Alternative Specifications for Instrumental Variable Analysis in Structural Equation Modeling: First Steps Toward Latent Analysis of Symmetrically Predicted Endogenous Subgroups

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Background

- Dissertation was on a method called Analysis of Symmetrically Predicted Endogenous Subgroups (ASPES).

- ASPES came from the evaluation literature (Peck, 2003; Peck, 2013) and has been called a “cousin” of principal stratification.

- It leverages random assignment of an intervention in order to identify average causal effects for each level of a post-random assignment (endogenous) variable of interest that was only observed in one group (e.g., compliance, quality of implementation).

Setting

- Let's call this post-treatment variable M since it is an intermediate variable. If it had observations in both the treatment and control groups, we could treat it as a causal mediator.

- However, when M is only observable in one group, it's impossible to treat it as a causal mediator because we have no information on one of the potential mediators (M_0 or M_1).

- Let's imagine M is a measure of implementation quality in the treatment group.

If *M* is discrete (e.g., low vs. high quality)…

- ASPES very much resembles well known methods of estimating principal stratum causal effects such as the complier average causal effect (CACE).

- Peck (2003) nonparametrically identified analogues to the CACE and NACE that can be estimated by ASPES.

- ASPES accomplishes this identification via a mean independence assumption, which I argue is best thought of as an exclusion restriction (ER) assumption, because it requires an instrumental variable.

If *M* is discrete (e.g., low vs. high quality)…

- ASPES essentially uses an instrument, *V*, for the interaction effect between treatment assignment (*T*) and *M* on the outcome, *Y*.

- The assumption used in ASPES ends up being mathematically equivalent to an assumption that there is no interaction effect between the chosen instrument and treatment assignment on the outcome.

- I call this the "interaction exclusion restriction" (IER) assumption because it is equivalent to using *T*V* as an instrument for *T*M*, requiring the usual ER assumption on the effect of *TV* on *Y*.

If *M* is discrete (e.g., low vs. high quality)…

- In this setting, ASPES is a nonparametric version of the approach to principal stratification via mixture modeling proposed by Jo (2002).

ASPES (Peck, 2003)

Stage 1:

$$M = f(V, X)$$

(estimate this in the treatment group using cross-validation method to prevent overfitting, then compute $\widehat{M}$ for each unit)

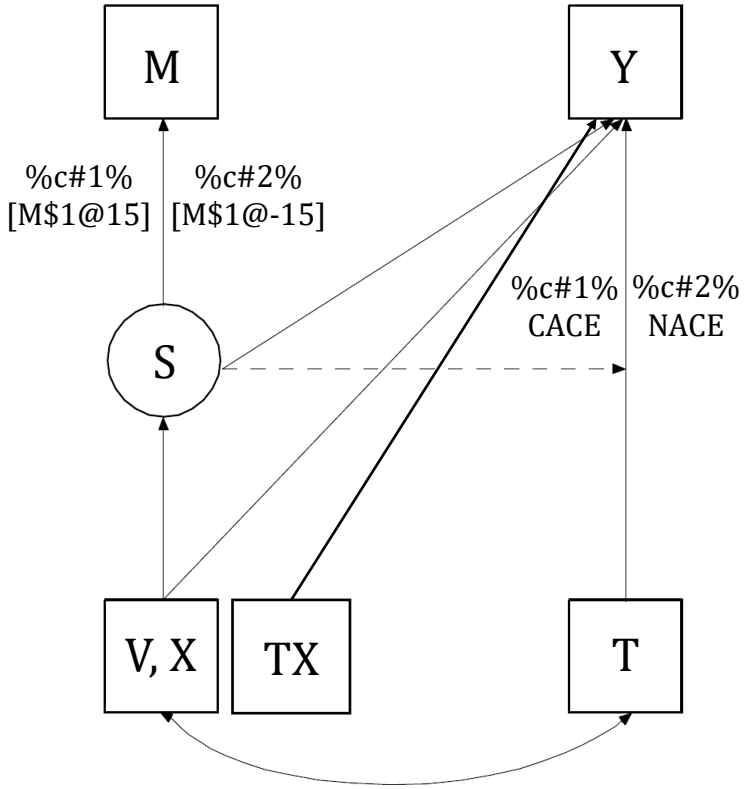
Stage 2:

