Summing the up and downs of life: The Bayesian Reservoir Model of Psychological Regulation

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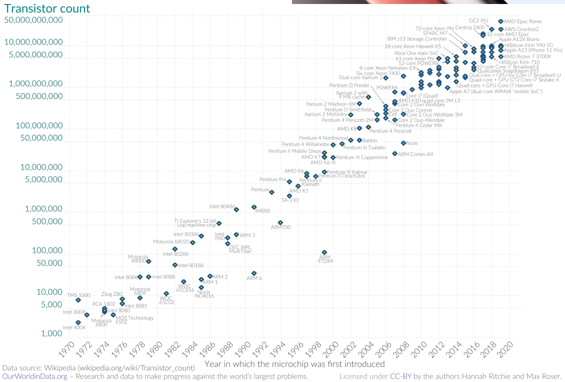
July 5, 2023

On a scale of 1-10
how was your mood
today?
8
1
10
lowest
highest
Done



Transistor count
50,000,000,000
10,000,000,000
5,000,000,000
1,000,000,000
500,000,000
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50,000,000
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500,000
100,000
50,000
10,000
5,000
1,000
1970 1972 1974 1976 1978 1980 1982 1984 1986 1988 1990 1992 1994 1996 1998 2000 2002 2004 2006 2008 2010 2012 2014 2016 2018 2020
Year in which the microchip was first introduced
Data source: Wikipedia (wikipedia.org/wiki/Transistor_count)
OurWorldinData.org – Research and data to make progress against the world's largest problems.
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Outline

1. Background
   - Modeling dynamics & dynamical systems theory overview
   - Dynamical systems theory in the social and behavioral sciences
   - Potential directions in the study of dynamics
2. Introduce the Reservoir Model
3. Bayesian Reservoir Model: Simulation 1
4. Substantive application of the Bayesian Reservoir Model
5. Multi-level Bayesian Reservoir Model: Simulation 2
6. Conclusions & Future Directions

- Dynamic systems: [1]
  - One or more components
  - Frequent (often reversible) changes over short time periods (as opposed to growth processes[2])
  - Dynamical systems theory: mathematical modeling of dynamic systems [1]
  - In psychology this often takes the form of difference/differential equation models [3]
    - differential equation: $\frac{dx}{dt} = Ax + G\frac{dW}{dt}$
    - difference equations: $x_{t+dt} = (A + 1)x_t + \epsilon$

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[1] Smith & Thelen, 2003; Thelen & Smith, 2006
[2] Nesselroade, 1991
[3] Kaplan & Glass, 1997; Boker, 2012

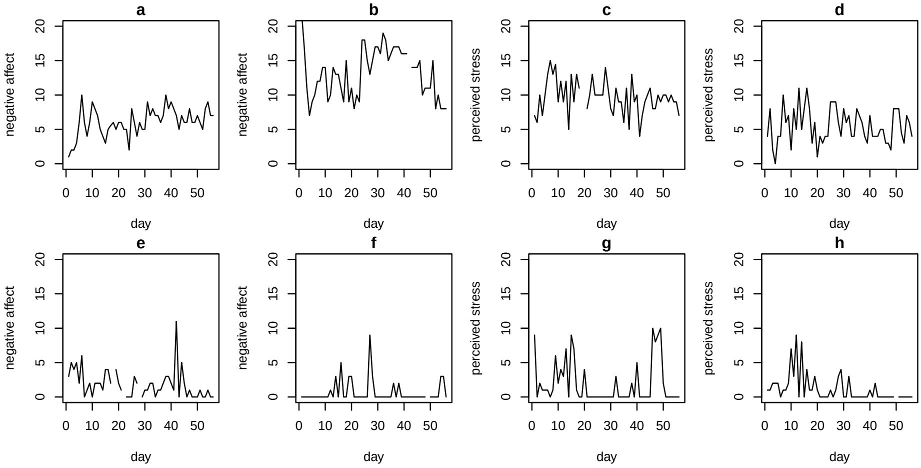
- Difference & differential equation applications in psychology
  - Time series analysis [4]
  - Cross-lagged panel models [5]
  - Continuous time models [6]
  - Dynamic Structural Equation Modeling (DSEM) [7]
  - Damped linear oscillator model [8]
  - Cusp-catastrophe model [9]
  - Among other variations

