

Power Analysis and Sample Size Planning in the Design of Two-Level Randomized Cost-Effectiveness Trials

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Modern Modeling Method

Outline of Talk

- **Background and Study Purpose**
- **(Briefly) Review of Basic Concepts and Methods**
 - Cost and cost-effectiveness analysis
 - Two-level designs, statistical analysis, and hypothesis testing
- **Power and minimum detectable effect size (MDES) computation**
- **Illustration – PowerUp!-CEA**
- **Discussion and Recommendations**

Experimental Design in Education Research

- **Gold standard for causal inference** (Imbens & Rubin, 2015)
 - Widely used in education for policy and program evaluations
 - Often involve nested data structures (e.g., students nested within schools)
 - Historically, educational researchers focused on assessing the effectiveness of educational interventions through multilevel randomized controlled trials (**MRCTs**)
 - Ignored the cost of achieving these effects
 - Recent discussions in education call for evaluating the cost as well as the effectiveness to facilitate better decision-making
 - Multilevel randomized cost-effectiveness trials (**MRCETs**)

Cost-Effectiveness Analysis

- **Policymakers and administrators usually request**
 - Achieve maximum effectiveness with a given budget
 - Attain a particular level of effectiveness at a minimal cost
- **Evaluations without a credible cost analysis can lead to misleading judgments**
 - Evaluation of class size reduction policy (e.g., Levin et al., 1987)
 - Evaluation of online teacher PD programs (e.g., Lay et al., 2020)
- **Major funding agencies in education started requiring that proposals for research include an economic evaluation**
 - Cost analysis (CA)
 - Cost-effectiveness analysis (CEA)

Motivation: Gaps in Literature and Practices

• **CEA Studies in Education**

- Usually collected cost data at the school level with a subsample
- Reported a descriptive measure (e.g., a cost-effectiveness ratio) without inference statistics such SEs or p-values
 - Limited discussion regarding how to conduct statistical inference for CEA in education

• **Statistical Power Analysis for MRCETs**

- Help researchers decide the sample size needed at each level to guarantee a good enough chance of detecting the effect of interest
- Literature in health science:
 - Do not distinguish the ingredients of costs at different levels
 - Do not consider covariates effects
 - Only consider random effect models and two-level designs
- Recent studies in education extended power analysis methods for MRCETs (e.g., Li, Dong, Maynard, 2020; Li et al., 2022)
 - Incorporate alternative design and analysis method

Purposes of This Study

- **There are three specific aims:**

1. Introduce the recent development in the design and analysis of MRCETs
 - a) How to conduct hypothesis testing for CEA
 - b) Sample size planning for the design of MRCETs
2. Demonstrate power analysis using a free and user-friendly tool – PowerUp!-CEA (Li et al., 2022)
3. Provide recommendations on sample size planning for MRCETs