$$\text{CACE}^P = E\left[Y|T=1, \widehat{M}=1\right] - E\left[Y|T=0, \widehat{M}=1\right]$$
$$\text{NACE}^P = E\left[Y|T=1, \widehat{M}=0\right] - E\left[Y|T=0, \widehat{M}=0\right]$$

$$\text{CACE} = \frac{E\left[1-\widehat{M}|M=0\right]\text{CACE}^P - \left(1-E\left[\widehat{M}|M=1\right]\right)\text{NACE}^P}{E\left[1-\widehat{M}|M=0\right] + E\left[\widehat{M}|M=1\right] - 1}$$

$$\text{NACE} = \frac{E\left[\widehat{M}|M=1\right]\text{NACE}^P - \left(1-E\left[1-\widehat{M}|M=0\right]\right)\text{CACE}^P}{E\left[1-\widehat{M}|M=0\right] + E\left[\widehat{M}|M=1\right] - 1}$$

Principal Stratification via Mixture Modeling (Jo, 2002)



If *M* is continuous…

- There isn't a lot of guidance on how to appropriately incorporate *M* in our analyses when it's continuous.

- The principal stratification framework struggles to accommodate an *M* with many levels (even many discrete levels).

- Often, researchers will artificially discretize *M* in this case so that they can apply the well-established methods for the discrete *M* case.

If *M* is continuous…

- ASPES can actually be used with a continuous *M* similarly to how two stage least squares (TSLS) methods for IV often handle continuous causal variables of interest.

If M is continuous…

Stage 1:

$$M = f(V, X) + \delta$$
$$\delta \sim N(0, \sigma_M^2)$$

(estimate this in the treatment group using cross-validation method to prevent overfitting, then compute $\widehat{M}$ for each unit)

Stage 2:

$$Y = g\left(T, \widehat{M}, X\right) + \epsilon$$
$$\epsilon \sim N(0, \sigma_Y^2)$$

This can be done with TSLS, using linear models in both stages if those functional form assumptions are reasonable.

If M is continuous…

Stage 1:

$$M = \alpha_0 + \alpha_1 V + \alpha_2 X + \delta$$
$$\delta \sim N(0, \sigma_M^2)$$

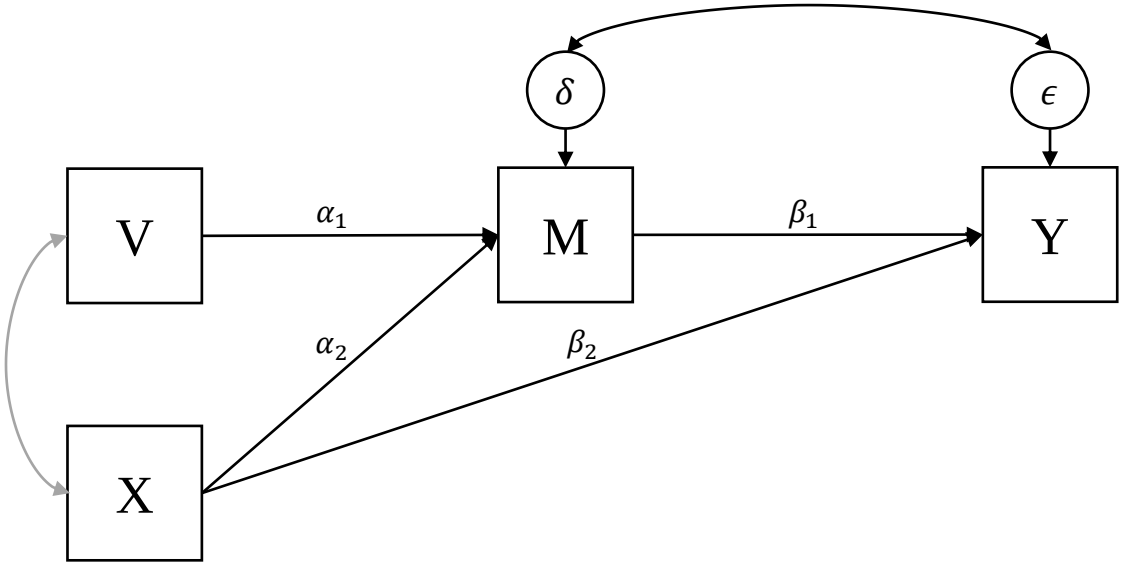
(estimate this in the treatment group using cross-validation method to prevent overfitting, then compute $\widehat{M}$ for each unit)