[4] Box et al., 2015; Shumway, Stoffer, & Stoffer, 2000
[5] Hamaker, Kuiper, & Grasman, 2015; Newsom, 2015
[6] Deboeck & Preacher, 2016; Oud & Jansen, 2000; Voelkle et al., 2012; Voelkle & Oud, 2015
[7] Asparouhov, Hamaker, & Muthén, 2018
[8] Montpetit et al., 2010; Boker & Laurenceau, 2006; Nicholson et al., 2011
[9] Chow, Witkiewitz, Grasman, & Maisto, 2015; Oliva & McDade, 2008

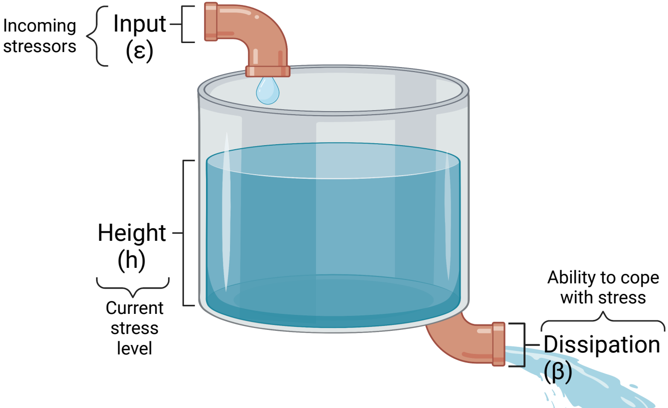
- Although there are several variations of these models
  - The range of dynamics explored is relatively limited
  - Especially given the wide range of processes studied in the social and behavioral sciences (i.e. perception, child development, suicide ideation, romantic partnerships, implicit bias, social support)
- The dynamical systems literature more broadly highlights the wide range of models needed to describe the rich variety of dynamic systems
  - Thus, a wider variety of models may be needed to match the wide range of processes studied in the social and behavioral sciences

- There is increasing acknowledgement that one-size-fits all approaches have substantial drawbacks [10]
  - Accounting for individual differences and providing individualized estimates is becoming increasingly popular [11]
  - Yet, "individualizing" or tailoring models to the expected dynamics of particular processes is less common

[10] Molenaar, 2004; Nesselroade & Ram, 2004
[11] Hoffman & Rovine, 2007; Nesselroade & Ram, 2004

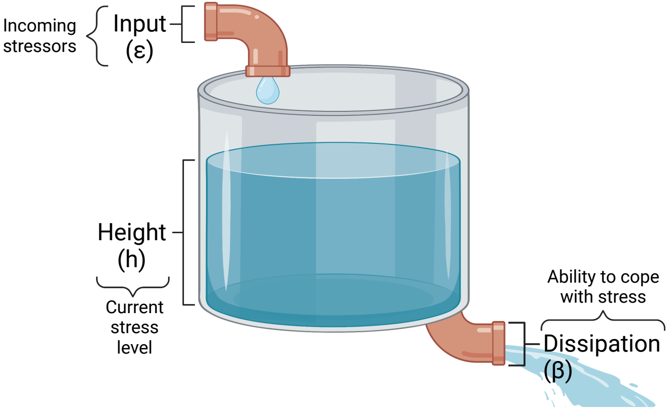


a
b
c
d
e
f
g
h
negative affect
perceived stress
day
0
5
10
15
20
0
10
20
30
40
50



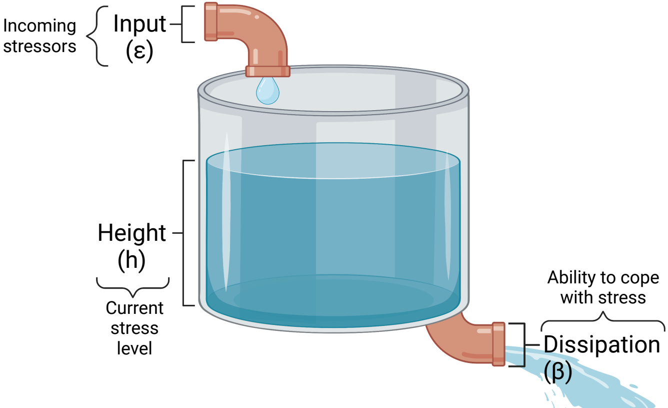
$$1).\ \frac{dh}{dt} = -\beta h + \epsilon$$