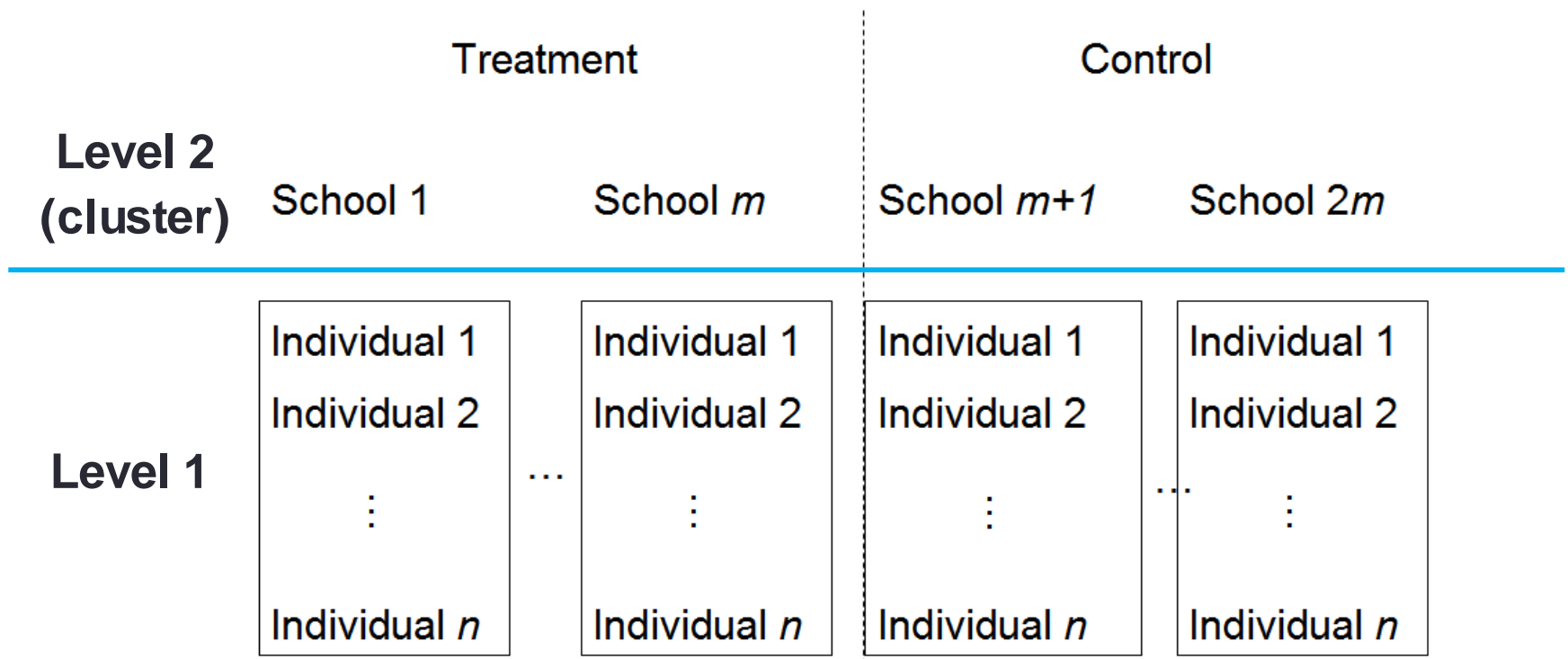
Ten Alternative Designs

Study Design	Label	Level of Clustering	Level of Treatment Assignment
Two-level Multisite Randomized Cost-Effectiveness Trials	Design 1	2	1
	Design 2	2	1
Two-level Cluster Randomized Cost-Effectiveness Trials	Design 3	2	2
	Design 4	2	2
Three-level Multisite Randomized Cost-Effectiveness Trials	Design 5	3	1
	Design 6	3	1
	Design 7	3	2
	Design 8	3	2
Three-level Cluster Randomized Cost-Effectiveness Trials	Design 9	3	3
	Design 10	3	3

Review of Basic Concepts: Two-Level Design and CEA

Cluster Randomized Design

- Schools are randomly assigned to one of two treatments, all students within schools receive the treatment:



Measures of CEA: INMB

- **Incremental Net Monetary Benefit (INMB)**

$$INMB = \kappa\Delta E - \Delta C \quad (1)$$

- ΔE : the incremental effect
- ΔC : the incremental cost
- κ : the monetary value that the decisionmaker assigns to a unit change in the outcome or their “willingness-to-pay (WTP)”
 - Both the effects and costs of interventions are scaled onto the same monetary scale through WTP
- Link the cost of implementing an intervention to its cost
- Advantages
 - Unbiased estimator
 - Easier interpretation: interventions with a positive INMB would always be deemed cost-effective
 - Facilitate statistical inference and power computation

Cluster Design: Two-Level Random Effect Model to Estimate the Incremental Effect

- **Level 1 (individual level):**

$$Y_{ij} = \beta_{0j} + r_{ij}, \quad r_{ij} \sim N(0, \sigma^2)$$

- **Level 2 (school level):**

$$\beta_{0j} = \gamma_{00} + \gamma_{01}T_j + u_{0j}, \quad u_{0j} \sim N(0, \tau^2)$$

- **Combined Model:**

$$Y_{ij} = \gamma_{00} + \gamma_{01}T_j + u_{0j} + r_{ij} \quad (2)$$

- T_j is the intervention indicator, γ_{01} represents the incremental effects (i.e., average treatment effect or ATE) on the effectiveness measures (e.g., test scores)

Cluster Design: Two-Level Random Effect Model to Estimate INMB

- J schools and n students within each school; School-level intervention

$$E_{ij} = \gamma_{00}^e + \boldsymbol{\gamma}_{A01}^e T_j + X_{ij}^e \Gamma_{10}^e + Z_j^e \Gamma_{02}^e + r_{A0j}^e + \varepsilon_{Aij}^e \quad (3)$$

$$C_{ij} = \gamma_{00}^c + \boldsymbol{\gamma}_{A01}^c T_j + X_{ij}^c \Gamma_{10}^c + Z_j^c \Gamma_{02}^c + r_{A0j}^c + \varepsilon_{Aij}^c \quad (4)$$