Stage 2:

$$Y = \beta_0 + \beta_1 T + \beta_2 \widehat{M} + \beta_3 T\widehat{M} + \beta_4 X + \beta_5 TX + \epsilon$$
$$\epsilon \sim N(0, \sigma_Y^2)$$

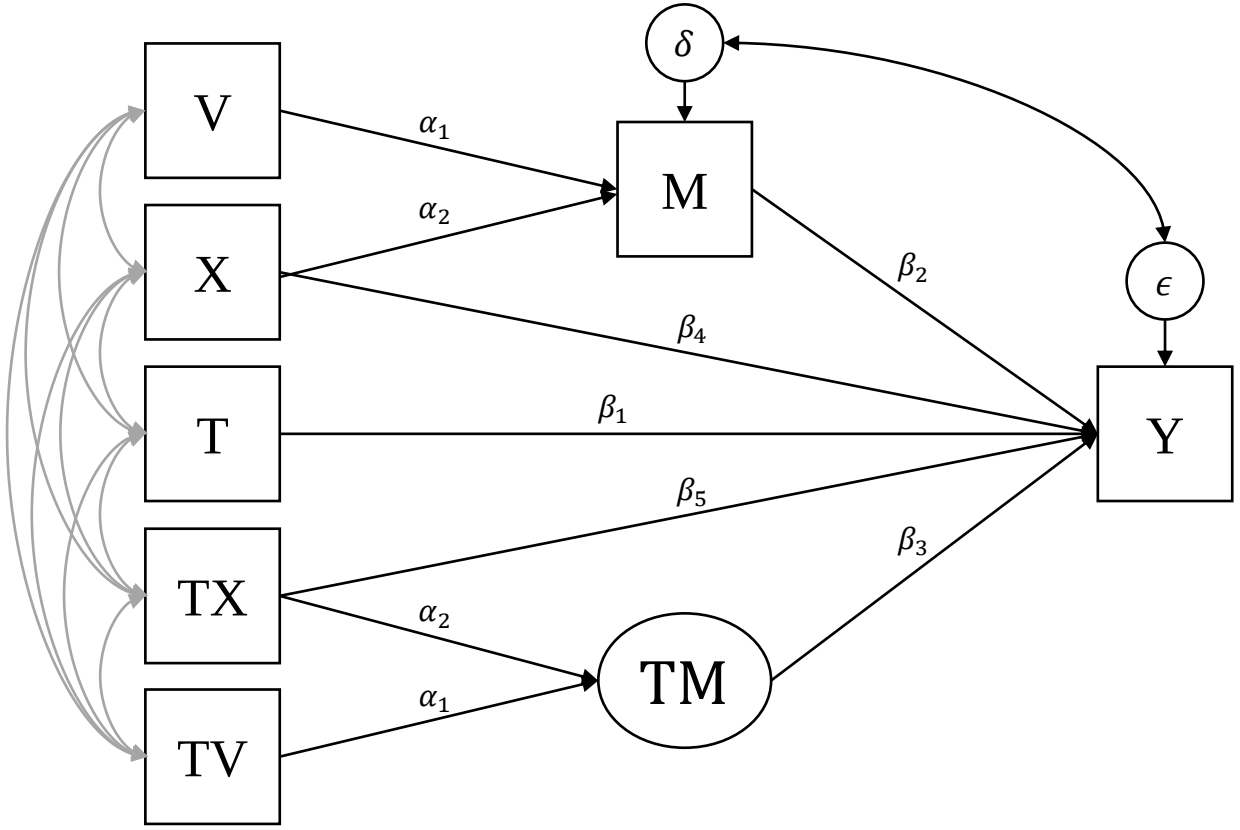
Translating to FIML-estimated Model

$$M = \alpha_0 + \alpha_1 V + \alpha_2 X + \delta$$
$$Y = \beta_0 + \beta_1 \widehat{M} + \beta_2 X + \epsilon$$