[10] Deboeck & Bergeman, 2013



1). $\frac{dh}{dt} = -\beta h + \epsilon$

[10] Deboeck & Bergeman, 2013



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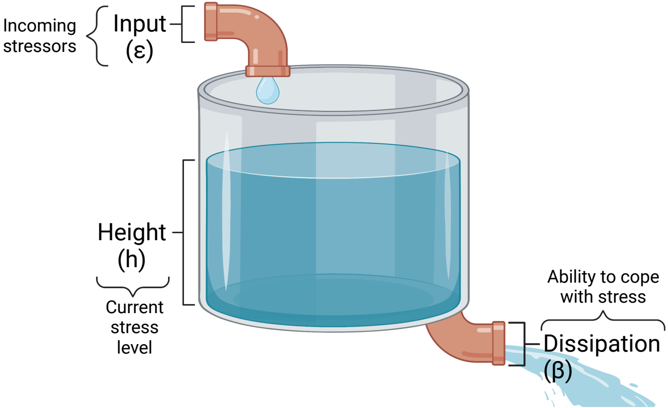

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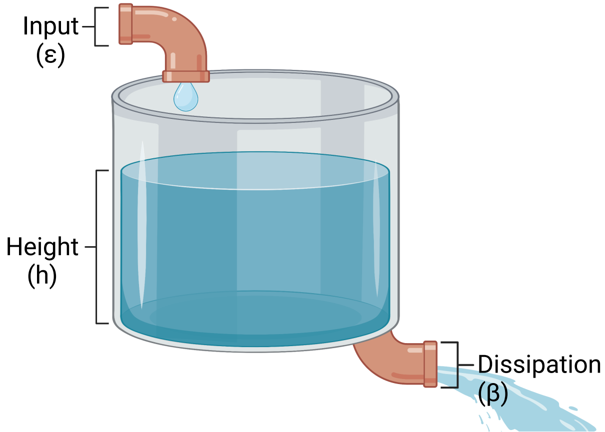


$$h_t = h_{t-1} - (\beta)(h_{t-1}) + \epsilon_t$$

[10] Deboeck & Bergeman, 2013

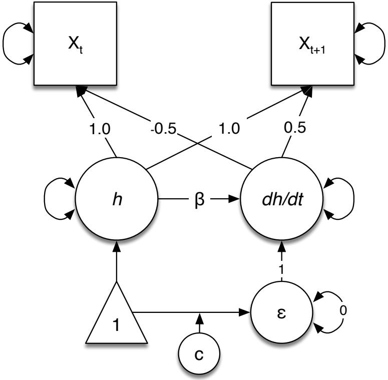


Figure: SEM model of the Reservoir Model. The variable dh/dt is regressed onto h, both of which are estimated using Latent Differential Equation Modeling. The distribution of errors ϵ makes use of Latent Distribution Modeling to produce a distribution of all-positive values. For each class c the mean of ϵ consists of a value greater or equal to zero; variations in the probability of the classes allow for different, all-positive distributions.

$$h_t = h_{t-1} - (\beta)(h_{t-1}) + \epsilon_t \quad (1)$$

$$x_t \sim N(h_t, \sigma) \quad (2)$$

Priors:

$$\beta \sim Exp(\lambda_{\beta,prior}) \text{ *bounded from 0-2} \quad (3)$$

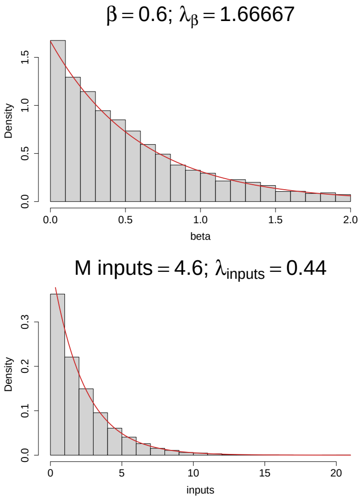
$$M_{inputs} \sim Exp(\lambda_{MeanInputs,prior})$$

$$1/\sigma^2 \sim Exp(\lambda_{Precision,prior}).$$

Note: M_{inputs} = mean ϵ distribution

$$\begin{aligned} \beta &\sim Exp(\lambda_{\beta,prior}) \text{ *bounded from 0-2} \\ M_{inputs} &\sim Exp(\lambda_{MeanInputs,prior}) \\ 1/\sigma^2 &\sim Exp(\lambda_{Precision,prior}). \end{aligned}$$