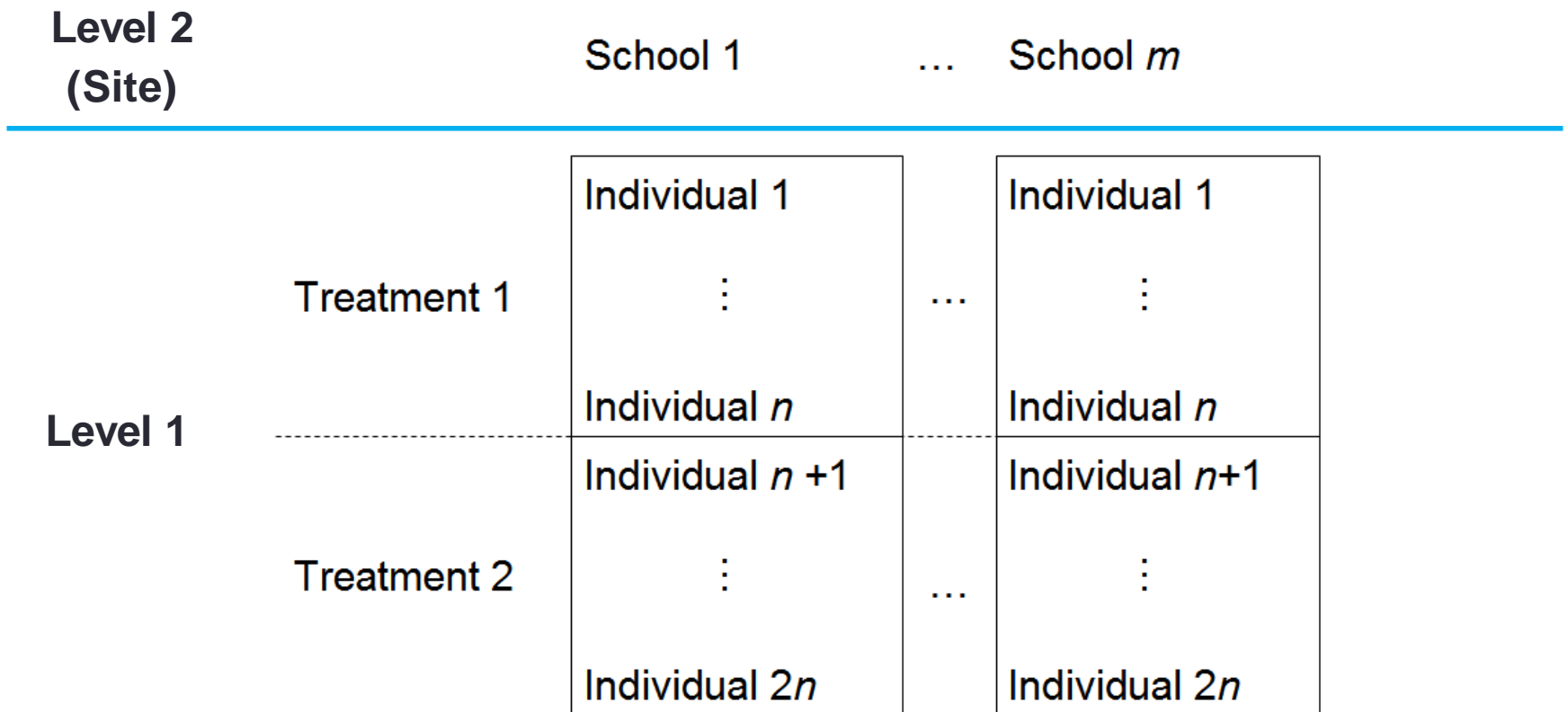
- The estimated γ_{A01}^e (denoted as $\hat{\gamma}_{A01}^e$) and γ_{A01}^c (denoted as $\hat{\gamma}_{A01}^c$) represent ΔE and ΔC , and then

$$\widehat{INMB} = \kappa \hat{\gamma}_{A01}^e - \hat{\gamma}_{A01}^c \quad (5)$$

$$\text{Var}(\widehat{INMB}) = \kappa^2 \times \text{Var}(\hat{\gamma}_{A01}^e) + \text{Var}(\hat{\gamma}_{A01}^c) - 2\kappa \times \text{Cov}(\hat{\gamma}_{A01}^e, \hat{\gamma}_{A01}^c) \quad (6)$$