Translating to FIML-estimated Model

$$M = \alpha_0 + \alpha_1 V + \alpha_2 X + \delta$$
$$Y = \beta_0 + \beta_1 T + \beta_2 \widehat{M} + \beta_3 T\widehat{M} + \beta_4 X + \beta_5 TX + \epsilon$$



Translating to FIML-estimated Model

- Full information maximum likelihood (FIML) handles the missing data in *M* under the assumption of missingness at random (MAR), which is satisfied by design due to the missingness in *M* being perfectly explained by *T*.

- Obviously, this limits the ASPES approach by making parametric assumptions, but these could be relaxed, and estimating the model with FIML circumvents the cumbersome stage-1 cross-validation and opens ASPES up to all the benefits/possibilities of latent variable modeling.

Preliminary Simulation Results

- Preliminary simulations have thus far been confirming that this FIML approach reproduces the estimates found with the original ASPES two-stage implementation using linear models, and that both approaches accurately recover the *TM* interaction effect when the IER assumption holds.

- Additionally, as long as the IER assumption was satisfied, the *TM* effect is accurately recovered even when *M* and *Y* share a confounder that mediates and/or moderates the treatment effect.

Causal Moderation?

- Although this clearly cannot be considered causal mediation, the question still stands: is this *TM* interaction effect an average causal *moderation* effect?

- Or is it simply an unbiased estimate of the heterogeneity in the average causal effect that happens to relate to *M*?

- In potential outcomes terms, is this the unbiased average causal effect of *M* on $Y_1 - Y_0$? Or is this just a measure of the relationship between *M* and $Y_1 - Y_0$ that may solely be due to an omitted confounder?

Causal Moderation?

- Let's consider VanderWeele's (2014) decomposition of causal mediation/interaction:

$$\text{ITT} = Y_1 - Y_0 = \text{CDE}(0) + \text{INT}_{ref} + \text{INT}_{med} + \text{PIE}$$

Controlled Direct Effect $\big(\text{CDE}(0)\big) = Y_{10} - Y_{00}$

Reference Interaction $\big(\text{INT}_{ref}\big) = \big((Y_{11} - Y_{10}) - (Y_{01} - Y_{00})\big)(M_0)$

Mediated Interaction $(\text{INT}_{med}) = \big((Y_{11} - Y_{10}) - (Y_{01} - Y_{00})\big)(M_1 - M_0)$

Pure Indirect Effect $(\text{PIE}) = (Y_{01} - Y_{00})(M_1 - M_0)$

Causal Moderation?

- If we define M as the value that would be realized when assigned to the treatment group, then there is no difference between M_0 and M_1; $M_0 = M_1 = M$

$$\text{ITT} = Y_1 - Y_0 = \text{CDE}(0) + \text{INT}_{ref} + \text{INT}_{med} + \text{PIE}$$

Controlled Direct Effect: $\big(\text{CDE}(0)\big) = Y_{10} - Y_{00}$

Reference Interaction: $\big(\text{INT}_{ref}\big) = \big((Y_{11} - Y_{10}) - (Y_{01} - Y_{00})\big)(M)$

~~Mediated Interaction: $(\text{INT}_{med}) = \big((Y_{11} - Y_{10}) - (Y_{01} - Y_{00})\big)(M - M)$~~

~~Pure Indirect Effect: $(\text{PIE}) = (Y_{01} - Y_{00})(M - M)$~~

Causal Moderation, kind of?

- What we end up with is a kind of one-sided moderation effect.

- We haven't actually identified the causal effect of M on Y given T, but we have identified the causal effect of T on Y given M (obviously through random assignment) and the causal effect of M on Y for the treatment group.

- It's arguably causal moderation in the sense that M causing changes in Y_1 while causing no changes in Y_0 naturally means that M causes changes in $Y_1 - Y_0$.

Conclusions

- Still working on these simulations
- Next steps include exploring ASPES with a latent variables using the FIML approach, as well as other extensions such as multilevel versions of the approach.
- Thanks for listening!

References

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