Does Group-Mean Centering Always Inflate Type I Error Rates in Multiple Regression?

Hua Lin, B. Wade Brorsen, & Robert E. Larzelere

Oklahoma State University

MMM Conference, June 28, 2023

Challenges in Estimating Treatment Effects

B. Wade Brorsen
Hua Lin
Robert Larzelere

ANCOVA with Dual-Centered Data

Proposed solution to Lord's Paradox:

- **Dual-Centered ANCOVA**
- **Extension of Huitema's Quasi-ANCOVA**

$$Y_{1ij} - \overline{Y}_{0j} = \delta_0 + \delta_1 X_{ij} + \delta_2 (Y_{0ij} - \overline{Y}_{0j}) + \varepsilon$$

Center the posttest scores on pretest group means

Center the pretest scores on pretest group means

Results Using Dual-Centered Data

Data	Difference Scores		Residualized Change Score	
	d_1	$t(d_1)$	b_1	$t(b_1)$
Lord's example	-0.01	-0.01	-0.01	-0.01
Reversed	15.61***	16.17	15.61***	18.76
Sex costs talk	–0.08**	–2.77	–0.08***	–3.43
Reasoning	–0.03*	–2.35	–0.03*	–2.73
Hospitalization	0.16***	3.81	0.17***	4.91

More Power?
OR
Inflated α

Treatment		**Outcome**
Sex costs talk	→	Unprotected sex
Reasoning	→	Child aggression
Hospitalization	→	Physical health

Possible Advantages of Dual-Centered ANCOVA

Four possible advantages (when diffs-in-diffs is warranted):

1. Yields consistent results when Lord's paradox applies

2. Estimates pure within-person effects

3. Can it provide more power than standard difference-score analyses?

4. Can test Pretest X Treatment interactions within difference-score analysis

Lord's paradox applies to most longitudinal analyses

Wade Brorsen's Analysis: Goals

- Explain why Quasi-Ancova standard errors are too low
- What to do about endogenous treatment effects

A Common Language

- Anova
- Ancova
- Differences
- Quasi-Ancova
- Dual-Centered Ancova

Analysis of Variance (Anova)

(1) $$Y_{ij1} = \beta_0 + \beta_1 X_j + \varepsilon_{ij}$$

where Y_{ij1} is posttest score of ith person receiving jth treatment, X_j is an indicator variable for the jth treatment (j = 1, 2).

Differences Model

(2) $Y_{ij1} - Y_{ij0} = \delta_0 + \delta_1 X_j + \vartheta_{ij}$

where Y_{ij0} is the pretest score.

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Analysis of Covariance (Ancova)

$$(3) \qquad Y_{ij1} = \alpha_0 + \alpha_1 X_j + \alpha_2 Y_{ij0} + v_{ij}.$$

Problems

- The choice of model can dictate the answer
- Endogenous treatment effects

Monte Carlo Study (Lin, 2022)

Name	Treatment Effect	SE	MC SD	MSE
Anova	-0.0002	0.948	0.998	224.8
ANCOVA	-0.0068	0.413	0.411	42.7
Quasi-ANCOVA	-0.0002	0.413	0.998	42.7

Quasi-Ancova

Posttest Score | Treatment Dummy | Pretest Score | Group Means

$$(4)\ Y_{ij1} = \gamma_0 + \gamma_1 X_j + \gamma_2 (Y_{ij0} - \widehat{Y_{ij0}}) + v_{ij}$$

Quasi-Ancova

$$(4) Y_{ij1} = \gamma_0 + \gamma_1 X_j + \gamma_2 (Y_{ij0} - \widehat{Y_{ij0}}) + v_{ij}$$

$$(5) Y_{ij0} = \varphi_0 + \varphi_1 X_j + \tau_{ij}$$

Possibilities

Generated Regressor Problem

Two-Stage Least Squares (IV)

Calculate coefficients using predictions

Calculate standard errors using actuals

Quasi-Ancova

$$(4) Y_{ij1} = \gamma_0 + \gamma_1 X_j + \gamma_2 (Y_{ij0} - \widehat{Y_{ij0}}) + v_{ij}$$

$$(5) Y_{ij0} = \varphi_0 + \varphi_1 X_j + \tau_{ij}$$

Quasi-Ancova gives same estimate and same standard errors as Anova when standard errors are adjusted for generated regressors

Quasi-Ancova

$$(4) Y_{ij1} = \gamma_0 + \gamma_1 X_j + \gamma_2 (Y_{ij0} - \widehat{Y_{ij0}}) + v_{ij}$$

$$(5) Y_{ij0} = \varphi_0 + \varphi_1 X_j + \tau_{ij}$$

The added term is the error from the second equation. Both have same regressors, so no gain in using seemingly unrelated regression.

