Disentangling Longitudinal Treatment Effects by Regimes: A Comparison of Selection Bias Adjustment Approaches

Hanna Kim and Jee-Seon Kim

1. Study Design and Research Questions
- Multiple treatment regimes occur from longitudinal treatments implemented at multiple time points.
- Generalized propensity scores (GPS)
  \[ P(G_i = g|C) = \frac{\exp(\beta_0g + \beta_1g'C')}{1 + \sum_{g'}=1 \exp(\beta_0g + \beta_1g'C')} \]
- Weights computed by entropy balancing (Hainmueller, 2012)
  \[ \frac{w_{PS} (g - g_c)}{\text{MSE}} = \frac{\sum_{g'}=1 \text{E}_{w}[Y_i | g, c] - \text{E}_{w}[Y_i | g, c]}{\text{MSE}} \]
- HC3 standard errors computed for the contrasts
- E\[Y_2|C = g, C = c\] = \[\theta_0 + \theta_1g + \theta_2c' + \theta_3g \ast c'\]
- For valid comparison, we need to adjust for baseline covariates (C) and intermediate vocabulary skills (Y_i)

2. Adjusting for Selection Bias in Multiple Treatments
- First, we conceptualize all possible patterns of treatment participation as categories of a multiple treatment (Imbens, 2000).
- e.g., Head Start as a four-category treatment

\[
G
(\begin{array}{c}
1, 1 \\
1, 0 \\
0, 1 \\
0, 0
\end{array}
) \rightarrow
(\begin{array}{c}
Y_{11} \\
Y_{10} \\
Y_{01} \\
Y_{00}
\end{array}
)
\]
- e.g., Children’s vocabulary skills (Y_2) after staying in the Head Start program (A_1, A_2 = (1, 1)), combining Head Start with other childcare services (A_1, A_2 = (1, 0) or (0, 1)), or never attending Head Start (A_1, A_2 = (0, 0)) for two years
- For valid comparison, we need to adjust for baseline covariates (C) and intermediate vocabulary skills (Y_i)

3. Adjusting for Selection Bias in Sequential Treatment Regimes
- On the other hand, we consider a sequence of treatments at successive time points.
- Multiplicative inverse probability of treatment weights are produced, and potential outcomes are estimated for different sequences of treatment participation.
  \[ \logit \{P(A_1 = 1 | C = c)\} = \beta_0 + \beta_1c' \]
  \[ \logit \{P(A_2 = 1 | C = c, A_1 = a, Y_1 = y)\} = \beta_2 + \beta_3c' + \beta_4a + \beta_5y \]
- Longitudinal IPTW estimation
  \[ w(A_1 = a_1, A_2 = a_2) = \frac{P(A_1 = a_1)}{P(A_1 = a_1 | C = c)} \times \frac{P(A_2 = a_2 | A_1 = a_1)}{P(A_2 = a_2 | A_1 = a_1, C, Y_1)} \]
- \[ \sum_{i=1}^{N} w(A_1 = a_1, A_2 = a_2 | g, c) \cdot Y_i - \sum_{i=1}^{N} w(A_1 = a_1, A_2 = a_2 | g, c) \]
- Longitudinal TMLE presented for comparison

4. Real Data Analysis Results
- Head Start Impact Study (HSIS) data analysis
- Both GPS entropy balancing and longitudinal IPTW achieved covariate balance.

5. Discussion
- This study proposed two distinct ways of inverse probability weighting based on different conceptualization of longitudinal treatment regimes.
- Considering longitudinal Head Start as a multiple treatment produced smaller standard errors compared to the sequential treatment approach.
- However, time-varying covariates (e.g., Y_1) cannot be appropriately incorporated within the multiple treatment approach.
- Alternative estimators such as the LTMLE can help gain precision and double robustness.
- Further extensions may address partially or fully clustered data and evaluate the performances of multiple estimation methods with simulation studies.

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