Confidence Interval of Effect Size in Longitudinal Growth Models

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Abstract

When modeling longitudinal data, longitudinal growth models (LGMs) are commonly used. In this study, we examine the performance of the bootstrap confidence intervals (CIs) for R^2 measures in LGMs. Two bootstrap methods, parametric and non-parametric residual based bootstrapping are used. With each bootstrap method, the CIs are computed using four different ways: normal, basic, percentile, and BCa. Preliminary results indicate that the parametric bootstrap method performs better for all types of CIs, especially with a small sample size. A larger sample size leads to a higher 95% CI coverage rate and narrower width for both bootstrap methods. Logit-transformation improved the coverage rate for normal CI and basic CI.

Background

Longitudinal Growth Models

Level-1:

$$y_{ti} = \beta_{0i} + \beta_{1i}Time_{ti} + e_{ti} \quad e_{ti} \sim N(0, \sigma^2)$$

Level-2:

$$\beta_{0i} = \gamma_{00} + U_{0i} \qquad \begin{pmatrix} U_{0i} \\ U_{1i} \end{pmatrix} \sim MVN(\mathbf{0}, \mathbf{T})$$

$$\beta_{1i} = \gamma_{10} + U_{1i} \qquad \mathbf{T} = \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}$$

R^2 computation in LGMs

- Model-implied Variance:

$$R^2 = \frac{\text{Variance explained by the effect of interest}}{\text{Total Variance}}$$

- R^2 computation:

$$R^2 = \frac{\boldsymbol{\gamma}'(\mathbf{D}_f\boldsymbol{\Sigma}\mathbf{D}_f)\boldsymbol{\gamma} + \mathrm{tr}\big((\mathbf{D}_r\mathbf{T}\mathbf{D}_r)\boldsymbol{\Sigma}_r\big) + \boldsymbol{\mu}'(\mathbf{D}_r\mathbf{T}\mathbf{D}_r)\boldsymbol{\mu}}{\boldsymbol{\gamma}'\boldsymbol{\Sigma}\boldsymbol{\gamma} + \mathrm{tr}(\mathbf{T}\boldsymbol{\Sigma}_r) + \boldsymbol{\mu}'\mathbf{T}\boldsymbol{\mu} + \sigma^2}$$

Method

Table 1: Research Design

Bootstrap methods	Confidence interval computing methods	Transformation methods
Non-parametric residual-based bootstrapping (Carpenter et al., 1999)	Normal CI (Davison & Hinkley, 1997)	Original (no transformation)
Parametric residual-based bootstrapping (Goldstein, 2010)	Basic CI (Davison & Hinkley, 1997)	Logit
	Percentile CI (Efron, 1982)	Log
	The bias-corrected and accelerated CI (Efron, 1987)	
	Studentized CI (Davison & Hinkley, 1997)	

Data simulation conditions					Total
Effect size	Intra-class correlation (ICC)	Unit size (# of time points)	Cluster size (# of individuals)	Level-1 residual covariance:	3 × 2 × 2 × 3 = 36
0.1, 0.3, 0.7	0.2, 0.5	Balanced: 4, 6	50, 100, 300	Normal	1000 data replications
		Unbalanced: Sample attrition with 10%, 20%		Diagonal, CS, AR(1), CSH, ARH(1)	1000 bootstrap replications

Note: CS=Compound Symmetry, AR(1)=Homogeneous First-order Autoregressive, CSH=Heterogeneous Compound Symmetry, ARH(1)=Heterogeneous First-order Autoregressive

Results

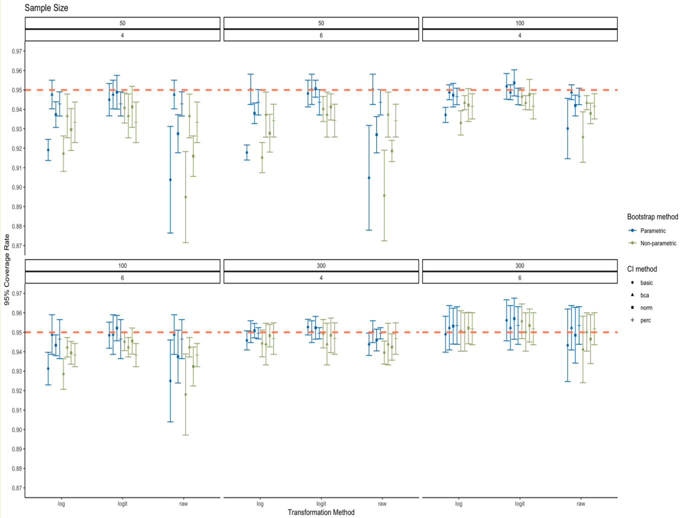

Figure 1: 95% CI coverage rate across sample size

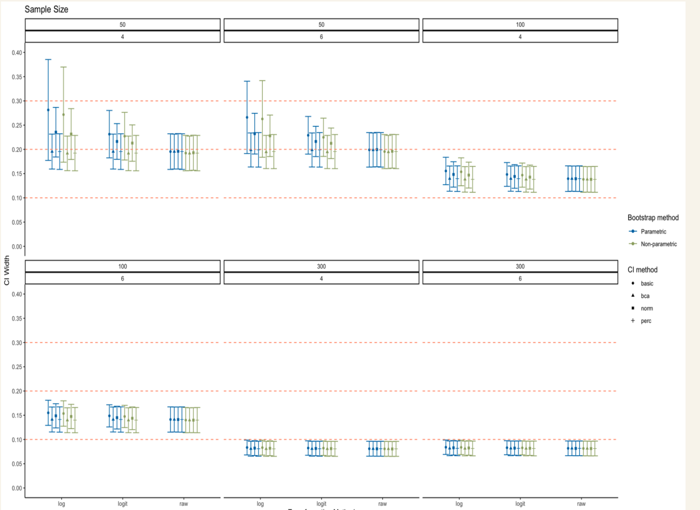

Figure 2: 95% CI width across sample size

Conclusion

Sample size

With a small sample size, the non-parametric bootstrap produces low 95% CI coverage rate for all types CIs

A larger sample size leads to a higher 95% CI coverage rate and narrower CI width for both parametric and non-parametric bootstrap methods

Transformation

Logit transformation will increase the coverage rate for normal CI and basic CI

Bootstrap method

The parametric bootstrap method produces a higher 95% CI coverage rate for all types CIs

ICC level and effect size level

Higher ICC and higher effect size lead to a higher 95% CI coverage rate for both parametric and non-parametric bootstrap CIs.