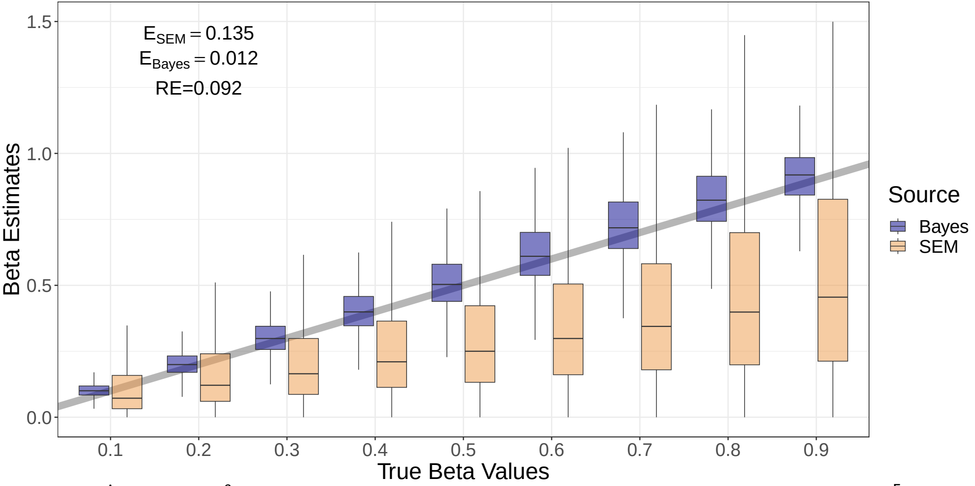
- $\lambda_{\beta,prior}$: $(\beta \text{ value})^{-1}$ where $dh/dt + (\beta)(h) \geq 0$
- $\lambda_{MeanInputs,prior} = (\frac{1}{2} * M(\text{value increase observations}))^{-1}$
- $1/\sigma^2$ was set to assume that 25% of the observed variance was measurement error



- Simulation conditions were similar to Deboeck & Bergeman (2013)
  - Time series lengths: 25, 50, or 100
  - Input values were generated from an exponential distribution with rates ranging from .50 to 1.50 in increments of .25
  - Values for β ranged from .10 to .90 in increments of .10
  - Measurement error was set to 10%, 30%, or 50%
  - In total 500 time series were generated for each of the 405 conditions
- Bayesian Reservoir Model: previous equations (slide 9) were implemented to estimate all parameters using R and STAN
- Original SEM Model: previous equations (slide 7) were implemented using the SEM approach (slide 8) via R and OpenMx