Two-Level Multisite Design

- Individuals are randomly assigned to one of two treatments within their school:



Multisite Designs: Two-Level Random Effect Model to Estimate INMB

- J schools and n students within each school; Student-level intervention

$$E_{ij} = \gamma_{00}^e + \boldsymbol{\gamma}_{A10}^e T_{ij} + X_{ij}^e \Gamma_{20}^e + W_j^e \Gamma_{01}^e + W_j^e \Gamma_{11}^e T_{ij} + u_{A0j}^e + u_{A1j}^e T_{ij} + \varepsilon_{Aij}^e \quad (7)$$

$$C_{ij} = \gamma_{00}^c + \boldsymbol{\gamma}_{A10}^c T_{ij} + X_{ij}^c \Gamma_{20}^c + W_j^c \Gamma_{01}^c + W_j^c \Gamma_{11}^c T_{ij} + u_{A0j}^c + u_{A1j}^c T_{ij} + \varepsilon_{Aij}^c \quad (8)$$

- Note that W_j^e and W_j^c are grand-mean centered
- The estimated γ_{A10}^e (denoted as $\hat{\gamma}_{A10}^e$) and γ_{A10}^c (denoted as $\hat{\gamma}_{A10}^c$) represent ΔE and ΔC , and then

$$\widehat{INMB} = \kappa \hat{\gamma}_{A10}^e - \hat{\gamma}_{A10}^c \quad (9)$$

$$\text{Var}(\widehat{INMB}) = \kappa^2 \times \text{var}(\hat{\gamma}_{A10}^e) + \text{var}(\hat{\gamma}_{A10}^c) - 2\kappa \times \text{Cov}(\hat{\gamma}_{A10}^e, \hat{\gamma}_{A10}^c) \quad (10)$$

Multisite Designs: Two-Level Constant/Fixed Effect Model to Estimate INMB

- **Constant Effect Model**

$$E_{ij} = \boldsymbol{\gamma}_1^e T_{ij} + X_{ij}^e \Gamma_3^e + \sum_{k=1}^J a_k^e \text{Site}_{k,ij} + \varepsilon_{Aij}^e$$
$$C_{ij} = \boldsymbol{\gamma}_1^c T_{ij} + X_{ij}^c \Gamma_3^c + \sum_{k=1}^J a_k^c \text{Site}_{k,ij} + \varepsilon_{Aij}^c$$

- **Fixed Effect Model**

$$E_{ij} = \boldsymbol{\gamma}_1^e T_{ij} + X_{ij}^e \Gamma_3^e + \sum_{k=1}^J a_k^e \text{Site}_{k,ij} + \sum_{k=1}^J \boldsymbol{b}_k^e \text{Site}_{k,ij} T_{ij} + \varepsilon_{Aij}^e$$
$$C_{ij} = \boldsymbol{\gamma}_1^c T_{ij} + X_{ij}^c \Gamma_3^c + \sum_{k=1}^J a_k^c \text{Site}_{k,ij} + \sum_{k=1}^J \boldsymbol{b}_k^c \text{Site}_{k,ij} T_{ij} + \varepsilon_{Aij}^c$$

- Note that Site dummy variables are grand-mean centered

Power and MDES Computation

Power Analysis

- We can test whether $\widehat{INMB} = 0$ using a t -test. Assuming the alternative hypothesis is true, the test statistic follows a non-central t -distribution, T' , with a non-centrality parameter:

$$\lambda = \frac{INMB}{\sqrt{Var(INMB)}} \quad (11)$$

- Under these specifications, the statistical power of a two-tailed test is (note $t_0 = t_{1-\frac{\alpha}{2}, df}$)

$$Power = 1 - P[T'(df, \lambda) < t_0] + P[T'(df, \lambda) \leq -t_0] \quad (12)$$

- MDES - the smallest true effect that has a good enough chance of being detected to be statistically significant

$$MDES = M_v * \sqrt{Var(INMB)} \quad (13)$$

- M_v - the sum of two t statistics (Bloom, 1995). For two-tailed tests, which are usually applied, $M_v = t_{\alpha/2} + t_{1-\beta}$, where α represents the Type I error and β represents the Type II error for the tests

Table 1. Summary of the Standardized Noncentrality Parameter, MDES, and Degrees of Freedom

