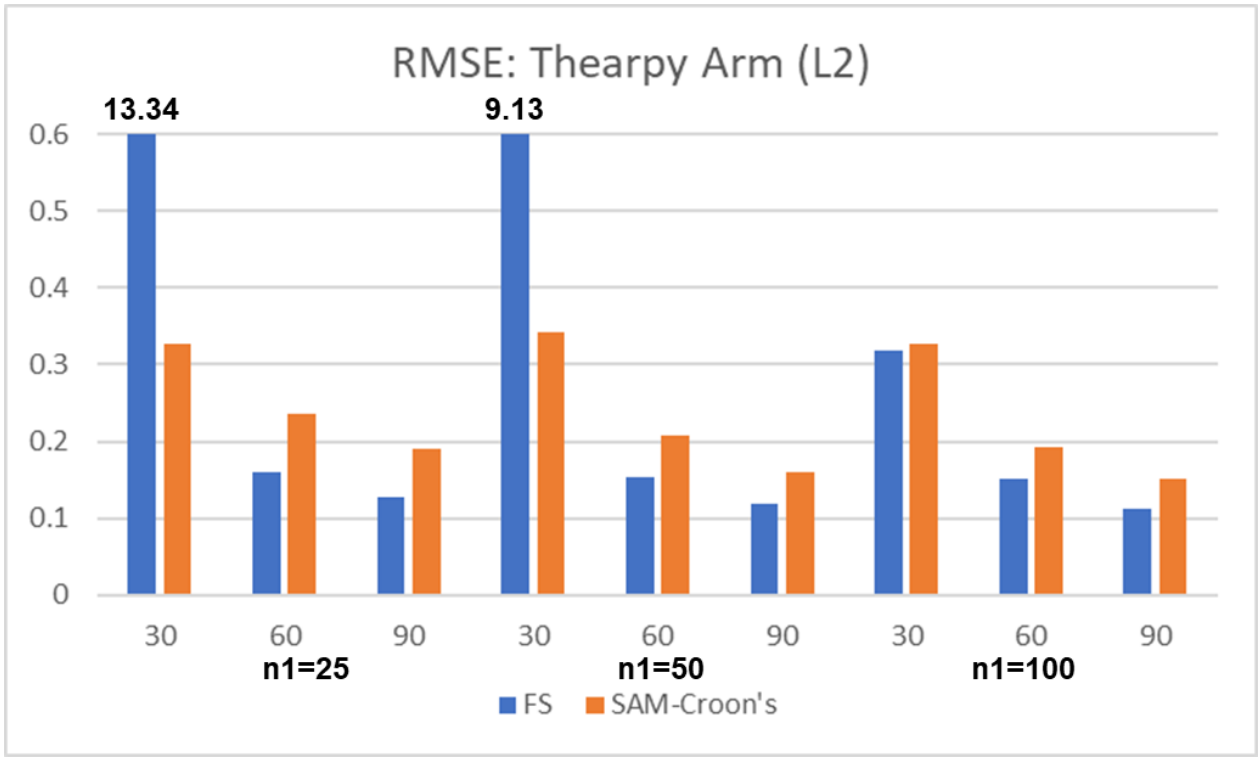
True path coefficients = 0.5
True latent interaction = 0.3