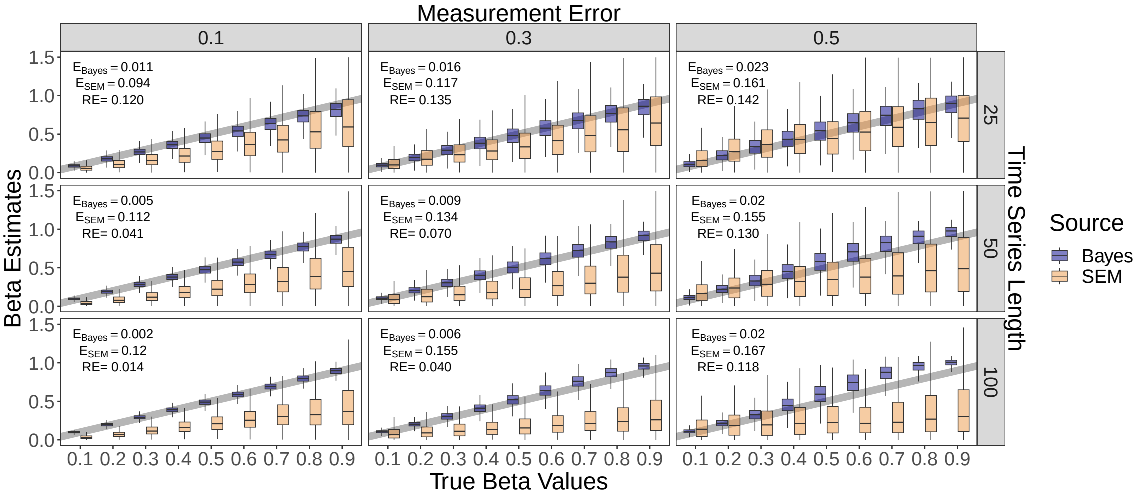
Bayesian Reservoir Model: Simulation 1

Results for β parameter



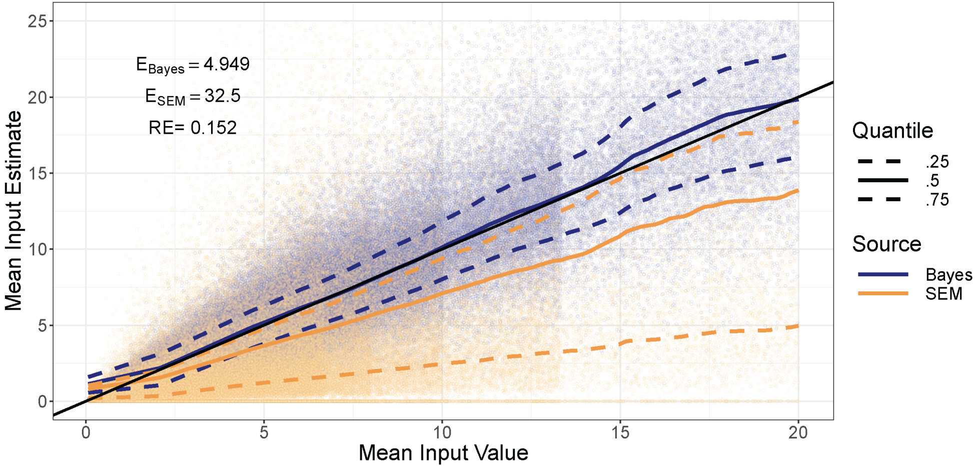
Note: Efficiency = $\frac{\Sigma(\text{Estimates}_k - \text{True}_k)^2}{N}$ for k time series; E_{Bayes} = Bayes Efficiency, E_{SEM} = SEM Efficiency, RE= Relative Efficiency ($\frac{E_{Bayes}}{E_{SEM}}$)

Bayesian Reservoir Model: Simulation 1
Results for β parameter



Simulation 1: Input Results

Results for M_{inputs} parameter



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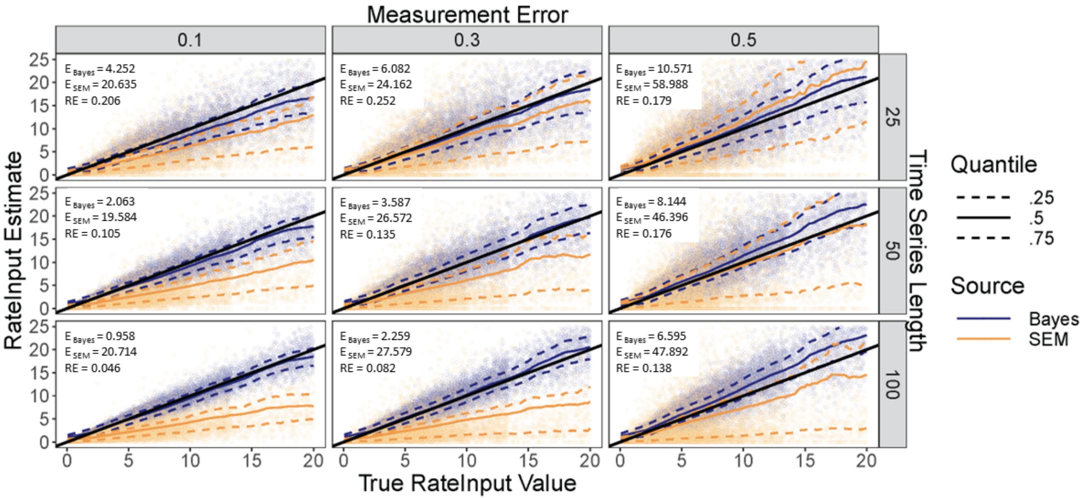
Simulation 1: Input results