Model Name	Models	Standardized Noncentrality Parameter (λ) and MDES	Degrees of Freedom
Cluster Design	$E_{ij} = \gamma_{00}^e + \gamma_{A01}^e T_{ij} + X_{ij}^e \Gamma_{10}^e + Z_j^e \Gamma_{02}^e + r_{A0j}^e + \varepsilon_{Aij}^e,$ $C_{ij} = \gamma_{00}^c + \gamma_{A01}^c T_{ij} + X_{ij}^c \Gamma_{10}^c + Z_j^c \Gamma_{02}^c + r_{A0j}^c + \varepsilon_{Aij}^c.$	<p>Standardized Noncentrality Parameter (λ):</p> $INMB \sqrt{\frac{P(1-P)nj}{\kappa^2[(nw_2^e - w_1^e)\rho_e] + \psi_c[(nw_2^c - w_1^c)\rho_c] + (\kappa^2 w_1^e + \psi_c w_1^c) - 2\kappa\sqrt{\psi_c}(nw_2^{ec}r_2 + w_1^{ec}r_1)}}$ <p>MDES:</p> $\frac{M_{J-2-q}}{\sqrt{P(1-P)nj}} \sqrt{\kappa^2[(nw_2^e - w_1^e)\rho^e] + \psi_c[(nw_2^c - w_1^c)\rho^c] + (\kappa^2 w_1^e + \psi_c w_1^c) - 2\kappa\sqrt{\psi_c}(nw_2^{ec}r_2 + w_1^{ec}r_1)}$	$J-2-q$
Multisite Design: Random Effect Model	$E_{ij} = \gamma_{00}^e + \gamma_{A10}^e T_{ij} + X_{ij}^e \Gamma_{20}^e + W_j^e \Gamma_{01}^e + W_j^e \Gamma_{11}^e T_{ij} + u_{A0j}^e + u_{A1j}^e T_{ij} + \varepsilon_{Aij}^e,$ $C_{ij} = \gamma_{00}^c + \gamma_{A10}^c T_{ij} + X_{ij}^c \Gamma_{20}^c + W_j^c \Gamma_{01}^c + W_j^c \Gamma_{11}^c T_{ij} + u_{A0j}^c + u_{A1j}^c T_{ij} + \varepsilon_{Aij}^c.$	<p>Standardized Noncentrality Parameter (λ):</p> $INMB \sqrt{\frac{Jn}{\kappa^2(n\eta_e w_2^e - \phi w_1^e)\rho_e + \psi_c(n\eta_c w_2^c - \phi w_1^c)\rho_c + \phi(\kappa^2 w_1^e + \psi_c w_1^c) - 2\kappa\sqrt{\psi_c}(n\eta_{ec}r_2 w_2^{ec} + \phi r_1 w_1^{ec})}}$ <p>MDES:</p> $\frac{M_{J-1-q}}{\sqrt{Jn}} \sqrt{\kappa^2(n\eta_e w_2^e - \phi w_1^e)\rho_e + \psi_c(n\eta_c w_2^c - \phi w_1^c)\rho_c + \phi(\kappa^2 w_1^e + \psi_c w_1^c) - 2\kappa\sqrt{\psi_c}(n\eta_{ec}r_2 w_2^{ec} + \phi r_1 w_1^{ec})}$	$J-1-q$
Multisite Design: Constant Effect Model	$E_{ij} = \gamma_1^e T_{ij} + X_{ij}^e \Gamma_3^e + \sum_{k=1}^J a_k^e Site_{k,ij} + \varepsilon_{Aij}^e,$ $C_{ij} = \gamma_1^c T_{ij} + X_{ij}^c \Gamma_3^c + \sum_{k=1}^J a_k^c Site_{k,ij} + \varepsilon_{Aij}^c.$	<p>Standardized Noncentrality Parameter (λ):</p> $INMB \sqrt{\frac{P(1-P)Jn}{\kappa^2 w_1^e(1-\rho_e) + \psi_c(1-\rho_c) - 2\kappa r_1 w_1^{ec} \sqrt{\psi_c}}}$ <p>MDES:</p> $\frac{M_{J(n-1)-1-q}}{\sqrt{P(1-P)nj}} \sqrt{\kappa^2 w_1^e(1-\rho_e) + \psi_c(1-\rho_c) - 2\kappa r_1 w_1^{ec} \sqrt{\psi_c}}$	$J(n-1)-1-q$
Multisite Design: Fixed Effect Model	$E_{ij} = \gamma_1^e T_{ij} + X_{ij}^e \Gamma_3^e + \sum_{k=1}^J a_k^e Site_{k,ij} + \sum_{k=1}^J b_k^e Site_{k,ij} T_{ij} + \varepsilon_{Aij}^e,$ $C_{ij} = \gamma_1^c T_{ij} + X_{ij}^c \Gamma_3^c + \sum_{k=1}^J a_k^c Site_{k,ij} + \sum_{k=1}^J b_k^c Site_{k,ij} T_{ij} + \varepsilon_{Aij}^c.$	<p>Standardized Noncentrality Parameter (λ):</p> $INMB \sqrt{\frac{P(1-P)Jn}{\kappa^2 w_1^e(1-\rho_e) + \psi_c(1-\rho_c) - 2\kappa r_1 w_1^{ec} \sqrt{\psi_c}}}$ <p>MDES:</p> $\frac{M_{J(n-2)-1-q}}{\sqrt{P(1-P)nj}} \sqrt{\kappa^2 w_1^e(1-\rho_e) + \psi_c(1-\rho_c) - 2\kappa r_1 w_1^{ec} \sqrt{\psi_c}}$	$J(n-2)-1-q$