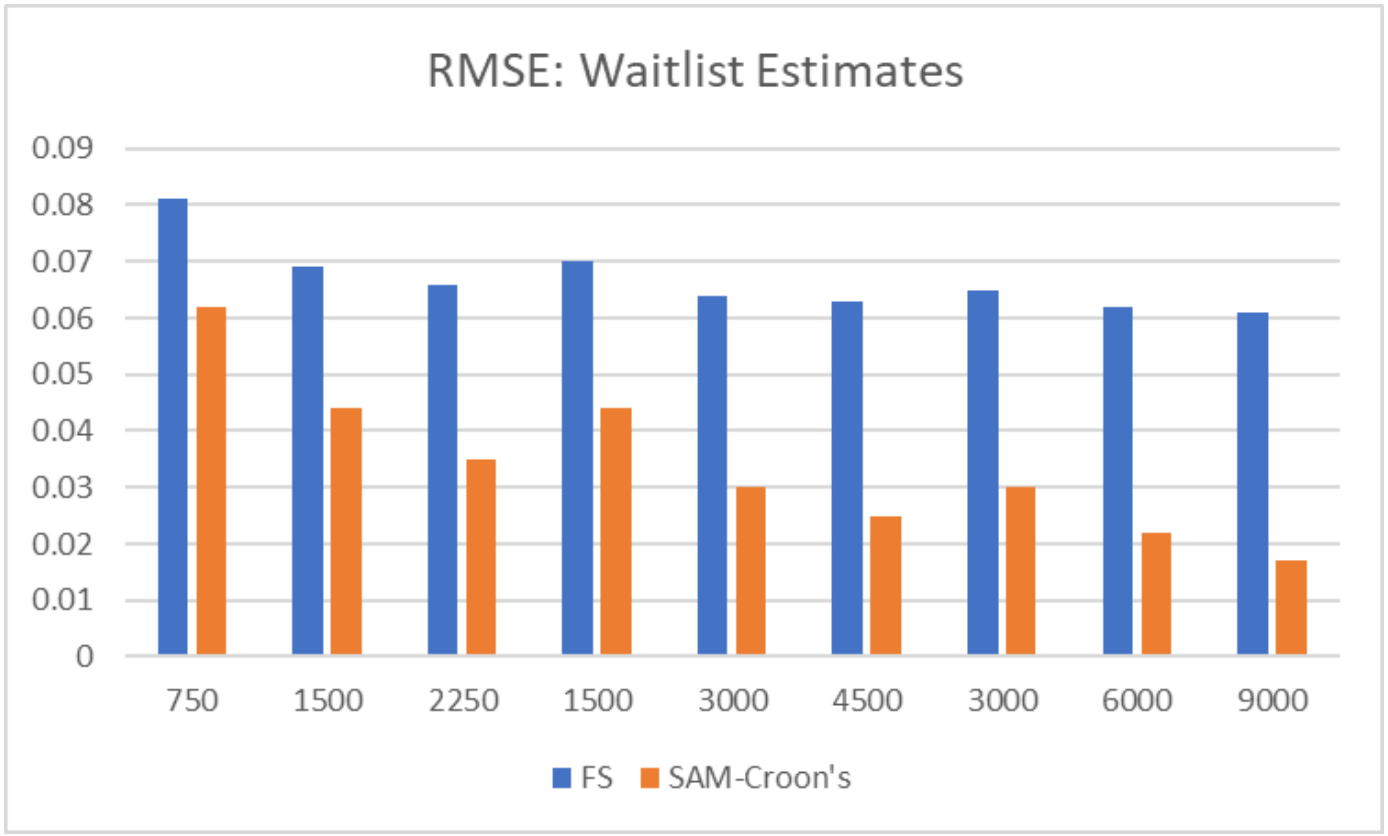


Results: RMSE Therapy Arm





Results: RMSE Waitlist Arm



Results…Conclusions and Implications

1) SAM-Croon's is an effective estimator for partially nested SEMs with latent interactions

2) SAM-Croon's can be employed with the full scope of latent interactions possible in 2/1 partially nested data including multiple latent interactions and when SEMs and latent interactions differ across study arms (full scope of study not presented here).

3) SAM-Croon's performed well even with smaller sample sizes in terms of convergence (time and avoiding failure), bias, and efficiency

Next Steps: SAM-Croon's for partially nested SEMs with latent interactions…

Estimator comparisons (maximum likelihood, EAP factor scores, and Bayesian estimation with and without informative priors)

Multiple latent interactions and/or different SEMs and latent interactions across study arms

Extend SAM-Croon's to partially nested SEMs that include latent moderated-mediation

Examine influence of other design factors (e.g., outcome variance decomposition; indicator variance decomposition; indicator reliability)

Examine robustness of SAM-Croon's to model misspecifications

Thank You!

Questions

Kyle Cox

kyle.cox@uncc.edu

Appendix

Other Results of note…

Similar results with n1=10

Bayes…

EAP Factor scores…