Results for M_{inputs} parameter



- Notre Dame Study of Health & Well-being [12]: 775 adults (age 40-91)
  - Rated their perceived stress [13] daily for 56 days
  - Completed one time assessments of: Environmental Mastery [14], Self-esteem [15], Control of Internal States [16], Dispositional Resilience [17], Ego Resilience [18], Social Coping [19], and Social Support from Family and Friends [20]
  - A missing data adapted version of the Bayesian Reservior Model was applied

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[12] Bergeman et al., 2021
[13] Cohen, 1988
[14] Ryff & Keyes, 1995
[15] Rosenberg, 2015
[16] Pallant, 2000
[17] Bartone, Ursano, Wright, & Ingraham, 1989
[18] Block & Kremen, 1996
[19] Carver, Scheier, & Weintraub, 1989
[20] Procidano & Heller, 1983

Table: Correlations between Reservoir Model Estimates and Reserve Capacity Resources

Resources	Stress Dissipation	Stress Input
Age	0.02	-0.16***
Environmental Mastery	0.32***	-0.12**
Self Esteem	0.30***	-0.07
Control of Internal States	0.25***	-0.13**
Dispositional Resilience	0.30***	-0.02
Ego resilience	0.24***	-0.01
Social Coping	0.24***	0.08
Support from Family	0.27***	0.04
Support from Friends	0.24***	0.00

Note: * $p<.01$; ** $p<.001$; *** $p<.0001$

$$
\begin{aligned}
h_{i,t} &= h_{i,t-1} - \beta_i h_{i,t-1} + \epsilon_{i,t} \\
\beta_i &\sim \text{Exp}(\lambda_\beta) \\
\epsilon_{i,t} &\sim \text{Exp}(\lambda_{\epsilon,i}) \\
\lambda_{\epsilon,i} &\sim \text{Exp}(\lambda_\epsilon).
\end{aligned} \tag{4}
$$

Priors:

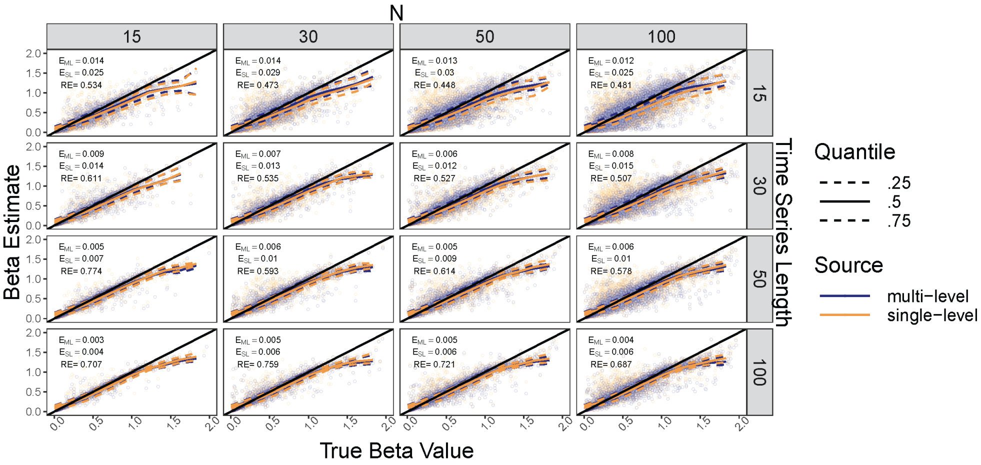
$$
\begin{aligned}
\lambda_\beta &\sim Exp(\lambda_{\beta,prior}) \\
\lambda_\epsilon &\sim Exp(\lambda_{\epsilon,prior}) \\
1/\sigma^2 &\sim Exp(\lambda_{Precision,prior}).
\end{aligned} \tag{5}
$$

Simulation conditions

- Evaluate model under varying multi-level data conditions
  - Time series lengths: 15, 30, 50 or 100
  - Number of participants (N): 15, 30, 50 or 100
  - β values were drawn from a gamma distribution and ranged from 0 to 2
  - Input values were drawn from a gamma distribution (2, 5)
  - Measurement error was set to 15%
- The equations on the previous slide (slide 16) were implemented to estimate all parameters using R and STAN