Note: (1) q is the number of level-2 covariates. (2) $\phi = \frac{1}{P(1-P)}$.

Design Parameters for Power Computation

- Adjustments common for all effectiveness studies:
 - Minimum relevant effect size
 - Sample size & allocation
 - Type I error
 - Nesting effects on outcome measures (i.e., ICCs)
 - Covariate adjustments
- Adjustments unique to MRCETs
 - Ratio of total variances of cost data & of effectiveness data
 - Nesting effects (i.e., ICCs) of cost data
 - Correlation between cost measures & effectiveness measures
 - Level of assignment & level of analysis

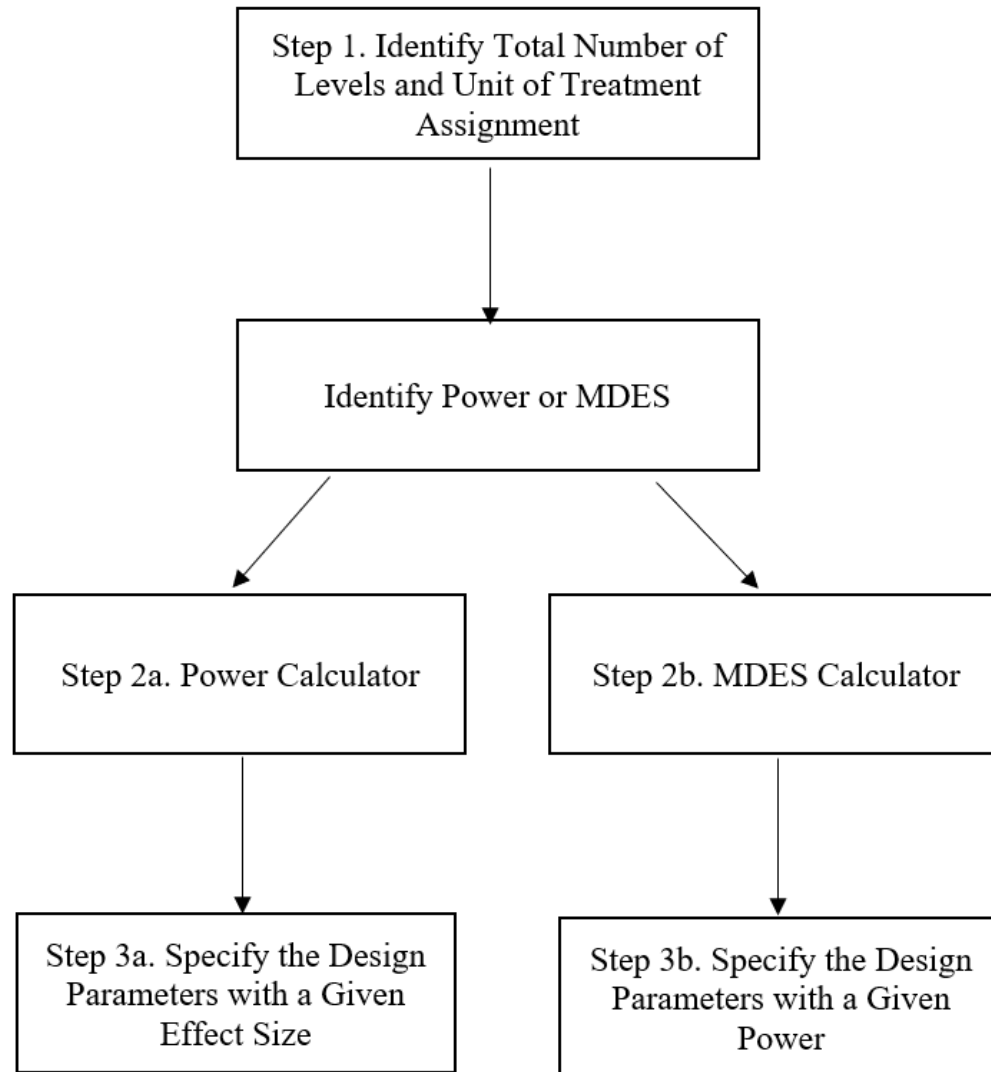
Illustration

Software

- **PowerUp!-CEA** (Li et al., 2022)
 - Free available from <https://www.causalevaluation.org/multilevel-cost-effectiveness-trials.html>
 - Compute power and MDES for two and three-level designs

Study Design	1	2	3	4	5	6	7
	Model Number	Number of Total Levels of Clustering	Unit of Treatment Assignment	Treatment Assignment Level	Cluster/Block Effect	Worksheet Name for:	
						Power Calculation	MDES Calculation
Multisite Random Assignment	1.1	2	Individual	1	Random	MIRA2_1r_Power	MIRA2_1r_MDES
	1.2	2	Individual	1	Cosntant	MIRA2_1c_Power	MIRA2_1c_MDES
	1.3	2	Individual	1	Fixed	MIRA2_1f_Power	MIRA2_1f_MDES
Cluster Random Assignment	2.1	2	Cluster	2	Random	CRA2_2r_Power	CRA2_2r_MDES
	2.2	3	Cluster	3	Random	CRA3_3r_Power	CRA3_3r_MDES

A Three-Step Process



Demonstration of Power Computation for Two-level Cluster Designs

Model 2.1: Power Calculator for Two-Level Cluster Random Assignment Design (CRA2_2)— Treatment at Level 2		
Assumptions		Comments
Alpha Level (α)	0.05	Probability of a Type I error
Two-tailed or One-tailed Test?	2	
Effect Size Difference	0.50	INMB standardized by the standard deviation of effectiveness data
Willingness to Pay (κ)	2.00	