Dual-Centered Ancova

$$(8)\quad Y_{ij1} - \bar{Y}_{j0} = \omega_0 + \omega_1 X_j + \gamma_2 (Y_{ij0} - \widehat{\mu_j}) + v_{ij}$$

$$(4)\quad Y_{ij1} = \gamma_0 + \gamma_1 X_j + \gamma_2 (Y_{ij0} - \widehat{Y_{ij0}}) + v_{ij}$$

$$(5)\quad Y_{ij0} = \varphi_0 + \varphi_1 X_j + \tau_{ij}$$

$$(9)\quad Y_{ij0} = \mu_j + \tau_{ij}$$

Dual-centered Ancova is the same as the differences model.

Endogenous Treatment Effects

- Spanking
- Obesity
- Depression
- Preventive antibiotics in feedlot

Endogenous Treatment Effects

- Randomized controlled trials
- Instrumental variables
- Matching
- Lewbel approach
- FIML with sample selection

Metaphylaxis

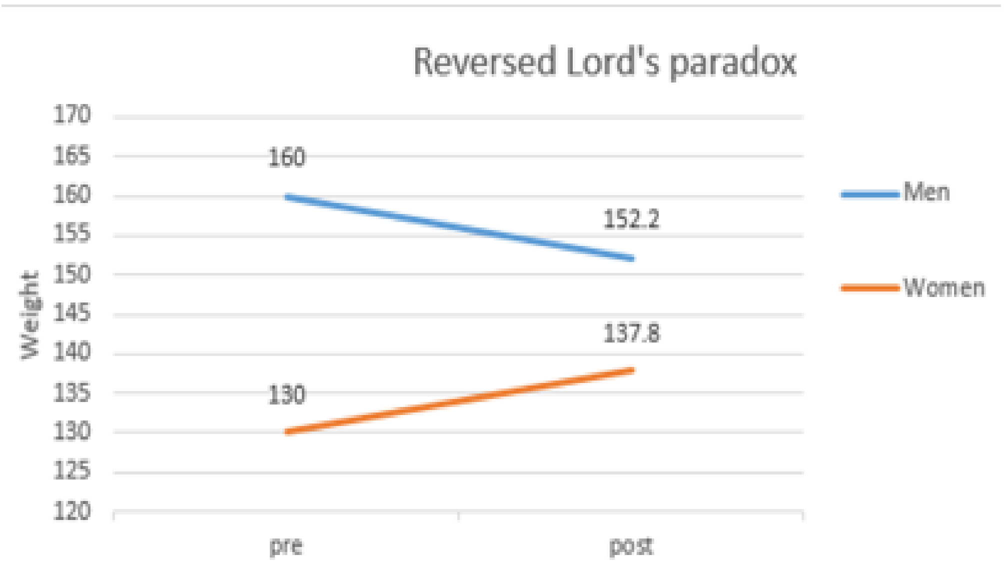
- Treated cattle have worse outcomes
- Treatments effective in experiments
- Propensity score still negative
- Lewbel (2012) can give zero effect (after pretesting)
- Need better selection variables

Summary

- Quasi-Ancova is same as Anova
- Dual-Centered Ancova is same as Differences approach
- Endogenous treatment effects- no clear answer

Graphical Explanation

Null Hypothesis for ANCOVA



When Does Group-Mean Centering Bias *se*'s of Tx Effects?

- Pagan (1984, *International Economic Review*) Generated regressors
- Multilevel modeling case (His Model 4)
  - Standard errors are correct at Level 1
  - Standard errors are biased at Level 2
- Do his conclusions apply only to OLS regression on Level 2 alone?
- Do multilevel modeling programs correct for this bias?
- Brorsen: Need 2SLS or Instrumental Variable approaches or maximum likelihood to get the correct standard errors.

Initial Simulation (Hua Lin)

- **Still working on simulating Multilevel Modeling to test whether Pagan (1984) is correct that se's are biased for treatment effects at Level 2**

Acknowledgements

- Expert consultation help from
  - Dave Kenny
  - Joshua Habiger
  - Tianyu Cao
  - Isaac Washburn
  - Brad Huitema
- Funding from
  - NICHD grant #5 R03 HD107307
  - OK State Univ. Parenting Professorship

- Help clarify this issue? Hua.lin@okstate.edu