Multi-level Bayesian Reservoir Model: Simulation 2

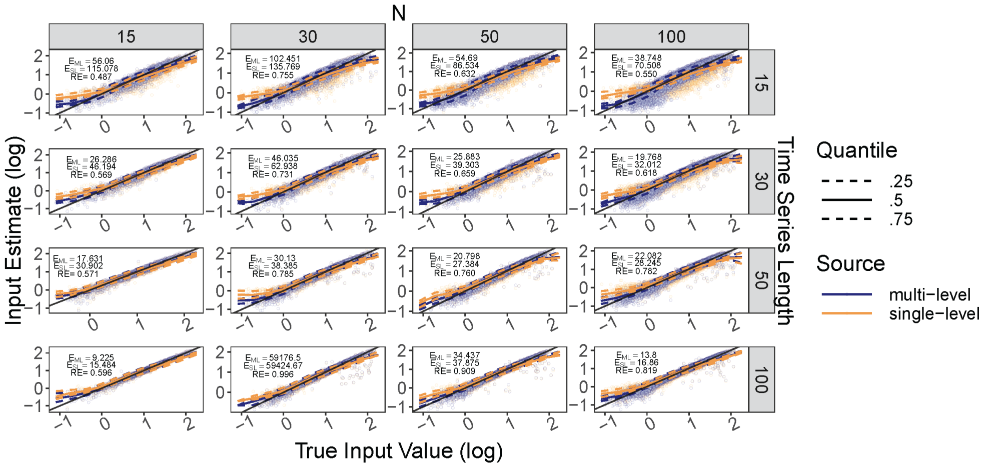
Results for β parameter



Note: Efficiency = $\frac{\Sigma(\text{Estimates}_k - \text{True}_k)^2}{N}$ for k time series; E_{ML} =multi-level efficiency, E_{SL} =single-level efficiency, RE= Relative Efficiency ($\frac{E_{ML}}{E_{SL}}$)

Multi-level Bayesian Reservoir Model: Simulation 2

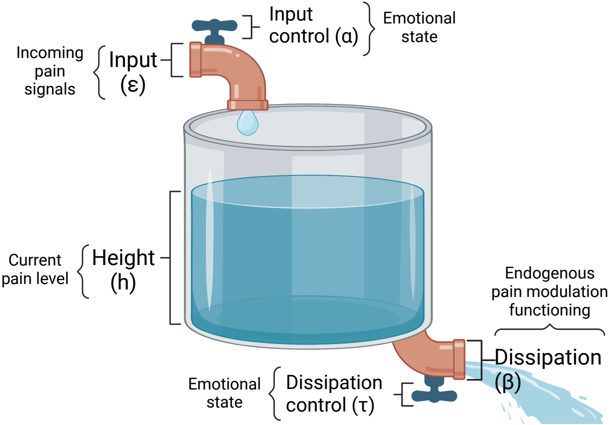
Results for M_{inputs} parameter



Note: Efficiency = $\frac{\Sigma(\text{Estimates}_k - \text{True}_k)^2}{N}$ for k time series; E_{ML} =multi-level efficiency, E_{SL} =single-level efficiency, RE= Relative Efficiency ($\frac{E_{ML}}{E_{SL}}$)

- The current adaptation of the Reservoir Model demonstrates the benefits of leveraging the combined strengths of Bayesian estimation and multi-level modeling to create a model tailored to self-regulatory processes (e.g., stress regulation)
  - Allows for the modeling of unique dynamics
  - Accommodates short time series and smaller samples
  - Aids applied researchers by broadening the models available to study dynamic processes like stress
    - Going beyond trait-level conceptualizations of adaptational processes like stress regulation and resilience
- Here we present a specific formulation of this model
  - Future work could expand or modify this model to reflect our conceptual and theoretical understanding of a variety of processes

For example...



Input
control (α)
Emotional
state
Incoming
pain
signals
Input
(ε)
Current
pain level
Height
(h)
Endogenous
pain modulation
functioning
Dissipation
(β)
Emotional
state
Dissipation
control (τ)

Thank you for your time and attention!

Questions, Comments, & Feedback are welcome!