P	0.50	Proportion of Level 2 units randomized to treatment: J_T / J
ψ_c / ψ_e	0.50	Ratio of the total variance of cost data to the total variance of effectiveness data
Parameters for Effectiveness Data		
ρ^e	0.23	Proportion of variance in effectiveness measures that is between clusters
R_{1e}^2	0.50	Proportion of variance of effectiveness data explained by level-1 covariates
R_{2e}^2	0.50	Proportion of variance of effectiveness data explained by level-2 covariates
Parameters for Cost Data		
ρ^c	0.23	Proportion of variance in cost measures that is between clusters
R_{1c}^2	0.50	Proportion of variance of cost data explained by level-1 covariates
R_{2c}^2	0.50	Proportion of variance of cost data explained by level-2 covariates
Parameters for Covariation between Effectiveness Data and Cost Data		
r_1	0.10	Standardized covariance between the effectiveness data and cost data at level-1
r_2	0.10	Standardized covariance between the effectiveness data and cost data at level-2
R_{1ec}^2	0.50	Proportion of the covariance explained by level-1 covariates
R_{2ec}^2	0.50	Proportion of the covariance explained by level-2 covariates
g^*	1	Number of Level 2 covariates
n (Average Cluster Size)	50	Mean number of Level 1 units per Level 2 cluster (geometric mean recommended)
J (Sample Size [# of Clusters])	60	Number of Level 2 units
$corr_1$	0.13	Correlation between the effectiveness data and cost data at level-1
$corr_2$	0.43	Correlation between the effectiveness data and cost data at level-2
Noncentrality Parameter	3.03	Automatically computed from the above assumptions
Power (1-β)	0.846	Statistical power (1-probability of a Type II error)

Note: (1)The parameters in yellow cells need to be specified. The power will be calculated automatically. (2) We always assume the effectiveness data are standardized with mean zero and standard deviation one.

