Confidence Interval of Effect Size in Longitudinal Growth Models  
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Abstract
When modeling longitudinal data, longitudinal growth models (LGMs) are commonly used. In this study, we examine the performance of the bootstrap confidence intervals (CIs) for $R^2$ measures in LGMs. Two bootstrap methods, parametric and non-parametric residual based bootstrapping are used. With each bootstrap method, the CIs are computed using four different ways: normal, basic, percentile, and BCa. Preliminary results indicate that the parametric bootstrap method performs better for all types of CIs, especially with a small sample size. A larger sample size leads to a higher 95% CI coverage rate and narrower width for both bootstrap methods. Logit transformation improved the coverage rate for normal CI and basic CI.

Background
Longitudinal Growth Models

Level-1:
$$y_{ti} = \beta_{0i} + \beta_{1i} \text{Time}_{ti} + e_{ti} \sim N(0, \sigma^2)$$

Level-2:
$$\begin{align*}
\beta_{0i} &= \gamma_{00} + U_{0i} \\
\beta_{1i} &= \gamma_{10} + U_{1i} \\
U_{1i} &\sim MVN(0, \mathbf{T}) \\
\mathbf{T} &= \begin{pmatrix} \tau_{00} & \tau_{01} \\ \tau_{10} & \tau_{11} \end{pmatrix}
\end{align*}$$

$R^2$ computation in LGMs

- Model-implied Variance:
  $$R^2 = \frac{\text{Variance explained by the effect of interest}}{\text{Total Variance}}$$

- $R^2$ computation:
  $$R^2 = \frac{\gamma'(\mathbf{D}_i \Sigma_i \mathbf{D}_i) \gamma + \text{tr}(\mathbf{D}_i \Sigma_i \mathbf{T}_i \mathbf{D}_i) + \mu'(\mathbf{D}_i \Sigma_i \mathbf{D}_i) \mu}{\gamma' \Sigma \gamma + \text{tr}(\Sigma \mathbf{T}) + \mu' \Sigma \mu + \sigma^2}$$

Results

Method

<table>
<thead>
<tr>
<th>Bootstrap methods</th>
<th>Confidence interval computing methods</th>
<th>Transformation methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Non-parametric residual-based bootstrapping (Carpenter et al., 1999)</td>
<td>Normal CI (Davison &amp; Hinkley, 1997)</td>
<td>Original (no transformation)</td>
</tr>
<tr>
<td>Parametric residual-based bootstrapping (Goldstein, 2010)</td>
<td>Basic CI (Davison &amp; Hinkley, 1997)</td>
<td>Logit</td>
</tr>
<tr>
<td></td>
<td>Percentile CI (Efron, 1982)</td>
<td>Log</td>
</tr>
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<td></td>
<td>The bias-corrected and accelerated CI (Efron, 1987)</td>
<td>Studentized CI (Davison &amp; Hinkley, 1997)</td>
</tr>
</tbody>
</table>

Data simulation conditions

<table>
<thead>
<tr>
<th>Effect size</th>
<th>Intra-class correlation (ICC)</th>
<th>Unit size (# of time points)</th>
<th>Cluster size (# of individuals)</th>
<th>Level-1 residual covariance:</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1, 0.3, 0.7</td>
<td>0.2, 0.5</td>
<td>Balanced: 4, 6</td>
<td>50, 100, 300</td>
<td>Normal</td>
<td>3 × 2 × 2 × 3 = 36</td>
</tr>
<tr>
<td></td>
<td>Unbalanced: Sample attrition with 10%, 20%</td>
<td></td>
<td></td>
<td>Diagonal, CS, AR(1), CSH, ARH(1)</td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Research Design

Note: CS=Compound Symmetry, AR(1)=Homogeneous First-order Autoregressive, CSH=Heterogeneous Compound Symmetry, ARH(1)=Heterogeneous First-order Autoregressive

Conclusion

Sample size
With a small sample size, the non-parametric bootstrap produces low 95% CI coverage rate for all types CIs. A larger sample size leads to a higher 95% CI coverage rate and narrower CI width for both parametric and non-parametric bootstrap methods.

Transformation
Logit transformation will increase the coverage rate for normal CI and basic CI.

Bootstrap method
The parametric bootstrap method produces a higher 95% CI coverage rate for all types CIs.

ICC level and effect size level
Higher ICC and higher effect size lead to a higher 95% CI coverage rate for both parametric and non-parametric bootstrap CIs.

Figure 1: 95% CI coverage rate across sample size

Figure 2: 95% CI width across sample size