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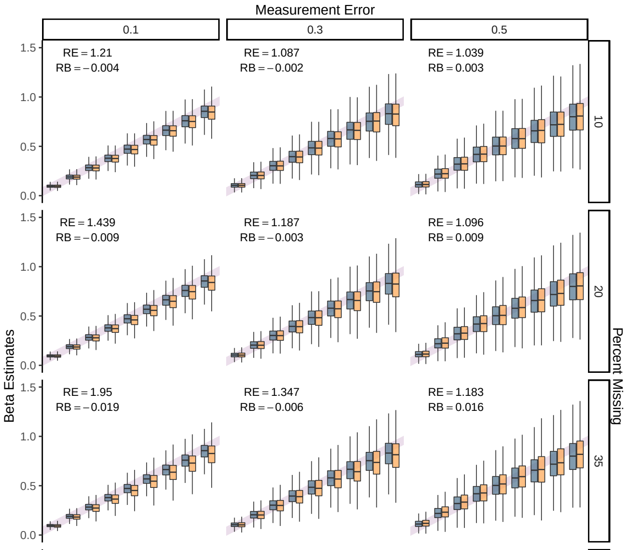
@WhitakerMirinda
model code is available at: osf.io/6qh37/

Graphics created using R and BioRender.com

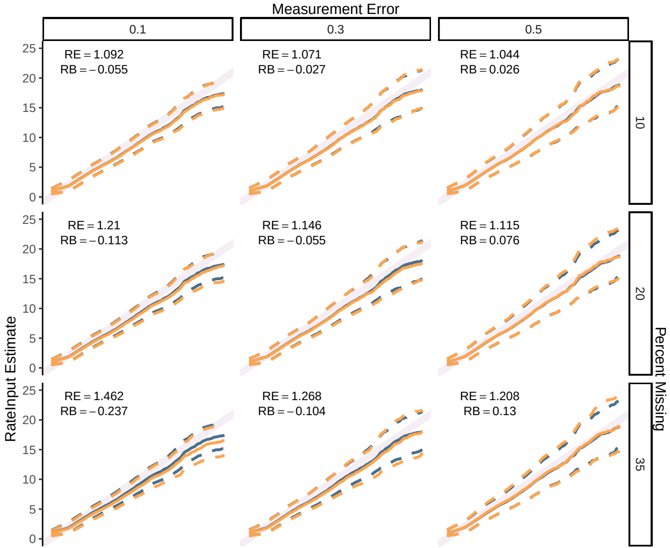
Data Censoring

$$p(x_t|h_t) = \prod_{t:x_t>0} N(x_t|h_t,\sigma) \prod_{t:x_t=0} \Phi\left(\frac{h_t - 0}{\sigma}\right)$$

Missing Data Simulation: β

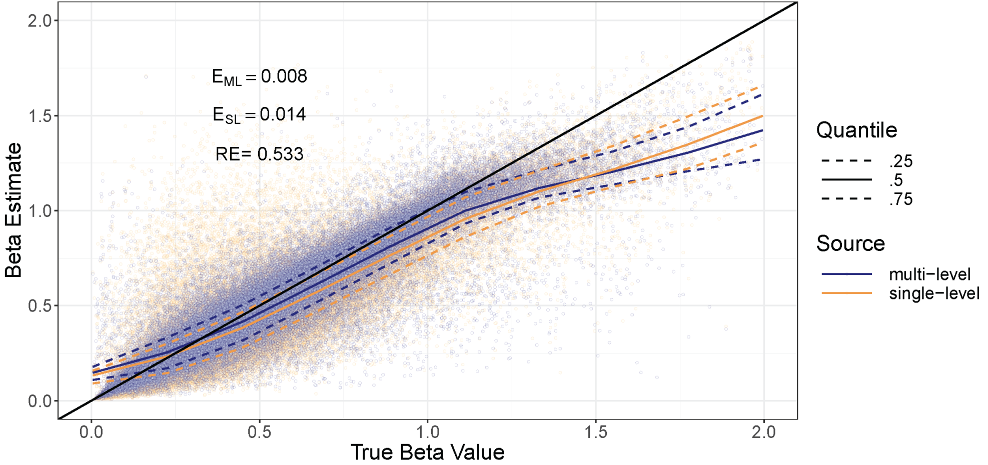


MISSING DATA SIMULATION: M_{inputs}



Multi-level Bayesian Reservoir Model: Simulation 2

results for β parameter



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Multi-level Bayesian Reservoir Model: Simulation 2

results for M_{inputs} parameter