Demonstration of MDES Computation for Two-level Multisite RCETs: Random Effect Model

Model 1.1: MDES Calculator for 2-Level Multisite Individual Random Assignment (MIRA2_1r) Designs— Random Effect Model		
Assumptions		Comments
Alpha Level (α)	0.05	Probability of a Type I error
Two-tailed or One-tailed Test?	2	
Power (1- β)	0.80	Statistical power (1-probability of a Type II error)
Willingness to Pay (K)	2.00	
P	0.50	Proportion of Level 1 units randomized to treatment: n_T / n
ϕ	4.00	$\phi = 1/P(1 - P)$
ψ_c / ψ_e	0.50	Ratio of the total variance of cost data to the total variance of effectiveness data
Parameters for Effectiveness data		
ρ^e	0.23	Proportion of variance in effectiveness measures that is between clusters
R_{1e}^2	0.50	Proportion of variance of effectiveness data explained by level-1 covariates
R_{2e}^2	0.50	Proportion of variance of effectiveness data explained by level-2 covariates
η_e	0.30	Proportions of the treatment by site variances to the total variance of effectiveness data at level-2
Parameters for Cost Data		
ρ^c	0.23	Proportion of variance in cost measures that is between clusters
R_{1c}^2	0.50	Proportion of variance of cost data explained by level-1 covariates
R_{2c}^2	0.50	Proportion of variance of cost data explained by level-2 covariates
η_c	0.30	Proportions of the treatment by site variances to the total variance of cost data at level-2
Parameters for Covariation between Effectiveness data and Cost Data		
r_1	0.10	Standardized covariance between the effectiveness data and cost data at level-1
r_2	0.10	Standardized covariance between the effectiveness data and cost data at level-2
η_{ec}	0.23	Proportions of the treatment by site covariances to the total covariance at level-2
R_{1ec}^2	0.50	Proportion of the covariance explained by level-1 covariates
R_{2ec}^2	0.50	Proportion of the covariance explained by level-2 covariates
g^*	1	Number of Level 2 covariates
n (Average Cluster Size)	60	Mean number of Level 1 units per Level 2 cluster (geometric mean recommended)
J (Sample Size [# of Clusters])	50	Number of Level 2 units
$corr_1$	0.13	Correlation between the effectiveness data and cost data at level-1
$corr_2$	0.43	Correlation between the effectiveness data and cost data at level-2
M (Multiplier)	2.86	Computed from T_1 and T_2
T_1 (Precision)	2.01	Determined from alpha level, given two-tailed or one-tailed test
T_2 (Power)	0.85	Determined from given power level
MDES	0.193	Minimum Detectable Effect Size Standardized by the Standard Deviation of the Effectiveness Data

Note: (1) The parameters in yellow cells need to be specified. The MDES will be calculated automatically. (2) We always assume the effectiveness data are standardized with mean zero and standard deviation one.

Discussion and Recommendations

Comparisons between Power Analyses for MRCTs and MRCETs

- **Multilevel randomized controlled trials (MRCTs)**
 - Measure of interests : $ATE = \Delta E \neq 0$
 - Power analysis only considers the variance of the effectiveness measure
- **Multilevel randomized cost-effectiveness trials (MRCETs)**
 - Measure of interest - $INMB = \kappa\Delta E - \Delta C \neq 0$
 - Power analysis considers both the variance of the effectiveness measure and the cost measure and their covariance
- **In general, the power for MRCETs is smaller with the same design parameters**
 - If the intervention could save cost or when cost and effectiveness are positively correlated, the power for MRCTs might be smaller

Recommendations (1)

- Account for cost variation and the nested structure of cost data when planning and analyzing MRCETs
- Including covariates adjustment is crucial for increasing power, and the covariates at higher-level have a larger impact on power than those at the first level
- Other things being equal, the power of a multisite design is larger than that of a cluster design
 - Power for random effect models is smaller than those from constant/fixed-effect models
 - Power is the same for constant and fixed effect models
- Balanced design is preferred when the budget for sampling treatment and control units is similar

Recommendations (2)

- When the budget for cost data collection is limited
 - Consider an unbalanced design, especially when the cost of sampling a control unit is smaller
 - Collect cost data at the cluster level
- Select appropriate references values of design parameters
 - Limited information regarding the benchmarks for cost data variation, ICC, correlations between cost and effectiveness measures
 - Identify a lower bound based on the plausible values of the design parameters

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Questions or Comments?

Thank you!

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<https://www.causalevaluation.org/multilevel-cost-effectiveness-trials.html>